Summary of Hydrodynamics Program

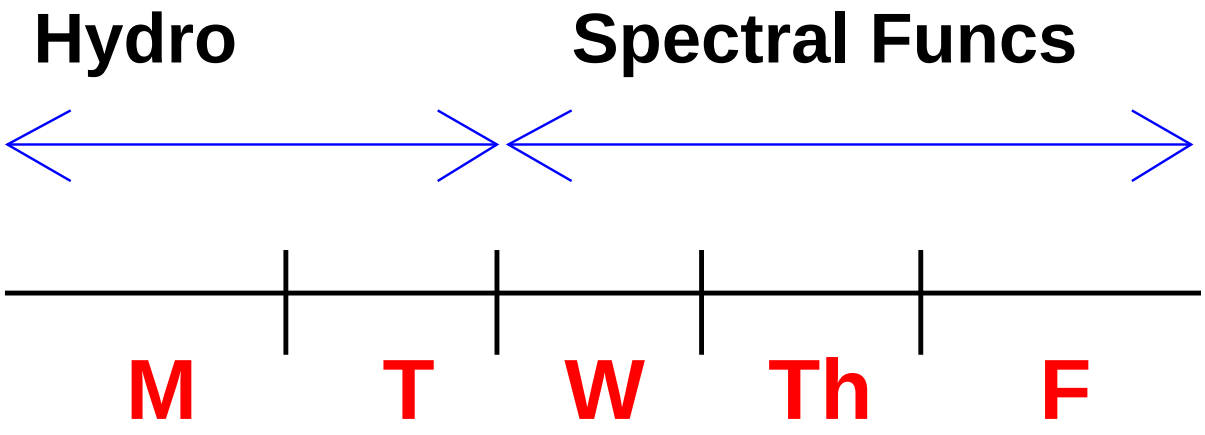
Derek Teaney

SUNY Stonybrook and RBRC Fellow

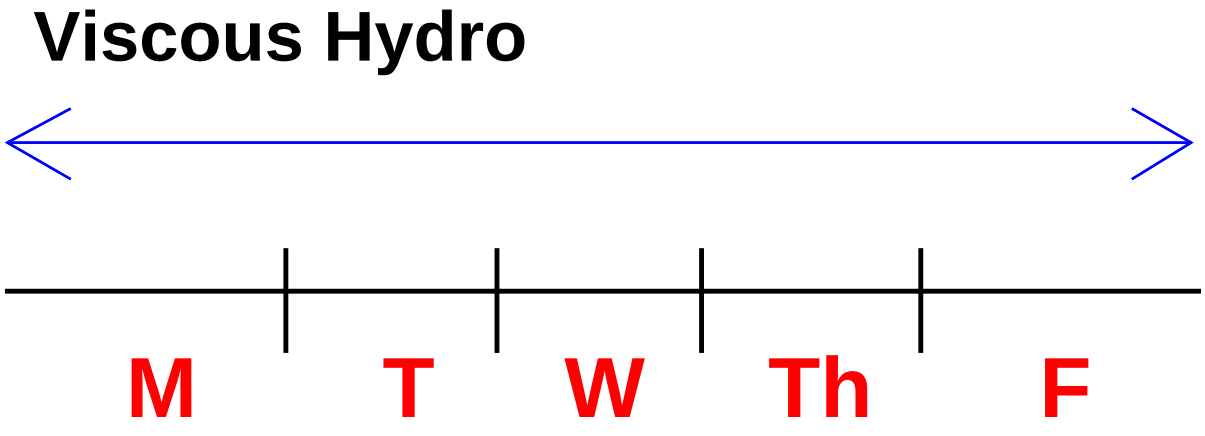
What, Where, When?

1. Two day short workshop on hydro applied to RHIC (RBRC)

2. Three day workshop on spectral functions (RBRC)



3. A week long seminar style program on viscous hydrodynamics (BNL)



Who? (a partial list)

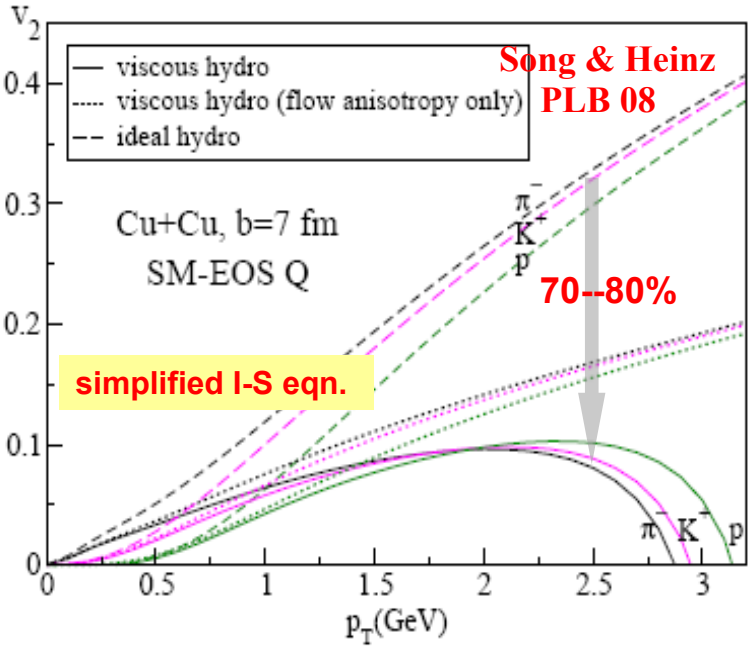
- Viscous Hydrodynamics
  - Paul Romatschke, Huichao Song, Kevin Dusling, Azwindini Muronga
- Experimental Aspects of Flow
  - J.Y. Ollitrault, Paul Sorenson, Peter Steinberg
- Approaching the hydrodynamic limit with Boltzmann equation
  - J.Y. Ollitrault, Pasi Huovinnen, Denes Molnar, Carsten Greiner, Zhe Xu, Andrej El
- Other participants
  - P. Petreczky, Dima Kharzeev, L. McLerran

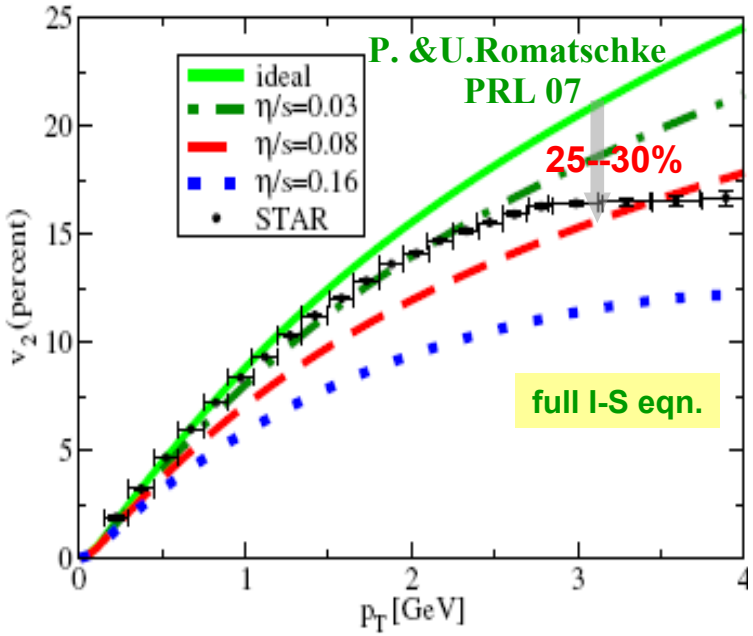
Major Physics Discussions

- Equation of Motion and Comparison of Results
  - Initial confusion by apparent differences.
  - Getting the first order viscous results
  - Second order corrections and convergence to first order
- Experimental Aspects of Flow
  - CuCu system
  - Flow Fluctuations and Non-flow
- Approaching the hydrodynamic limit with Boltzmann equation
- Other issues: Bulk Viscosity, Lattice EOS, Gluaber vs. KLN

Viscous Results are remarkably consistent with each other

Apparent Differences: I





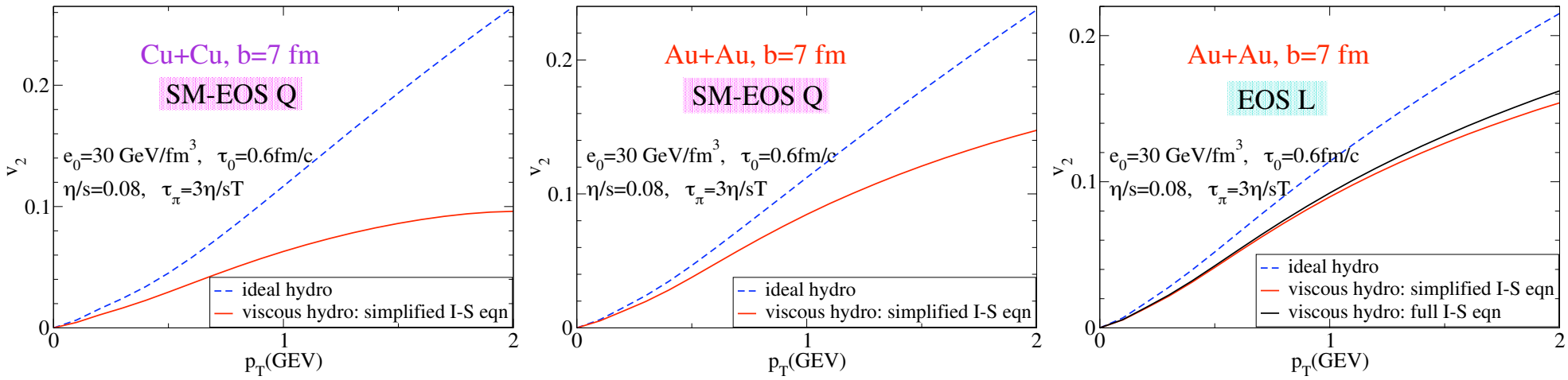
1. Different system size!

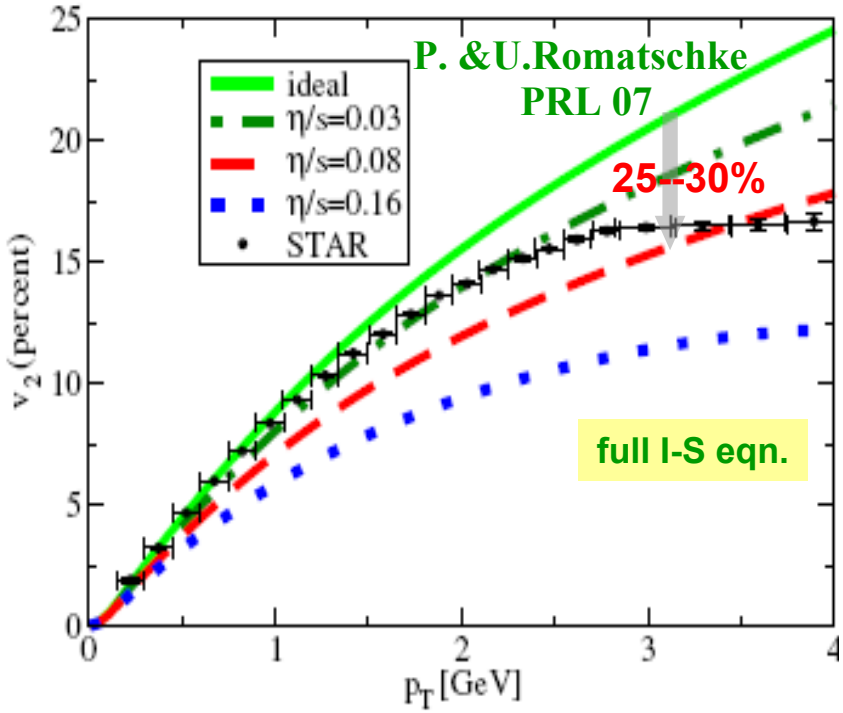
3. Different EOS (relatively minor)

2. Different equations (relatively minor)

The differences explain the difference

Resolution of Differences I: (H. Song and U. Heinz – appears today)





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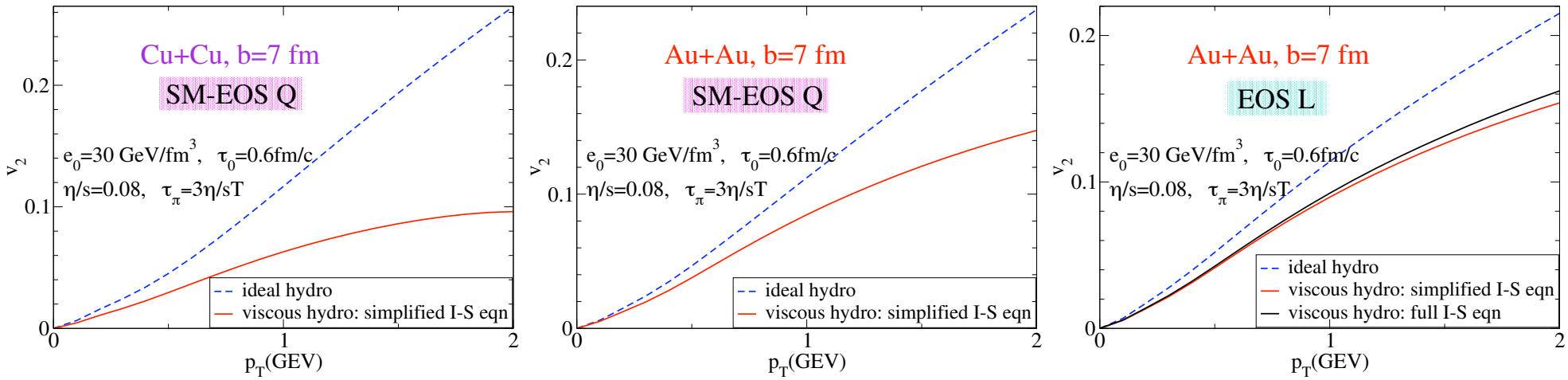
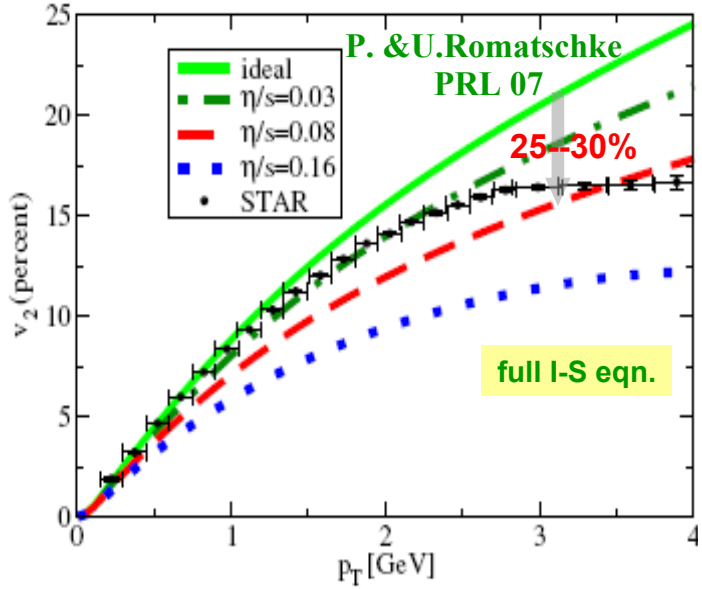
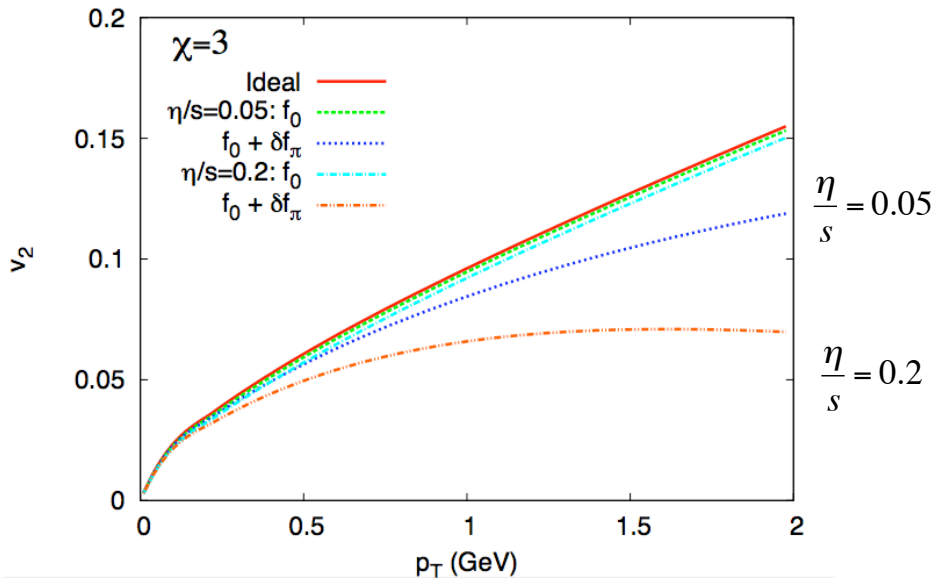






Viscous Equations

$$T^{\mu\nu} = eu^\mu u^\nu + pg^{\mu\nu} + \pi^{\mu\nu}$$

- First order navier stokes theory

$$\pi = \pi^{\mu\nu}_{(1)} \equiv -\eta \left(\nabla^\mu u^\nu + \nabla^\nu u^\mu - \frac{2}{3}\Delta^{\mu\nu}\nabla \cdot u \right)$$

- Second order theory

$$\pi^{\mu\nu} = \overbrace{\pi^{\mu\nu}_{(1)}}^{O(\epsilon)} + \overbrace{\text{second derivatives}}^{O(\epsilon^2)}$$

- Minimal second order theory (Simplified Israel-Stuart)

$$\pi^{\mu\nu} = \pi^{\mu\nu}_{(1)} - \tau_\pi D_t \pi^{\langle\mu\nu\rangle}$$

Important advance: Baier, Romatschke, Son, Starinets Stephanov

- Classified all possible second derivatives.
- For a conformally invariant fluid find a combo which must come together
  - (Full Israel Stuart)

$$\pi^{\mu\nu} = \pi^{\mu\nu}_{(1)} - \tau_\pi D_t(\pi^{\langle\mu\nu\rangle} + \frac{4}{3}\Delta^{\mu\nu}\nabla\cdot u) + \text{Other Derivs}$$

- Three possible forms in flat space which respect conformal invariance.
  - $\Omega_{\mu\nu} =$ Vorticity $= \nabla_\mu u_\nu - \nabla_\nu u_\mu$
  - Other Derivatives

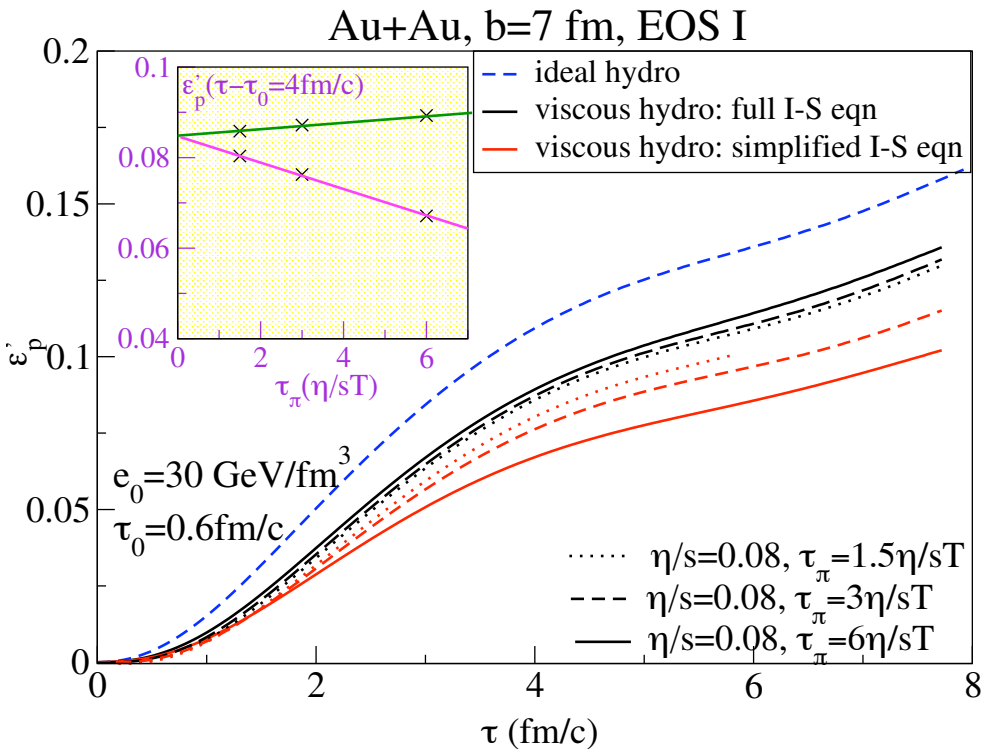
$$\pi^{\alpha\langle\mu}\pi^{\nu\rangle}_{\alpha} \qquad \pi^{\alpha\langle\mu}\Omega^{\nu\rangle}_{\alpha} \qquad \Omega^{\alpha\langle\mu}\Omega^{\nu\rangle}_{\alpha}$$

- Non-conformal terms – 11 total, for example $\pi_{\mu\nu}\nabla\cdot u$

Why this is important. (Huichao & Romatschke)

$$\epsilon_p = \frac{\langle T^{xx} - T^{yy} \rangle}{\langle T^{xx} + T^{yy} \rangle}$$

- $p = \frac{1}{3} e$ Equation of State

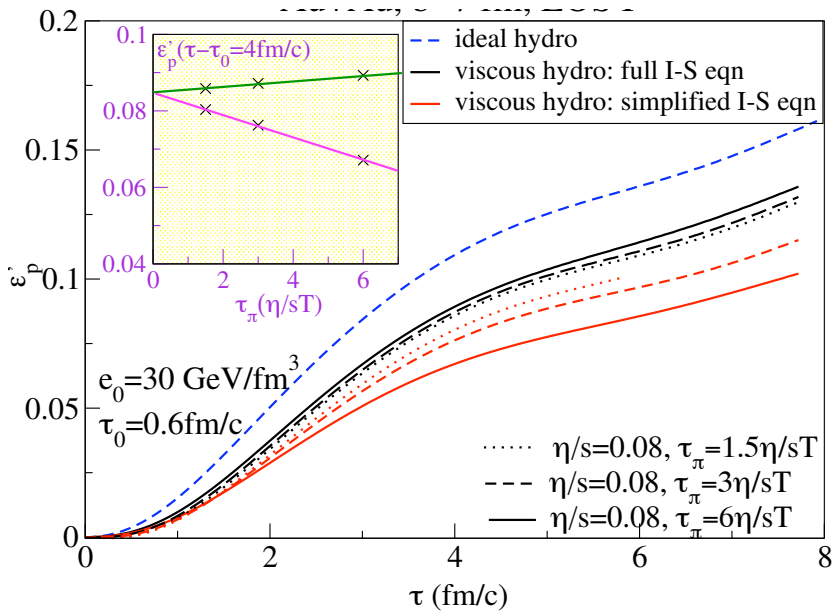


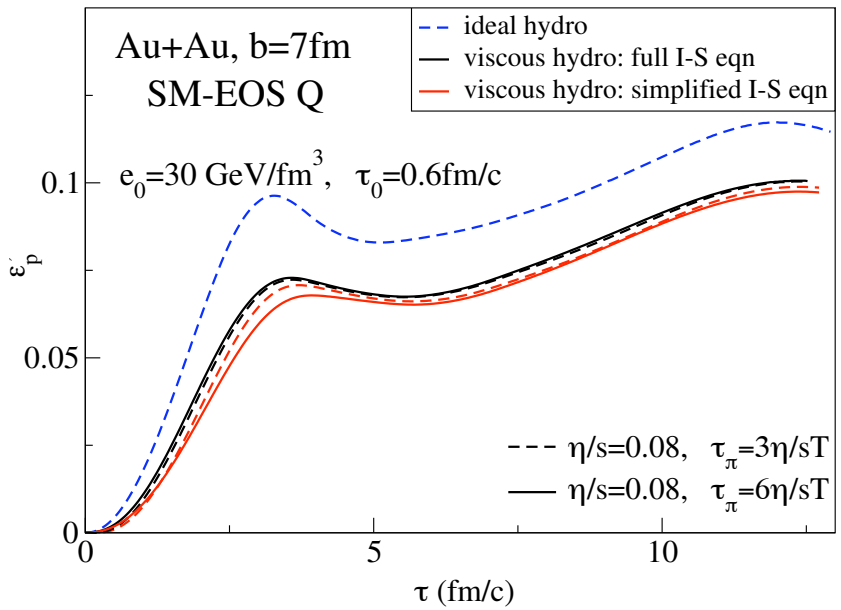
Rapidly converges to the first order theory with conformal terms

Why this is important. (Huichao)

$$\epsilon_p = \frac{\langle T^{xx} - T^{yy} \rangle}{\langle T^{xx} + T^{yy} \rangle}$$

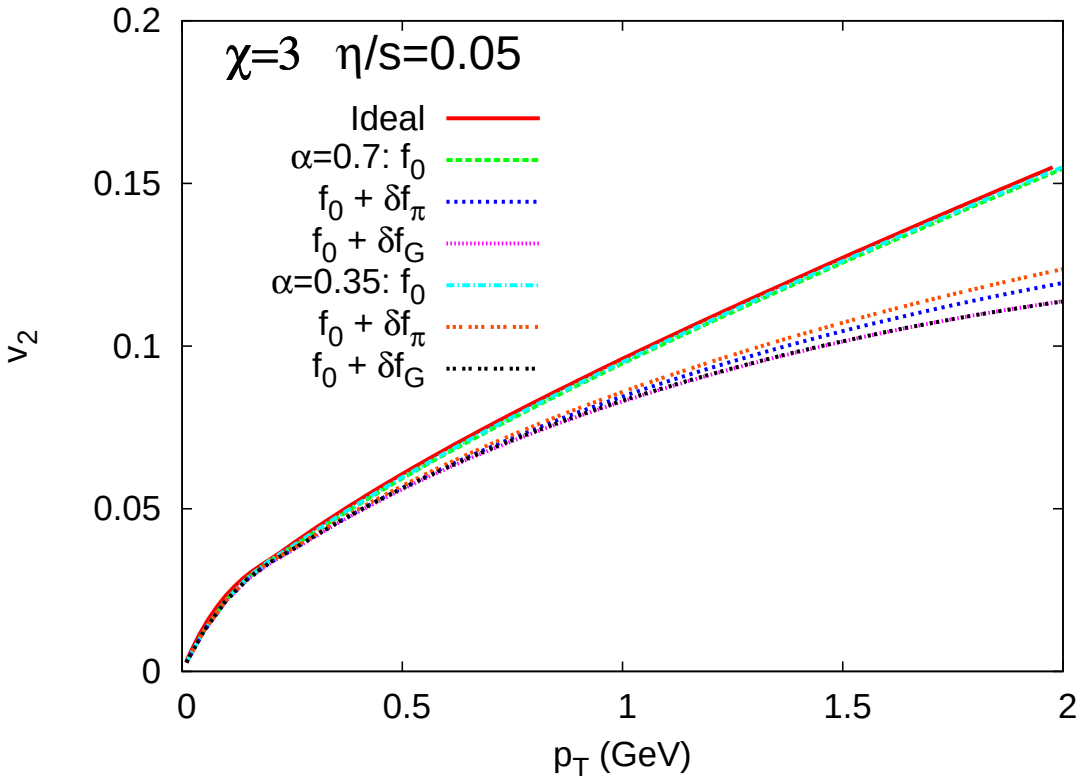
- $p = \frac{1}{3} e$ Equation of State & Lattice type EOS





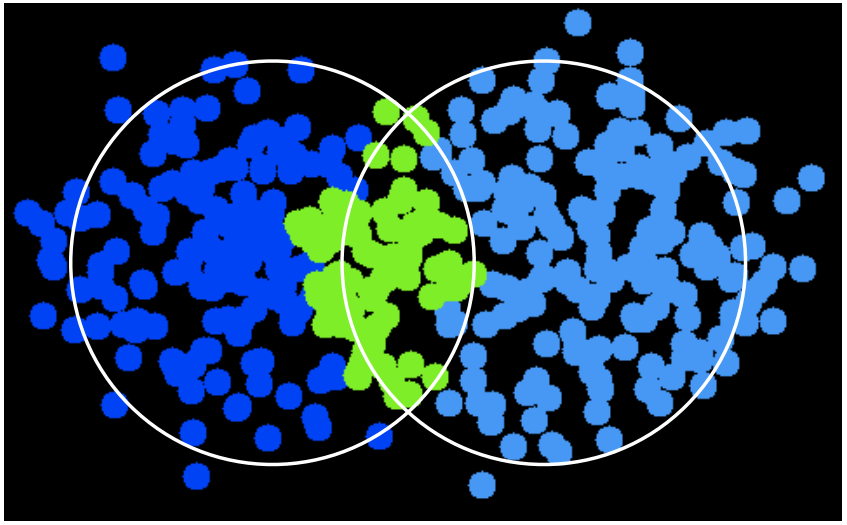
Rapidly converges to the first order theory with conformal terms

Dependence on short time

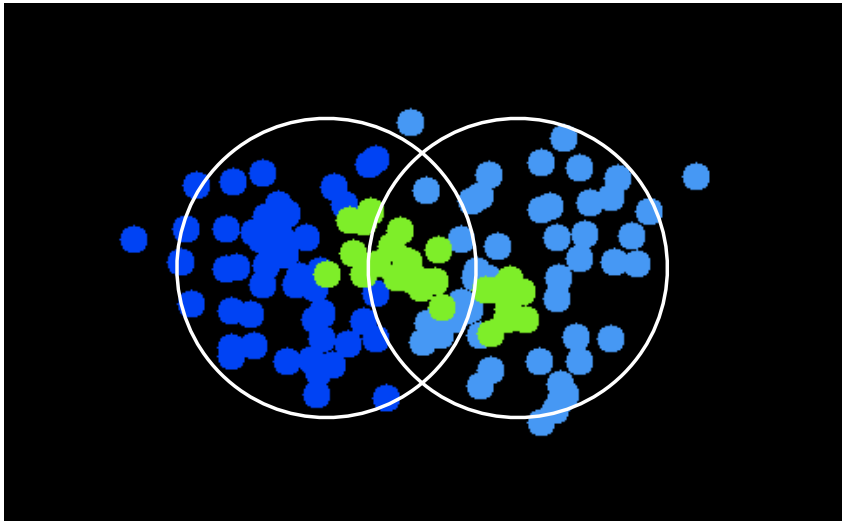


Experimental Aspects of Flow and Flow Fluctuations: (Picture by P. Steinberg)

Au + Au



Cu + Cu



- We want to use the smaller CuCu system to analyze elliptic flow
  - J.Y. Ollitrault, P. Romatschke, DT, K. Dusling, Pasi Huovinnen, Huichao Song, Zhe Xu
- Need to understand the systematics of fluctuations
  - J.Y. Ollitrault, Paul Sorenson, Peter Steinberg

The CuCu data and AuAu larger impact parameters are under good control

Flow Fluctuations. (Ollitrault, Steinberg, Voloshin, Sorenson)

- Flow from two particle cummulants. Pairs of particles

$$\begin{aligned} v_2\{2\}^2 &= \langle \cos 2\left(\phi_1 - \phi_2\right)\rangle \\ &\simeq \left\langle v_2^2 \right\rangle + \sigma_v^2 + \delta_2 \end{aligned}$$

- Flow from four particle cummulants

$$v_2\{4\}^4 = 2\left\langle \cos 2\left(\phi_i - \phi_2\right)\right\rangle - \left\langle \cos\left(\phi_1 + \phi_2 - \phi_3 - \phi_4\right)\right\rangle$$

  - Have the approximate relationship

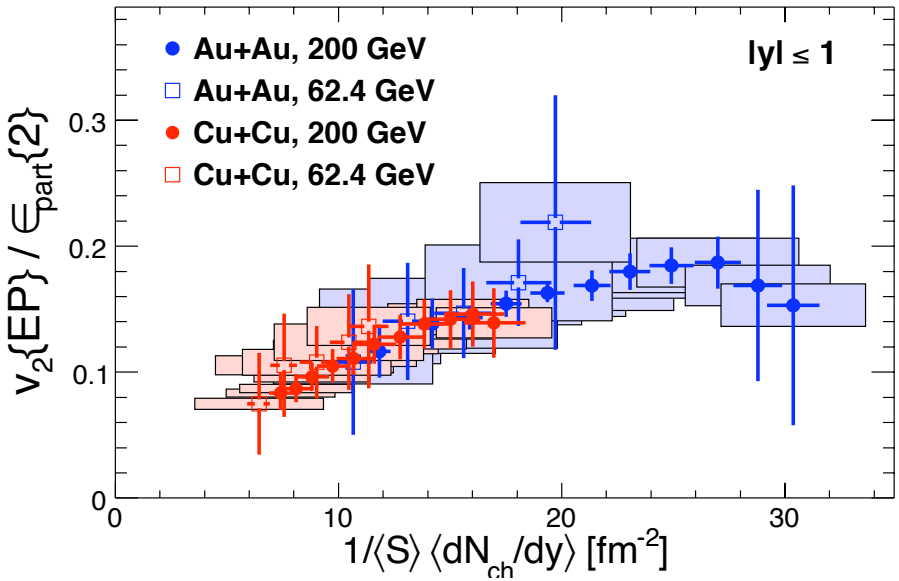
$$v_2\{4\}^2 = \sqrt{\langle v_2\rangle^4 - 2\sigma_v \langle v_2\rangle^2 + \sigma_v^4} \simeq \langle v_2\rangle^2 - \sigma_v^2$$

Can only measure flow fluctuations and nonflow together

$$\sigma^2_{\rm dyn} = v_2\{2\}^2 - v_2\{4\}^2 = \overbrace{2\sigma_v^2}^{\text{flucts}} + \overbrace{\delta}^{\text{non-flow}}$$

- The differences in $v_2\{2\}$ and $v_2\{4\}$ are from flow fluctuations (mostly)
- Can study smaller systems and make sense of the results

Want to make the best “Voloshin” plot ever



- What to divide by? Pre-Phobos answer

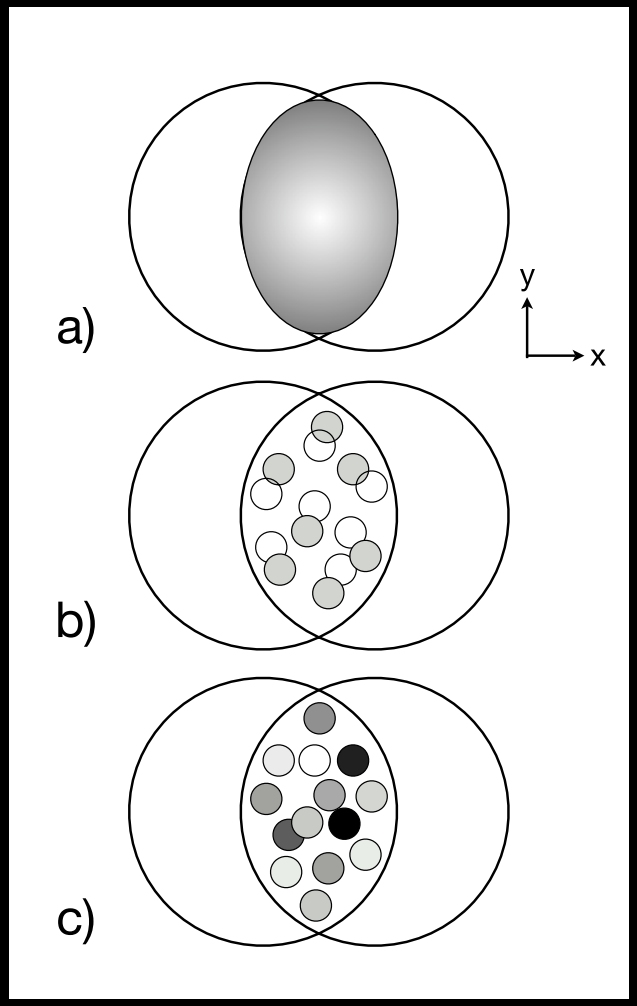
$$\frac{v_2\{2\}}{\epsilon\{2\}} \qquad \frac{v_4\{4\}}{\epsilon\{4\}}$$

Other?

- For $v_2\{EP\}$ the appropriate ϵ depends on the reaction-plane resolution

Next three slides from P. Steinberg describing PHOBOS results

Correlations in A+A



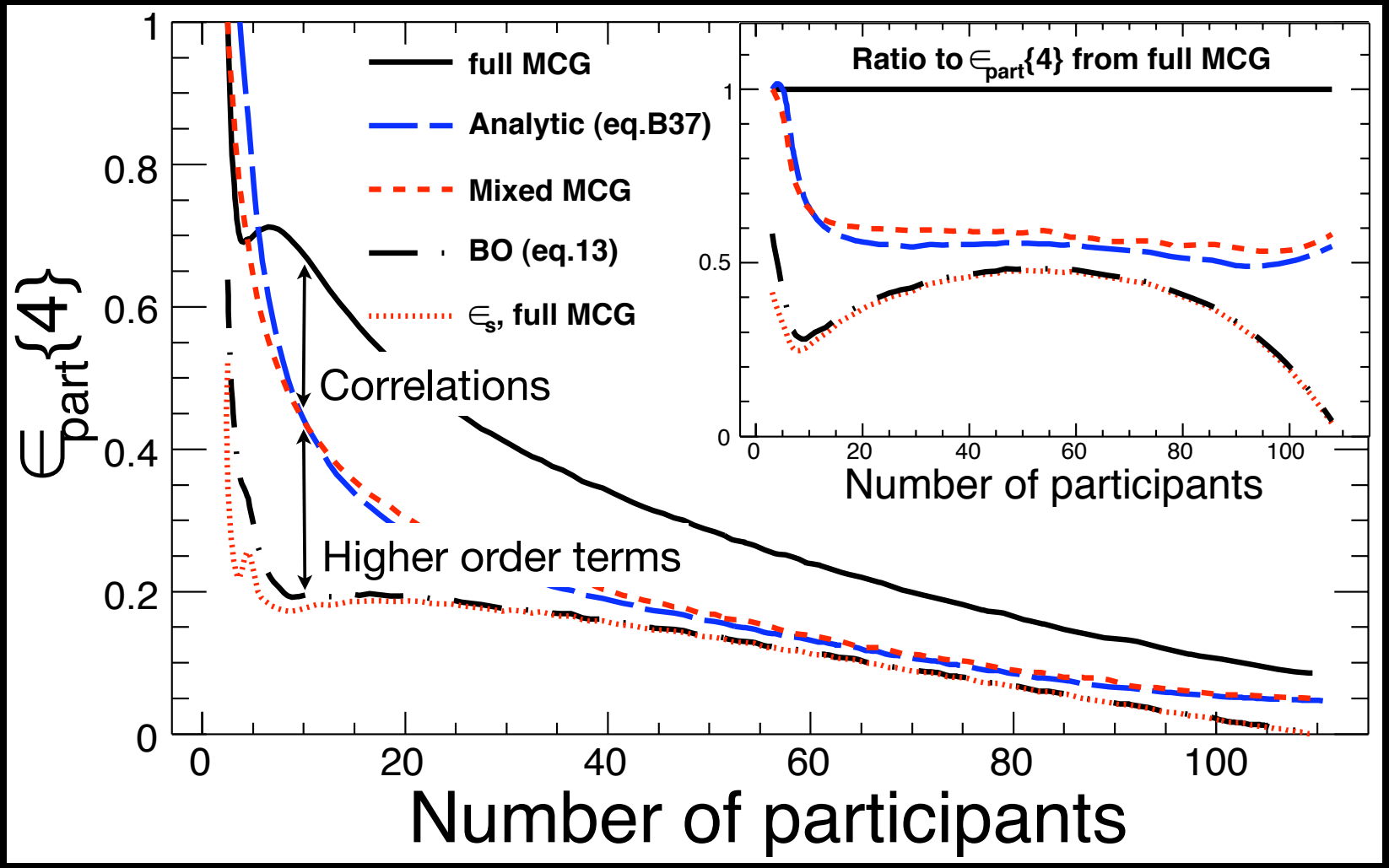
smooth densities
(leading to “standard” eccentricity)

standard Glauber MC
(nucleons collide in <u>pairs</u>,
fluctuations & <u>correlations</u>)

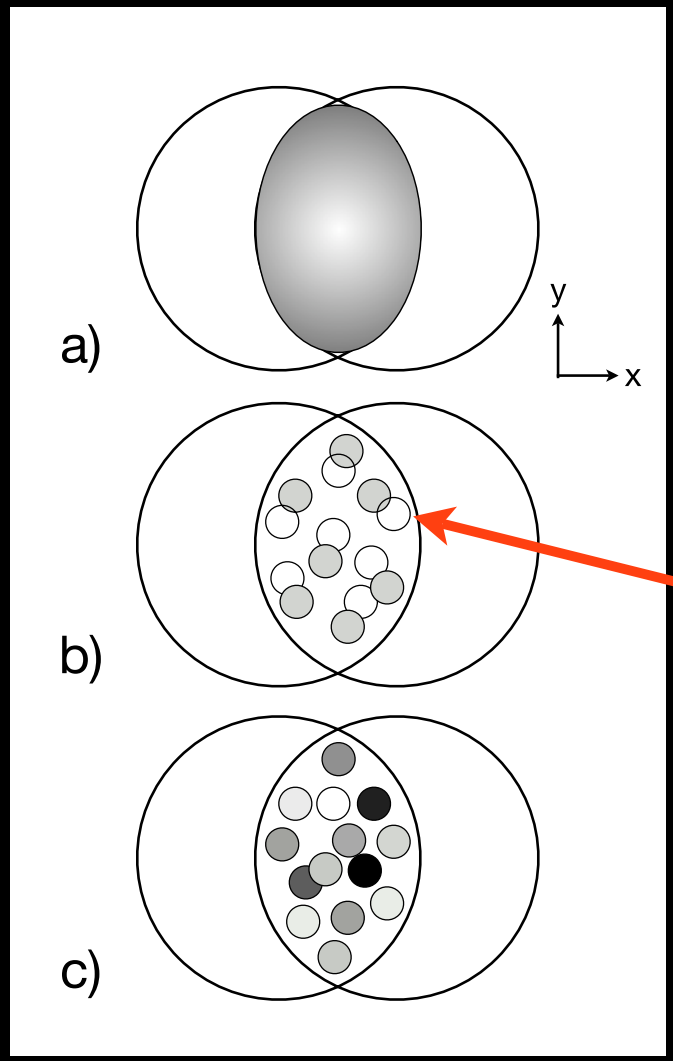
“mixed” Glauber MC
(sample nucleons from
different collisions,
fluctuations & ~~correlations~~)

NB: no correlations <u>between</u> nucleons in a nucleus

A Closer Look at Cu+Cu



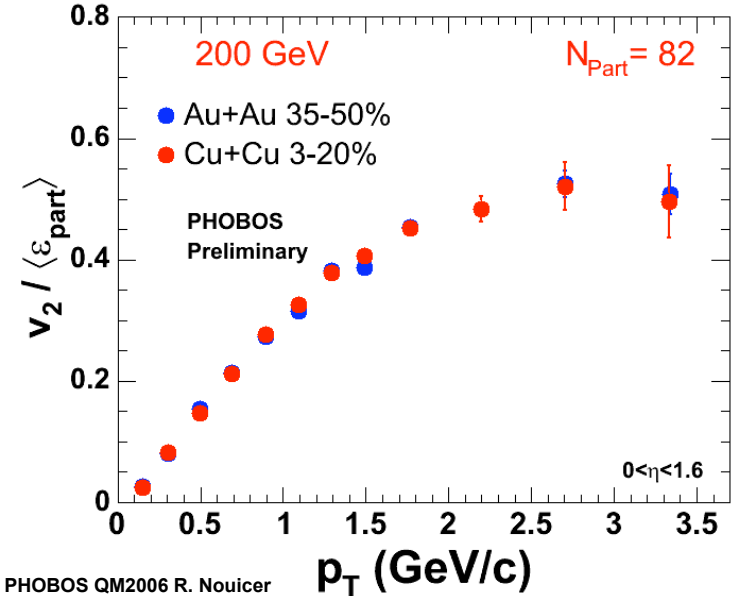
The Difference



The pairwise collisions of nucleons as the nuclei collide induce genuine **spatial correlations** which enhance the fluctuations

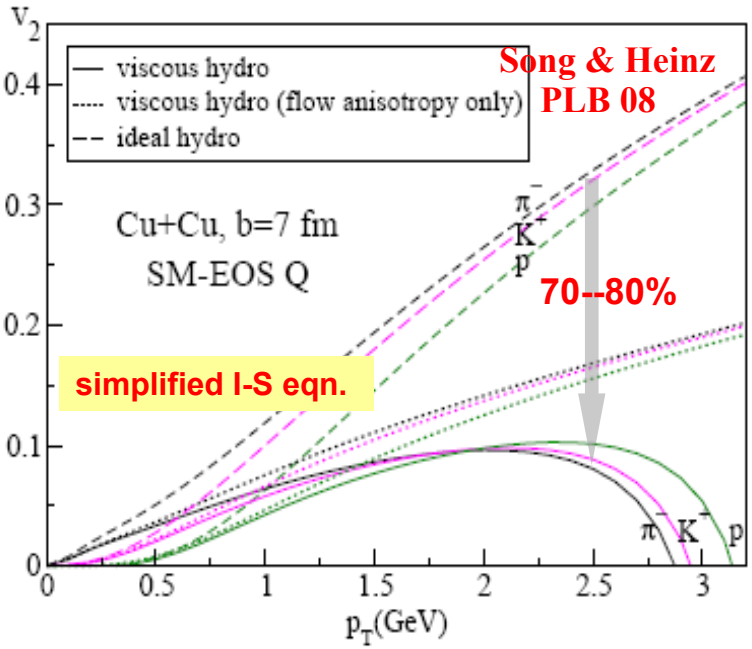
Example of potential physics

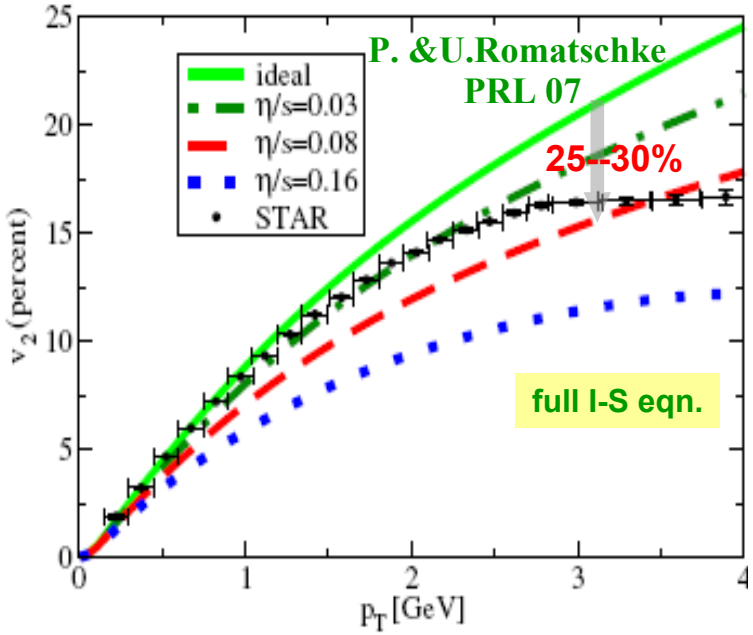
- Compare central CuCu to peripheral AuAu. Phobos



Want to see many more data like this

Potential Comparison with Viscous Hydro



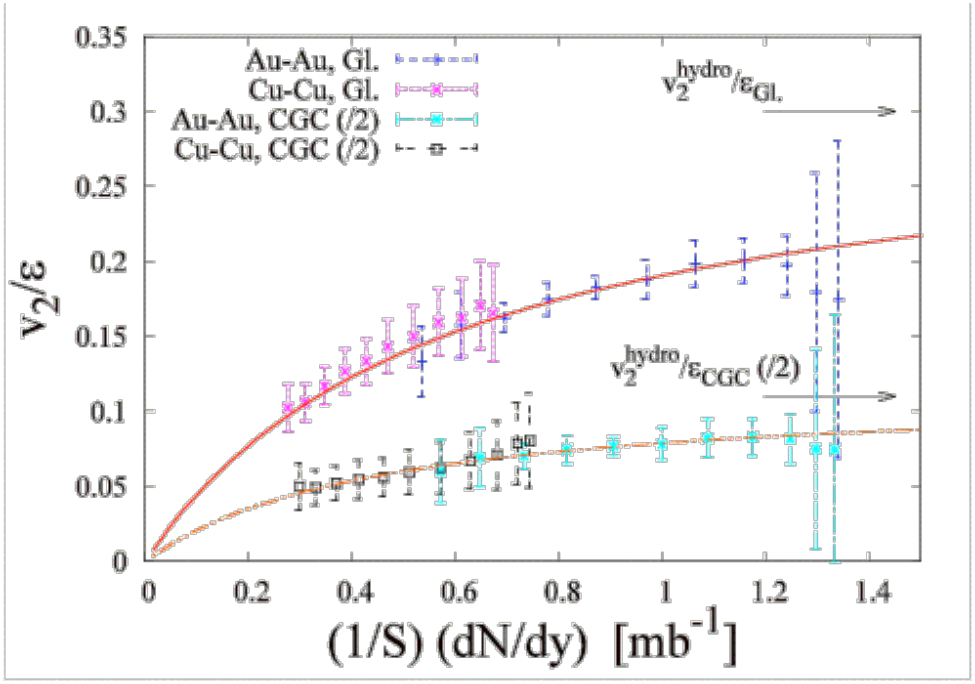


Physics with CuCu (Ollitrault et al)

- Interpolating ansatz between the low density and hydro limits

$$v_2 = \frac{v_2^{\mathrm{idealhydro}}}{1 + K/K_0}$$

- Two parameter fit of elliptic flow results



Ollitrault finds a $K/K_0 = 30\%$ reduction from ideal hydro

Questions?

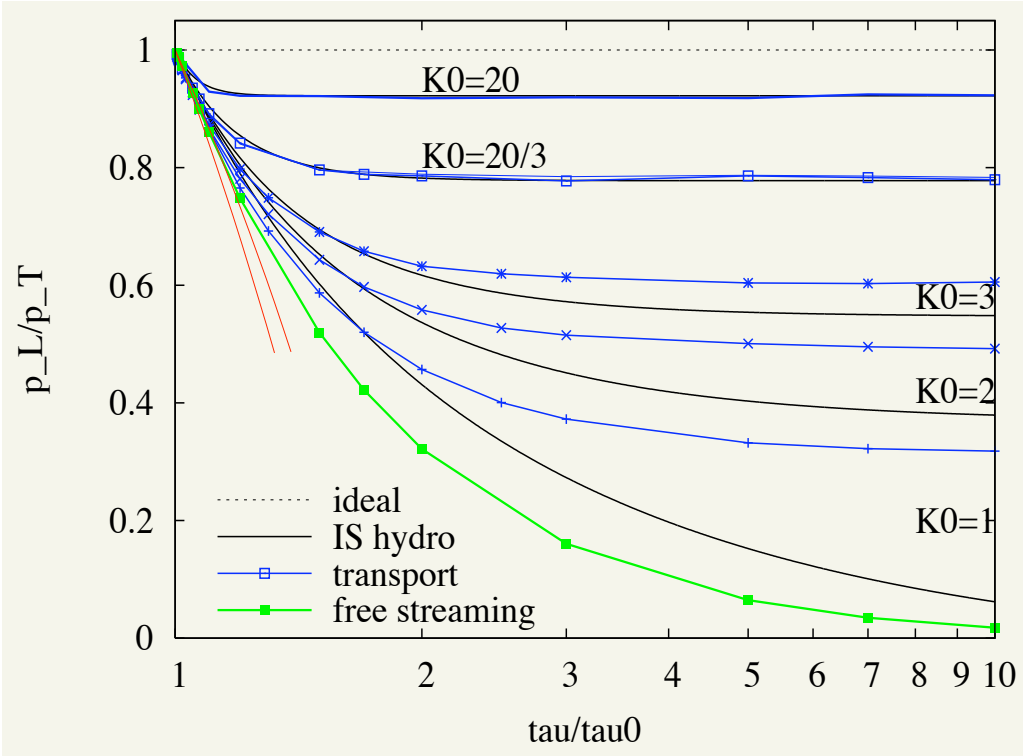
- Can we understand the 30% quantitatively with viscous hydro?
- Beyond the ansatz can we compare the Boltzmann equation to the CuCu data? To hydro?

Comparing viscous hydro to Boltzmann equation. P. Huovinnen and D. Molnar

- Basic Prediction of viscous hydro

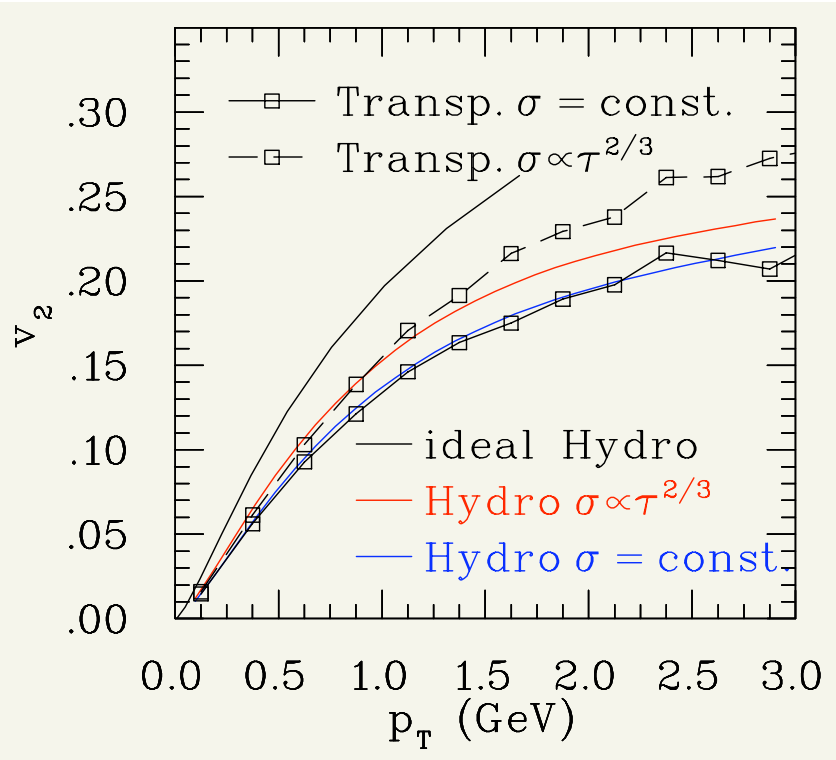
$$T^{zz} = p - \frac{4}{3}\frac{\eta}{\tau} \qquad T^{xx} = p + \frac{2}{3}\frac{\eta}{\tau}$$

- Need the Boltzmann equation and IS relax to this form



Navier Stokes sets in after $(\Delta\tau)/\tau_0 \gtrsim 2/K$

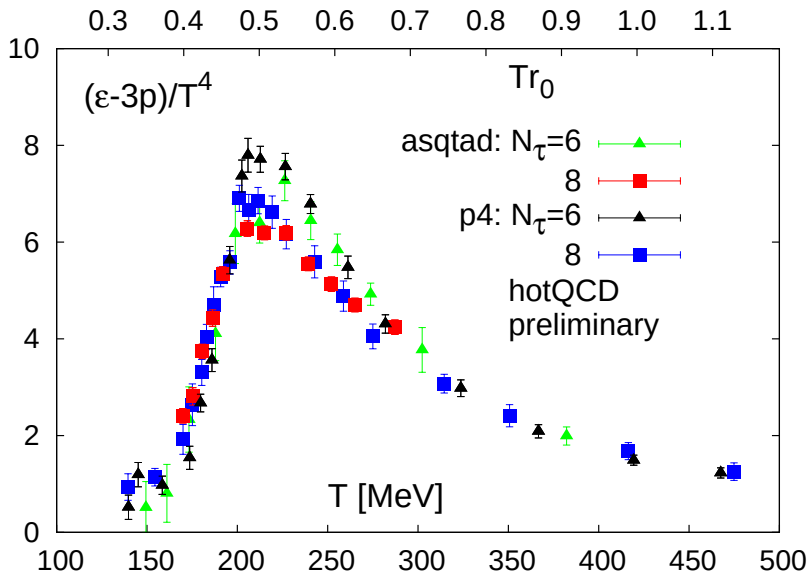
More Detailed Comparisons in Progress

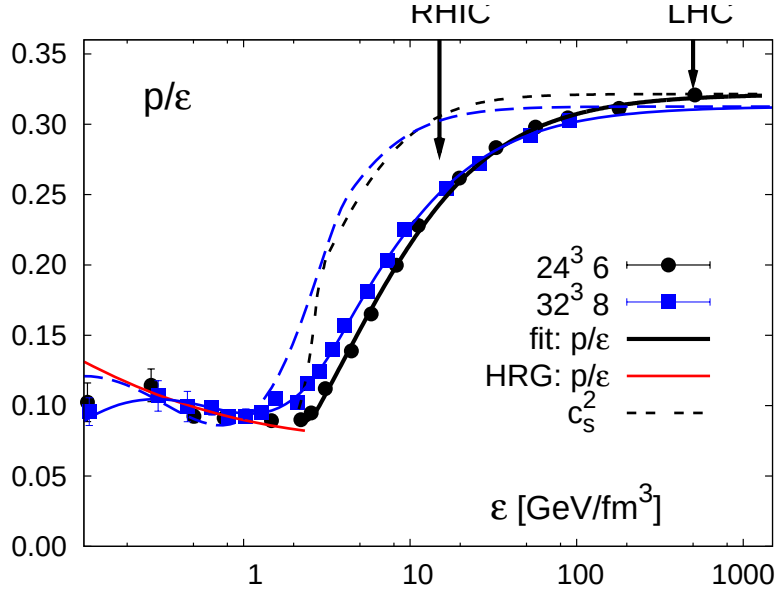


See Pasi's Talk

Putting More QCD into Hydro Simulations

- Lattice EOS: (Used by Romatschke& Romatschke and Heinz& Song)



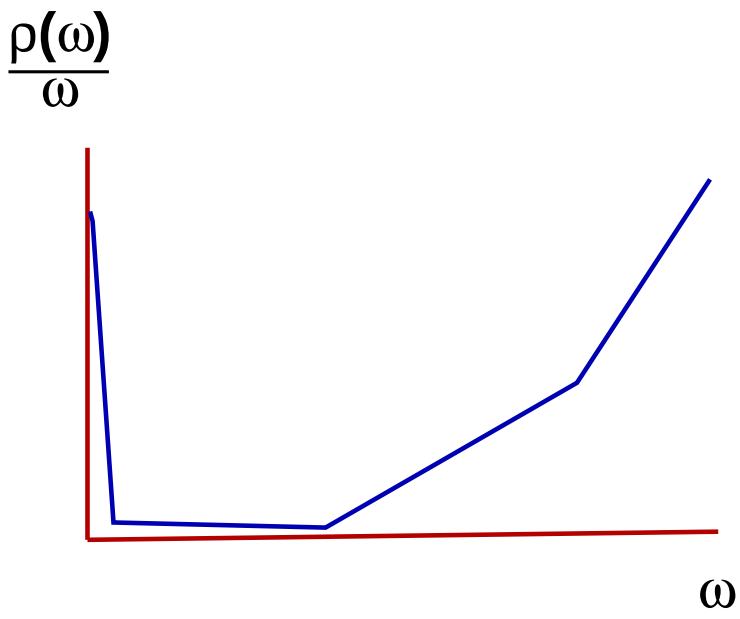


  - RBC-Bielefeld, HotQCD, MILC, Wuppertal – all agree with controllable errors
  - Quite different from the old “Bag Model” equations of state used in the past.
  - The theory is not conformal for the relevant temperatures

Bulk Viscosity. (Dima Kharzeev, Kay Hubener, Ranier Fries)

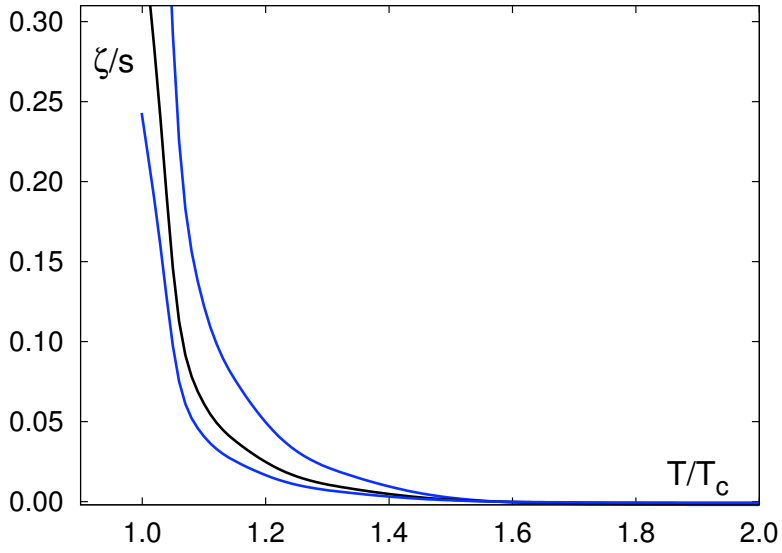
- There is good lattice evidence that the Bulk Viscosity gets large near T_c.

  (Based on discussion with F. Karch)



- This is the expectation of critical slowing down (Guy Moore)

- Extract a Bulk Viscosity



More things to study with hydro!

Conclusions

1. Viscous hydrodynamic codes seem very consistent with each other.
   - They are being compared to the data. Need

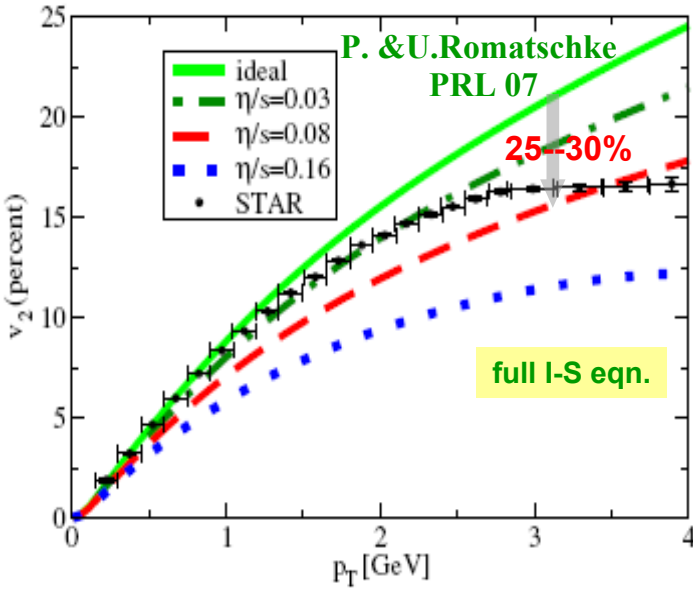
$$\frac{\eta}{s} \lesssim 0.5$$

   to run meaningfully

2. Ongoing analysis of small systems and larger impact parameters
   - Impressive experimental work. More needed.
   - Breakdown of hydro is very important

$$\frac{\eta}{s} \gtrsim ???$$

3. Need to place flow results in a larger context – e-loss



How is this $R_{AA}(\phi)$??

Thanks to all