Effect of Eccentricity Fluctuations and Nonflow on Elliptic Flow Methods

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TECHQM 15 Dec 08

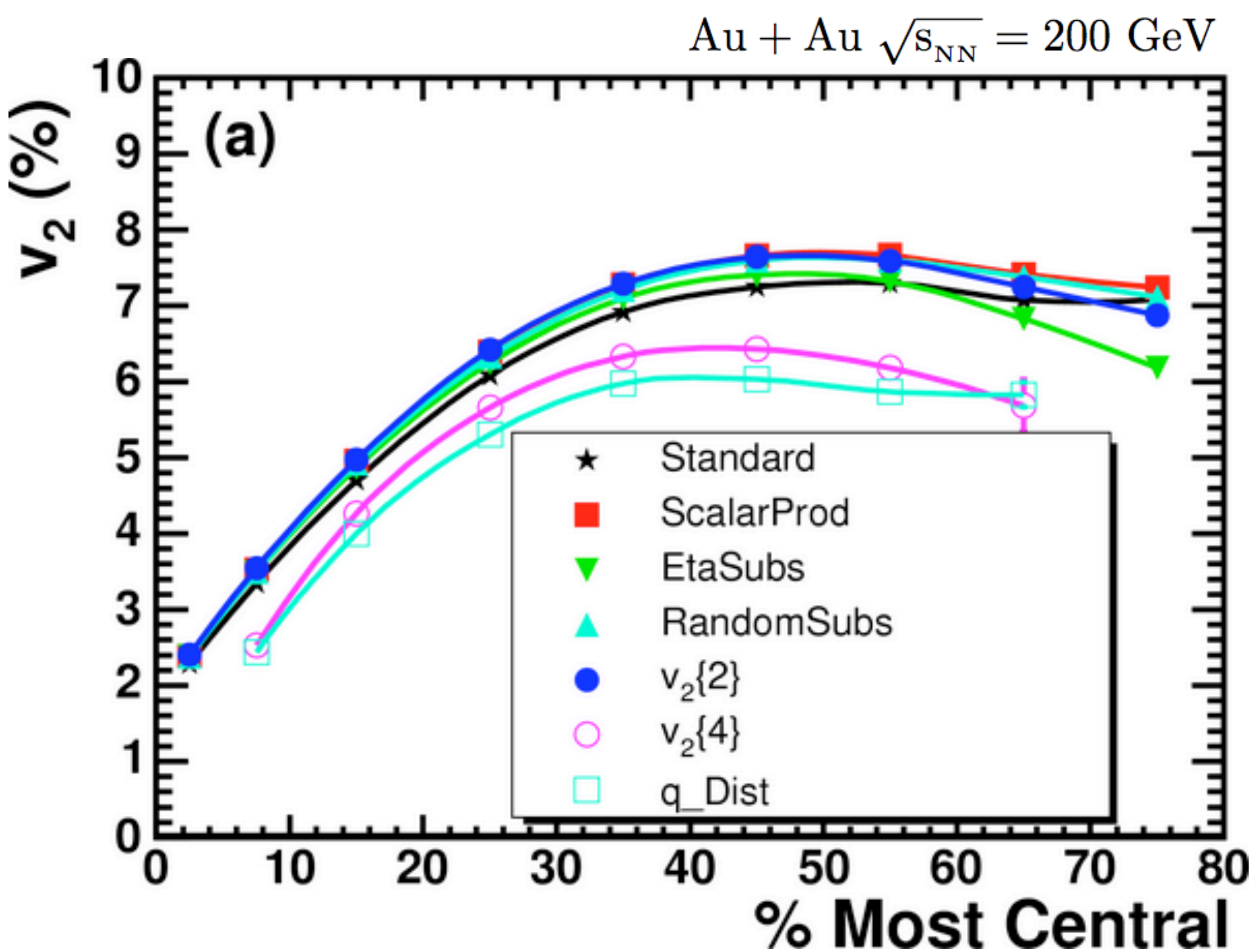
Methods

- "Two-particle":
  - v_2\{subEP\}: each particle with the EP of the other subevent
  - v_2\{EP\} "standard": each particle with the EP of all the others
  - v_2\{SP\}: same, weighted with the length of the Q vector
  - v_2\{2\}: each particle with every other particle

- Many-particle:
  - v_2\{4\}: 4-particle - 2 * (2-particle)^2
  - v_2\{q\}: distribution of the length of the Q vector
  - v_2\{LYZ\}: Lee-Yang Zeros multi-particle correlation

review of azimuthal anisotropy:
arXiv: 0809.2949

Integrated v_2



STAR, J. Adams et al., PRC **72**, 014904 (2005)

Methods Comparison (2005)

To expand scale, plot ratio to Standard Method:

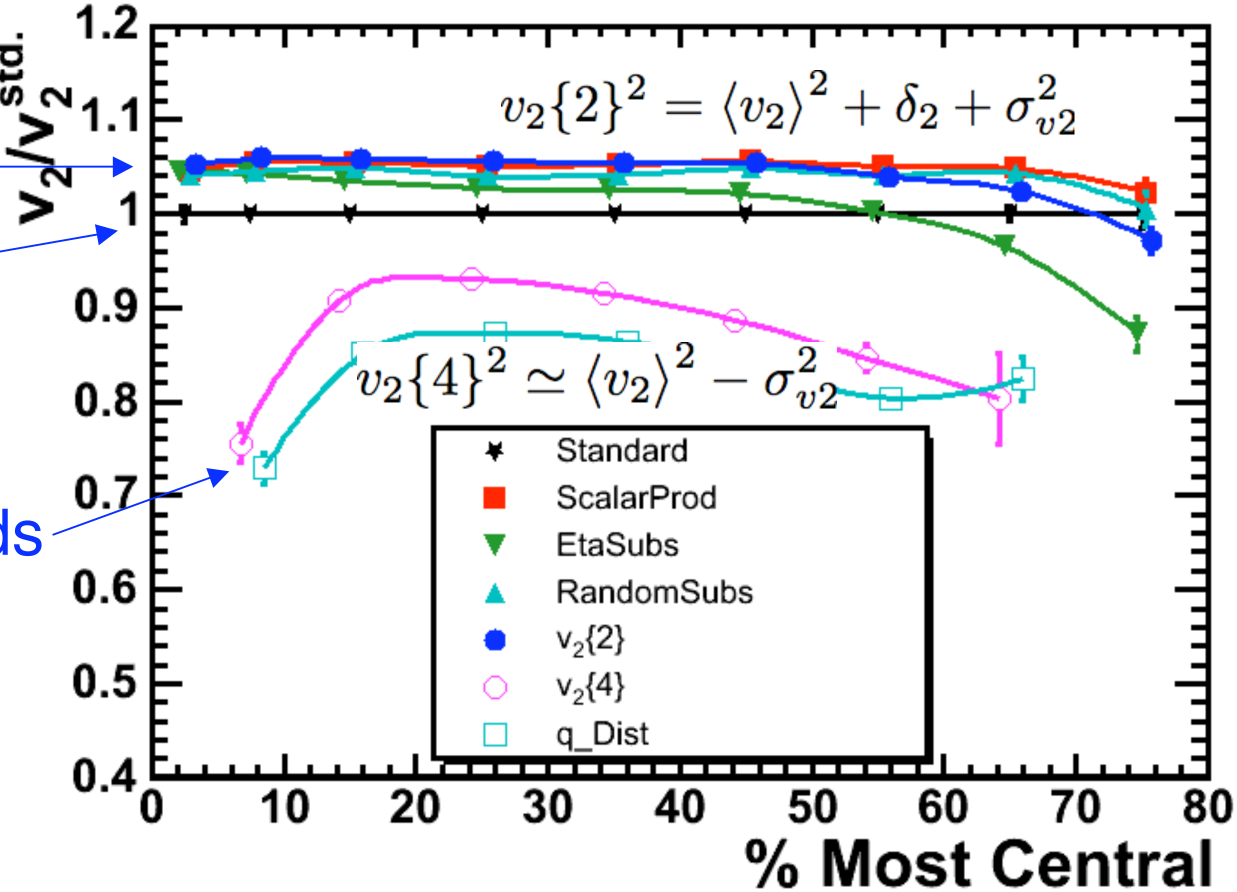
2-part. methods

std. method

5% low!

multi-part. methods

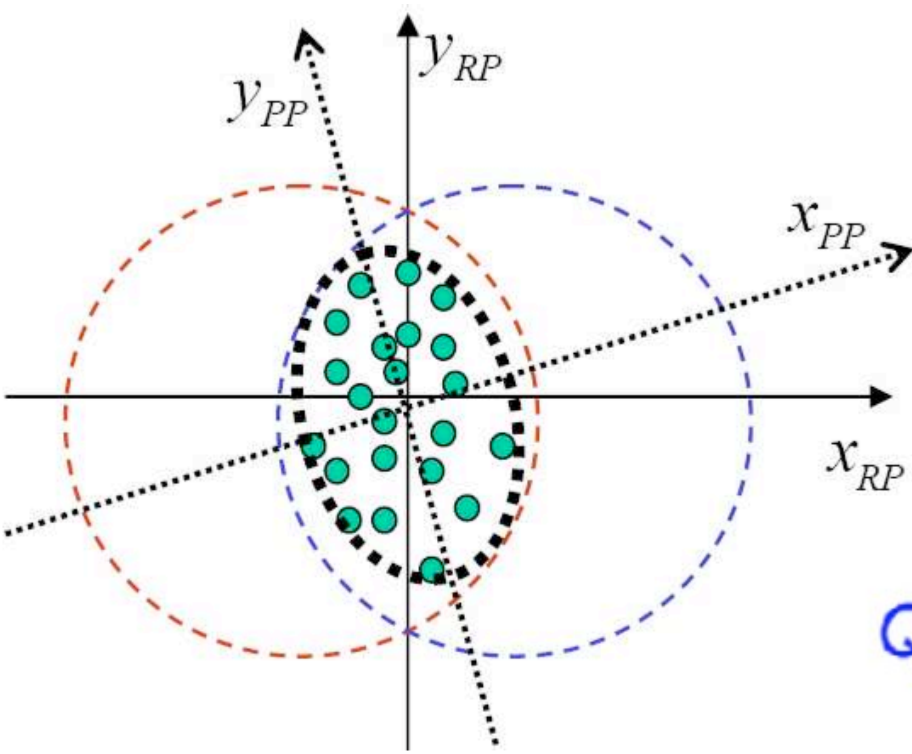
systematic error:
spread of methods



Because of nonflow and fluctuations the true v_2 lies between the lower band and the mean of the two bands.

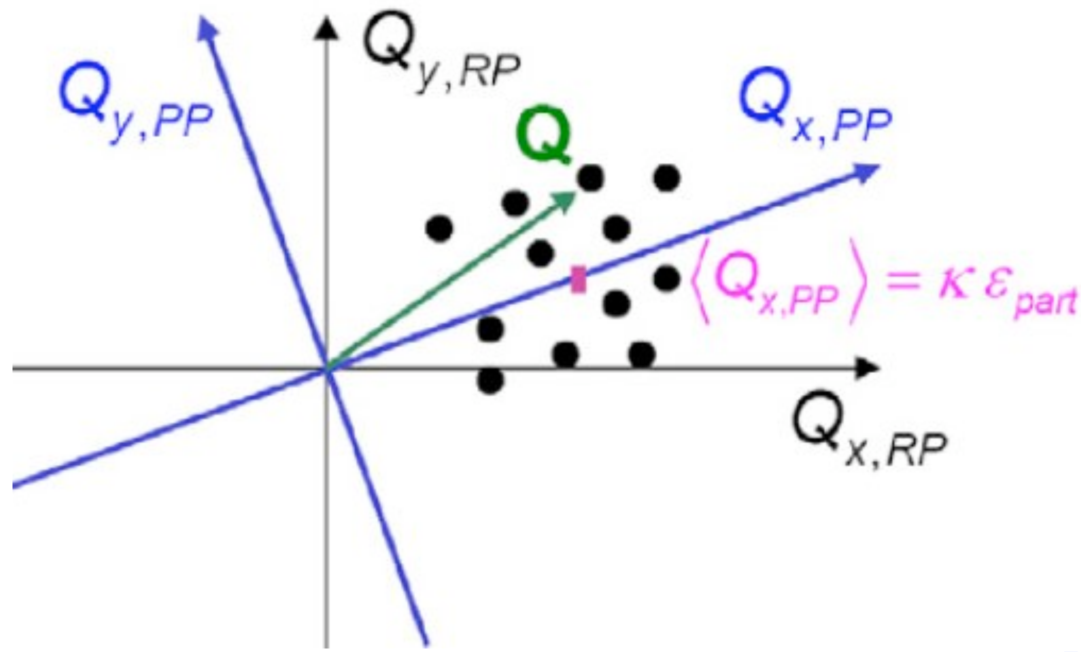
STAR, J. Adams et al., PRC **72**, 014904 (2005)

Reaction, Participant, and Event Planes



coordinate space

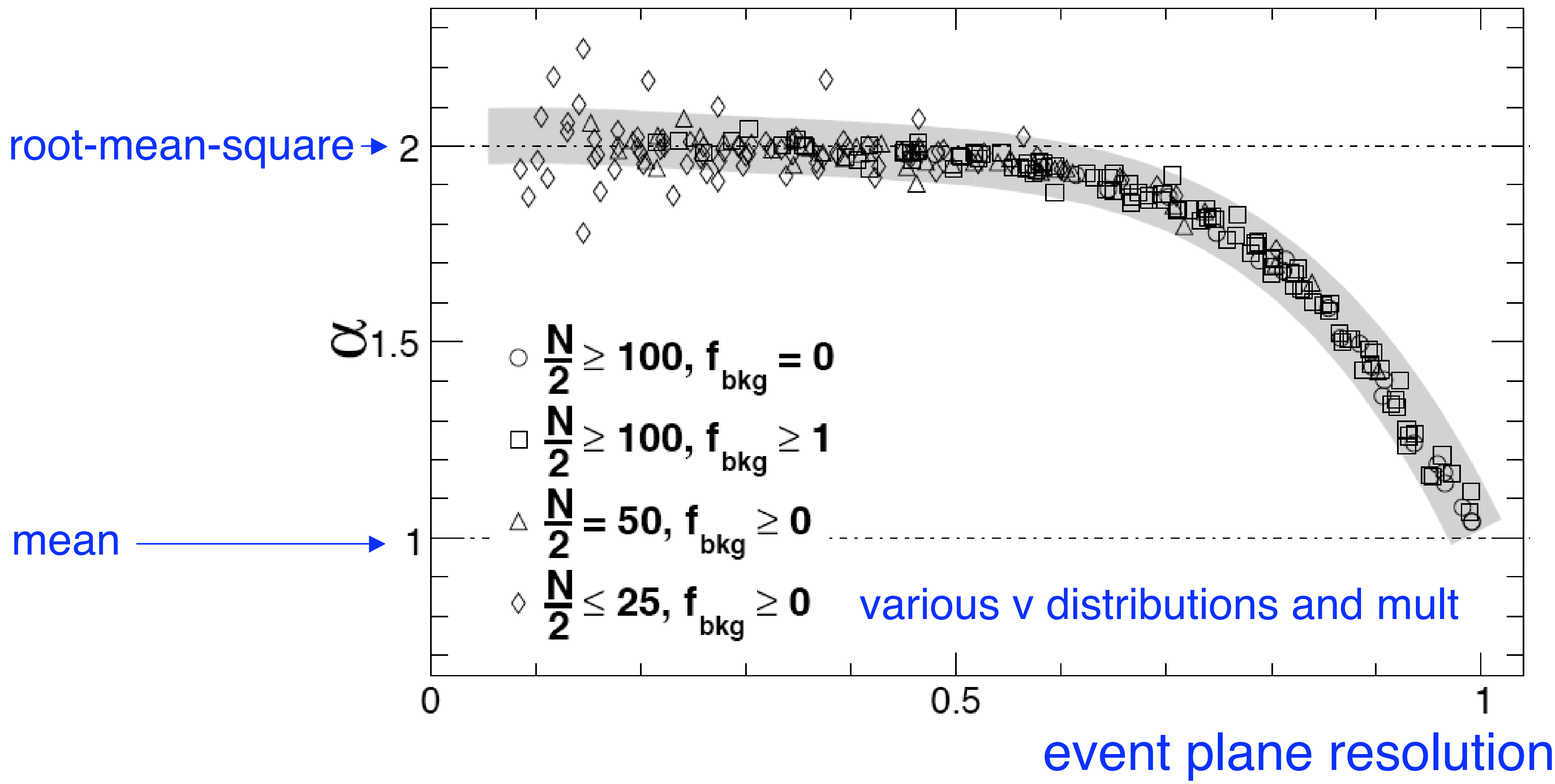
momentum space



participant plane

PHOBOS Simulations

$$v_2\{\mathrm{subEP}\} = \langle v_2^\alpha \rangle^{1/\alpha}$$



PHOBOS+Heinz, PRC **77**, 014906 (2008)

Include Fluctuations

$$v = \frac{v^{\mathrm{obs}}}{\mathrm{res}}$$

$$\langle \cos(\phi - \Psi_{\mathrm{EP}}) \rangle = \langle \cos(\phi - \Psi_{\mathrm{RP}}) * \cos(\Psi_{\mathrm{EP}} - \Psi_{\mathrm{RP}}) \rangle$$

$<v^{obs}>$ v res

$$v\{\mathrm{subEP}\} = \frac{\langle v \ \mathrm{res}(v) \rangle}{\sqrt{\langle \mathrm{res}^2(v) \rangle}}$$

square-root of
subevent corr.

$$= v$$

in absence of fluctuations

The α Equation for Subevents

$$\alpha = 1 + \frac{\langle v \rangle \text{res}'}{\text{res}} \left[2 - \frac{\langle v \rangle \text{res}'}{\text{res}} \right]$$

$$\chi = v \sqrt{M}$$ resolution parameter

$$x = \chi^2/2$$

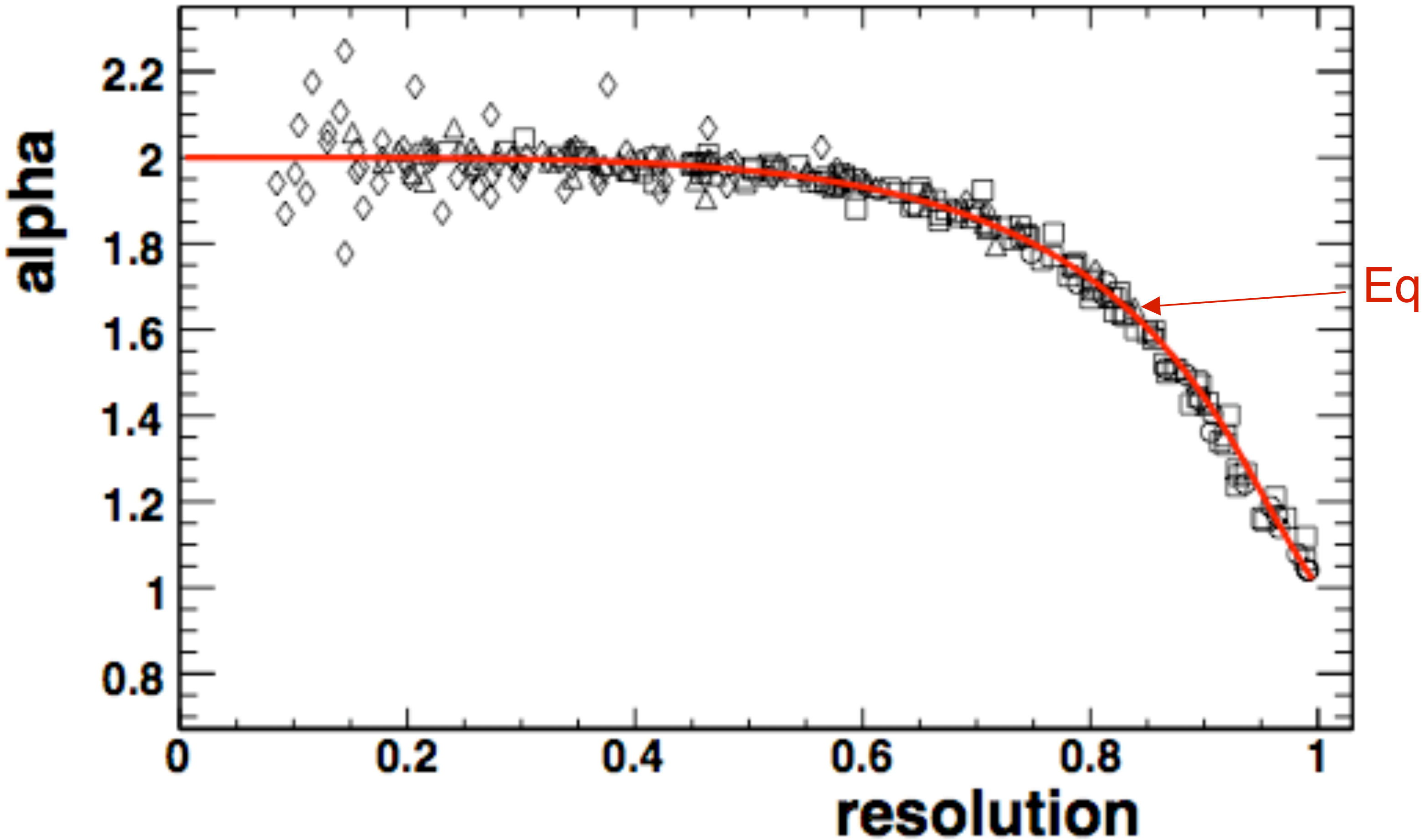
$$\text{res} = \sqrt{\pi}/2 \, \chi \exp(-x) \, (I_0(x) + I_1(x))$$

$$\alpha = 2 - \frac{4 I_1(x)^2}{[I_0(x) + I_1(x)]^2}$$ only function of χ

$I_{0,1}$ are modified Bessel functions

for full events, it is more complicated

PHOBOS+ with Equation



Fluctuations!

Experiment - Theory

- Experiment
  - correct for event plane resolution
  - nonflow problem
  - fluctuations: measure mean of some power of the distribution
- Theory
  - usually a value in the reaction plane
- We must communicate

Analytic Correction for Fluctuations

$$I_{0,1} = I_{0,1}(\chi^2/2) \qquad i_{0,1} = I_{0,1}(\chi_s^2/2)$$

$$v_2\{2\}^2 = \langle v_2 \rangle^2 + \sigma_{v2}^2$$

$$v_2\{4\}^2 \simeq \langle v_2 \rangle^2 - \sigma_{v2}^2$$

$$v\{\mathrm{subEP}\}^2 = \langle v \rangle^2 + \left(1 - \frac{4\,I_1^2}{(I_0 + I_1)^2}\right)\sigma_v^2$$

$$v\{\mathrm{EP}\}^2 = \langle v \rangle^2 + \left(1 - \frac{2(I_0 - I_1)}{I_0 + I_1}\left(\chi^2 - \chi_s^2 + \frac{2i_1^2}{i_0^2 - i_1^2}\right)\right)\sigma_v^2$$

method similar to momentum conservation correction:
N. Borghini, P.M. Dinh, J.-Y. Ollitrault, A.M. Poskanzer, and S.A. Voloshin,
PRC **66**, 014901 (2002)

Analytic Correction for Nonflow

$$\langle \cos(\phi_1 - \phi_2) \rangle = \langle v \rangle^2 + \delta$$

nonflow

$$v_2\{2\}^2 = \langle v_2 \rangle^2 + \delta_2$$

$$v_2\{4\} = \langle v_2 \rangle$$

$$v\{\text{subEP}\}^2 = \langle v \rangle^2 + \left(1 - \frac{2I_1^2}{(I_0 + I_1)^2}\right) \delta$$

$$v\{\text{EP}\}^2 = \langle v \rangle^2 + \left(1 - \frac{(I_0 - I_1)}{(I_0 + I_1)} \left(\chi^2 - \chi_s^2 + \frac{2i_1^2}{(i_0^2 - i_1^2)}\right)\right) \delta$$

$$\langle v \rangle = v$$

Differences of Measured v_2 Values

$$
\begin{aligned}
v\{2\}^2 - v\{4\}^2 &= \delta + 2\sigma_v^2 \\
v\{2\}^2 - v\{\mathrm{EP}\}^2 &= \frac{(I_0 - I_1)}{(I_0 + I_1)} \left(\chi^2 - \chi_s^2 + \frac{2 i_1^2}{(i_0^2 - i_1^2)} \right) (\delta + 2\sigma_v^2) \\
v\{2\}^2 - v\{\mathrm{subEP}\}^2 &= \frac{2 I_1^2}{(I_0 + I_1)^2} (\delta + 2\sigma_v^2) \\
v\{\mathrm{subEP}\}^2 - v\{\mathrm{EP}\}^2 &= \frac{(I_0 - I_1)}{(I_0 + I_1)} \left(\chi^2 - \chi_s^2 + \frac{2 i_1^2}{(i_0^2 - i_1^2)} - \frac{2 I_1^2}{(I_0^2 - I_1^2)} \right) (\delta + 2\sigma_v^2)
\end{aligned}
$$

All differences proportional to $\sigma_{\mathrm{tot}}^2 = \delta_2 + 2\sigma_{v2}^2$

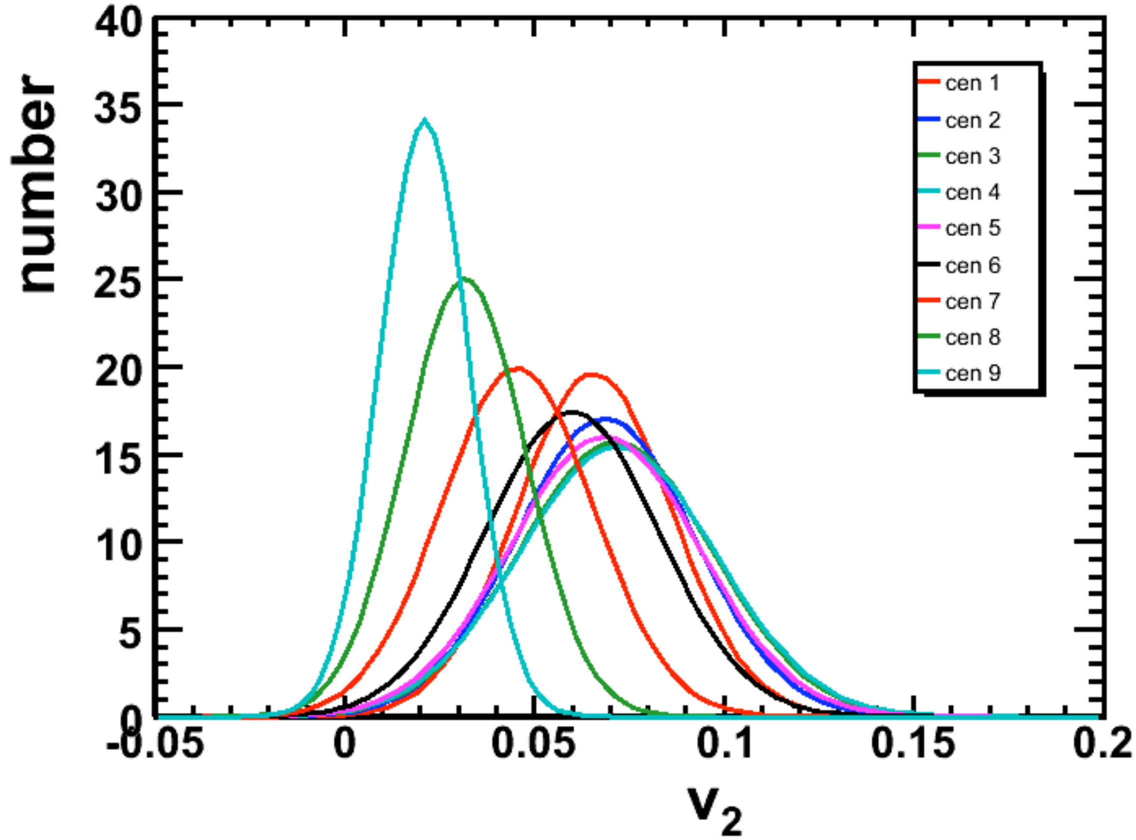
Without additional assumptions
can not separate nonflow and fluctuations

Gaussian Fluctuations

Assume Gaussian with same percent width as ε_{part}: $$\sigma_{v2} = \frac{\sigma_\varepsilon}{\langle\varepsilon\rangle}\langle v_2\rangle$$

σ_ε is from standard deviation of nucleon MC Glauber of ε_{part}

along the
participant plane axis



Application to Data

- **Assumptions**

$$\sigma_{v2} = \frac{\sigma_\varepsilon}{\langle \varepsilon \rangle} \langle v_2 \rangle$$

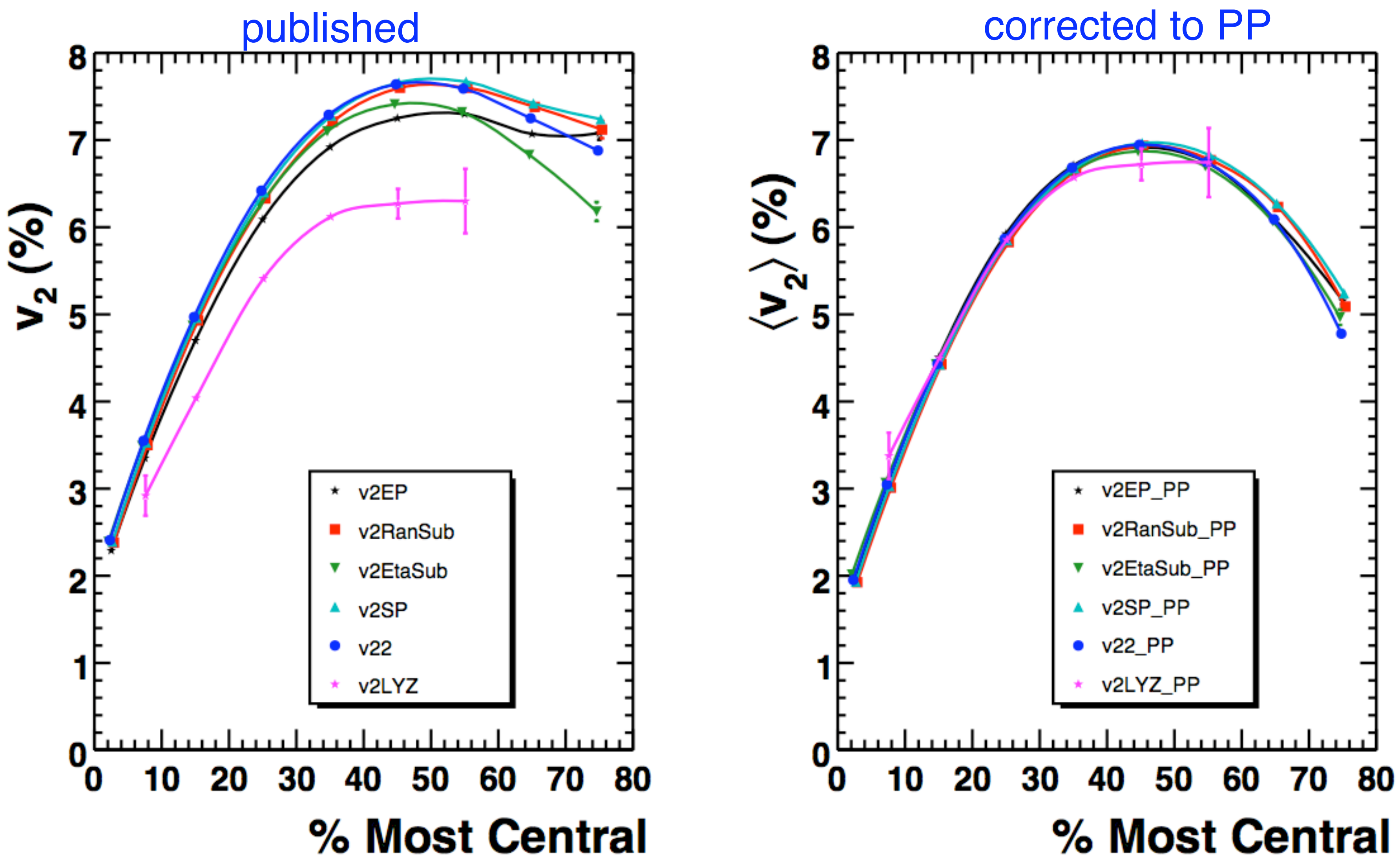
MC Glauber ε participant

$$\delta_2 = 2\ \delta_{pp}/N_{\text{part}} \qquad \delta_{pp} = 0.0145$$

$$\delta_{\text{etaSub}} = 0.5\ \delta_2$$

less nonflow

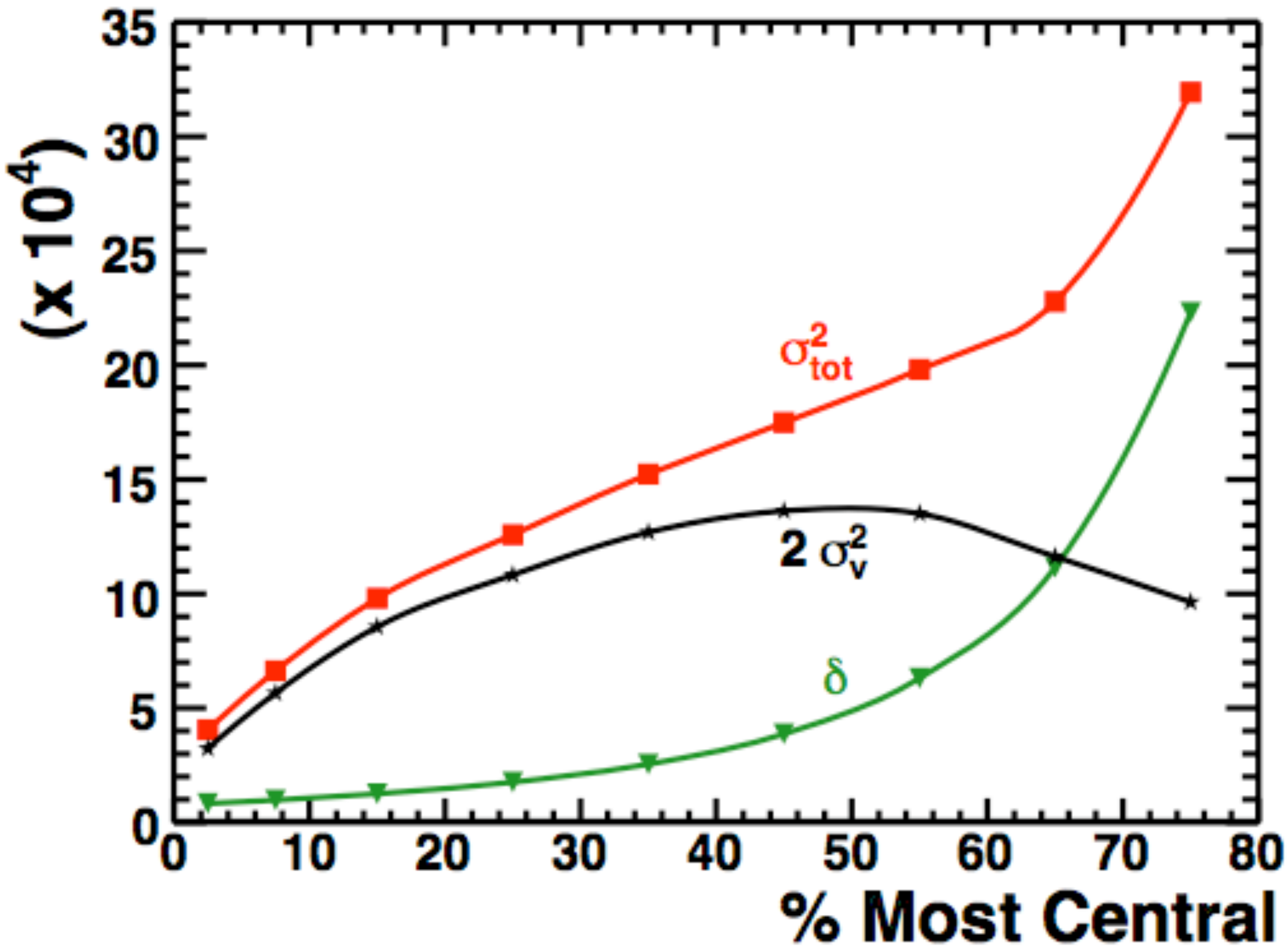
Data Corrected to $\langle v_2 \rangle$



agreement for mean v_2 in participant plane

Nonflow and Fluctuations

with my assumptions and parameters:



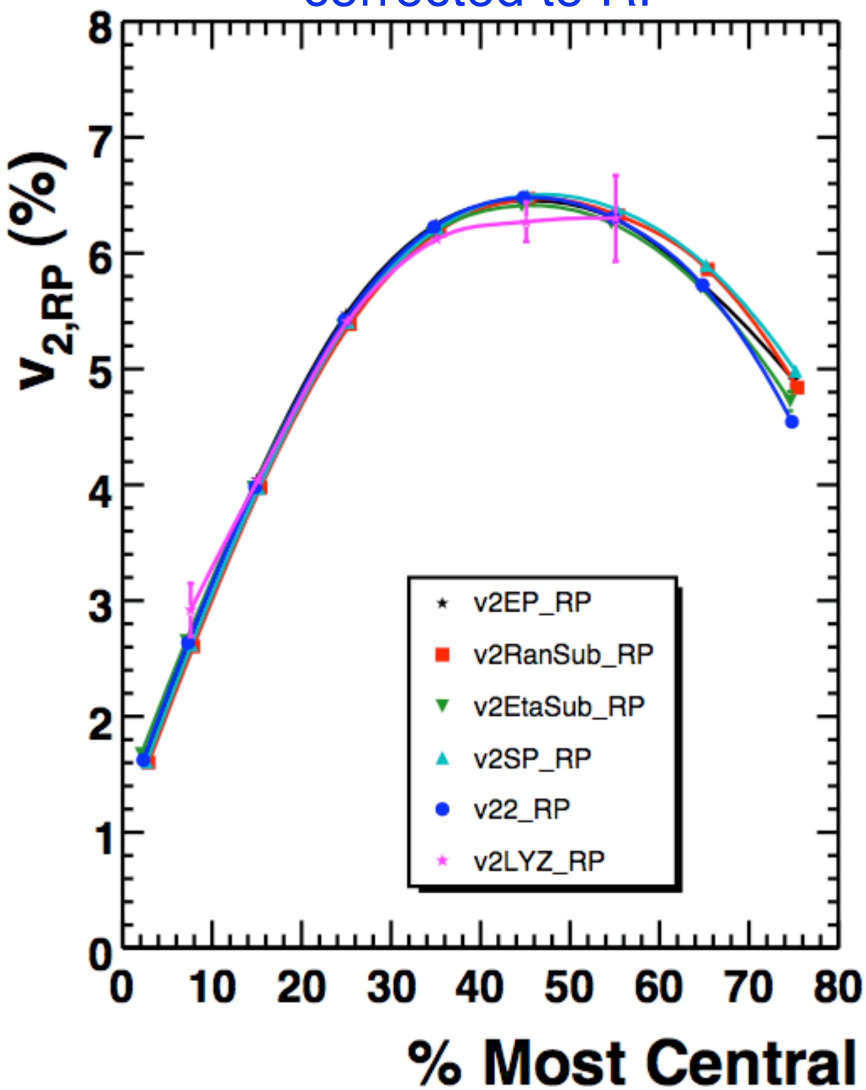
v_2 in the Reaction Plane

in Gaussian fluctuation approximatio

$$v_{2,\mathrm{PP}}^2 \simeq v_{2,\mathrm{RP}}^2 + \sigma_{v2}^2$$

a v_2 for theorists

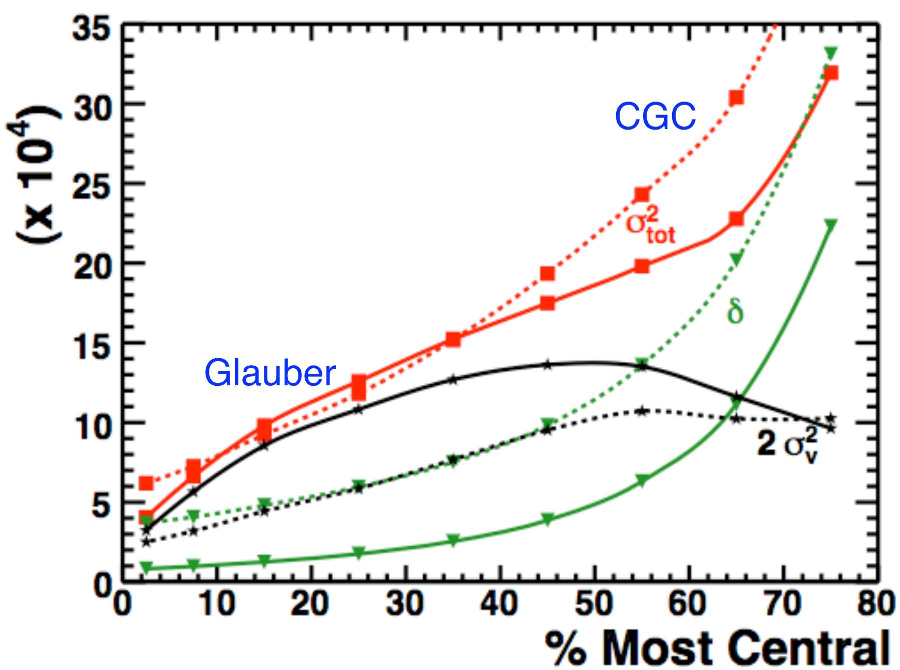
corrected to RP

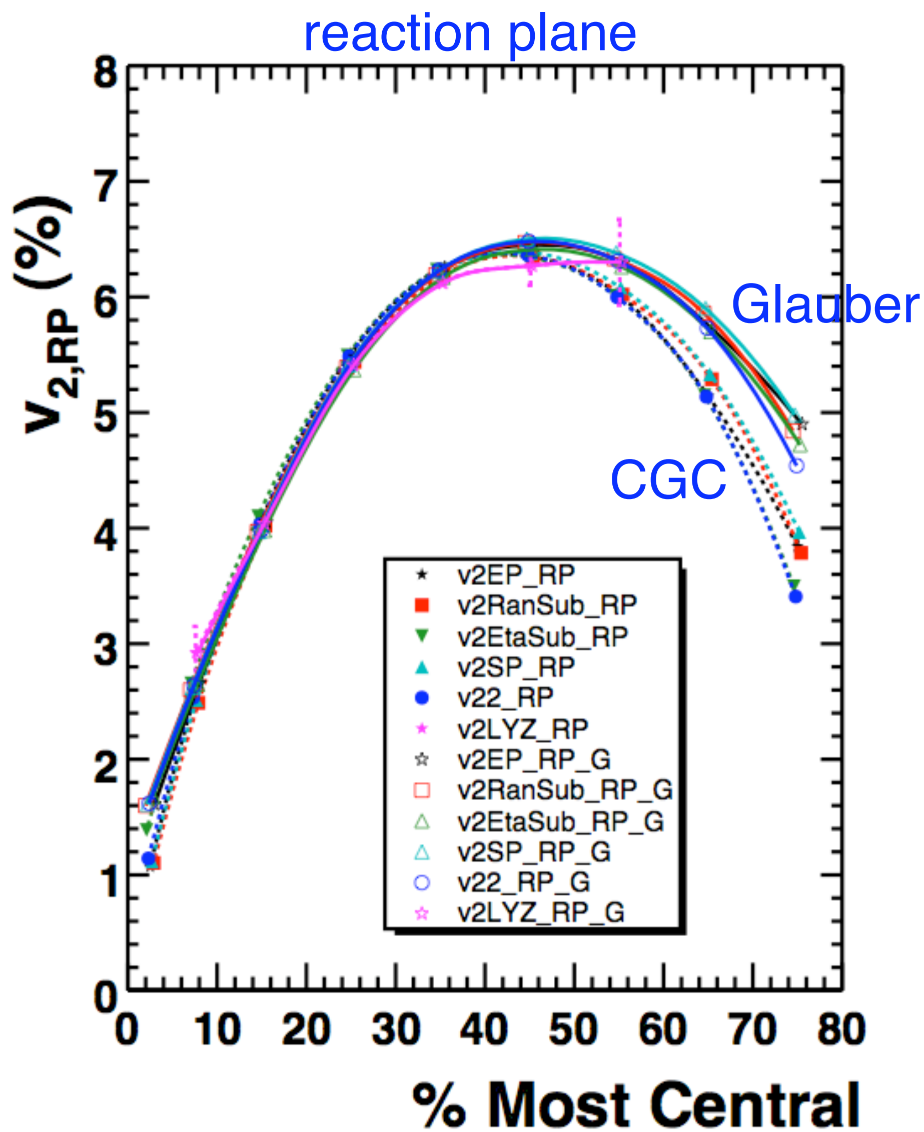


Voloshin, Poskanzer, Tang, and Wang, Phys. Lett. B **659**, 537 (2008)

CGC Model

Fluctuations from CGC
Nonflow from $\delta_{pp}/N_{\mathrm{part}}$
partly weighted by $N_{\mathrm{bin}}/N_{\mathrm{part}}$





Conclusions

- Understanding of effects of fluctuations and nonflow
- All corrections proportional to σ_{tot}^2
- Can not separate δ and σ_v without assumptions
- $<v_2>$ consistent from different measurements with
    - ε_{part} fluctuations from MC Glauber
    - $\delta = 2\,\delta_{pp}/N_{part}$ and etaSub less
- Other assumption also work with slightly different results
- Important for measurements with respect to event plane:
    - HBT, R_{AA}, conical emission, etc.
- Better multi-particle correlations are needed

Extra

Project Chronology

- I asked Constantin Loizides of PHOBOS
- He said Ollitrault understood
- I asked Jean-Yves
- Sergei rederived it elegantly
- I asked, does it explain why v_2\{EP\} is 5% low?
- I did numeric integrations
- Jean-Yves derived equations
- I applied them to published STAR data
- Sergei will do simulations to test equations

Numeric Correction for Fluctuations

If one knows <v> and σ:

$$v\{\mathrm{subEP}\} = \frac{\langle v\mathcal{R}\rangle}{\sqrt{\langle \mathcal{R}^2\rangle}} \qquad \chi = v\,\sqrt{M}$$

$$v\{\mathrm{subEP}\} = \frac{\langle v\ \mathcal{R}(v\ \sqrt{M/2})\rangle}{\sqrt{\langle[\mathcal{R}(v\ \sqrt{M/2})]^2\rangle}}$$

$$v\{\mathrm{EP}\} = \frac{\langle v\ \mathcal{R}(v\ \sqrt{M})\rangle}{\mathcal{R}\Big[\mathcal{C}\Big(\sqrt{\langle[\mathcal{R}(v\ \sqrt{M/2})]^2\rangle}\Big)\ \sqrt{2}\Big]}$$

$$v\{2,\mathrm{SP}\} = \sqrt{\langle v^2\rangle} \qquad \langle v\rangle = v\{2,\mathrm{SP}\}/\sqrt{1+\sigma_\varepsilon^2/\langle\varepsilon\rangle^2}$$

Assume $\sigma_{v2} = \frac{\sigma_\varepsilon}{\langle\varepsilon\rangle}\langle v_2\rangle$

and solve for $\langle v_2\rangle$

not restricted to a Gaussian