The Brick Problem in High-Twist Approximation

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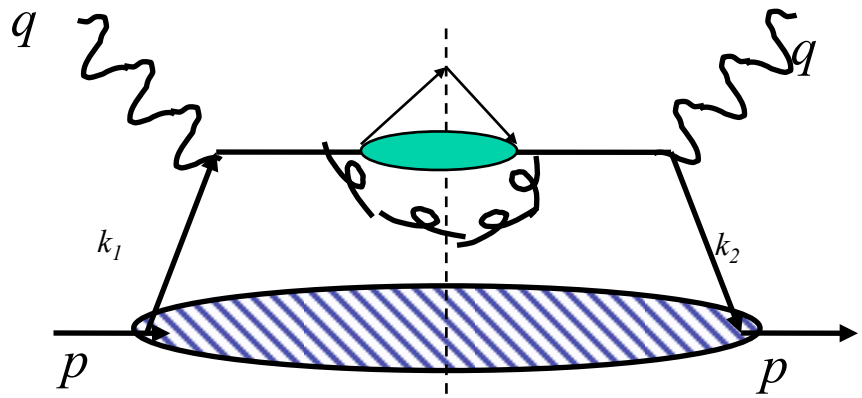
W.-T. Deng & XNW, arXiv:0910.3403

DGLAP Evolution in vacuum



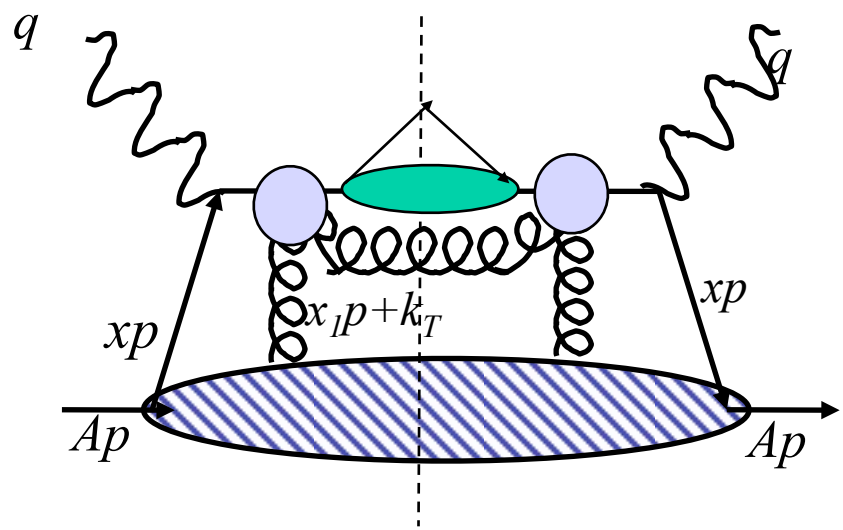
$$\Delta D_{q\to h}(z_h) = \frac{\alpha_s}{2\pi}\int^{\mu^2}\frac{d\ell_\perp^2}{\ell_\perp^2}\int_{z_h}^{1}\frac{dz}{z}\left[P_{q\to qg}(z)D_{q\to h}(\frac{z_h}{z}) + P_{q\to qg}(1-z)D_{g\to h}(\frac{z_h}{z})\right]$$

Splitting function $$P_{q\to qg}(z) = C_F\left[\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z)\right]$$

perturbative region: $$\ell_\perp^2 \geq Q_0^2$$

no matter how large is initial Q^2

Induced gluon emission in twist expansion



$$W^D_{\mu\nu} \propto \int d^2k_T e^{ik\cdot(y_1-y_2)} H^D_{\mu\nu}(p,q,k_T) \langle A | \bar{\psi}\gamma^+ A^+(y_1) A^+(y_2) \psi | A \rangle$$

k_T transverse momentum of medium gluon

Modified Fragmentation

$$\Delta D_{q\to h}(z_h, Q^2) = \frac{\alpha_s}{2\pi}\int_0^{Q^2}\frac{d\ell_\perp^2}{\ell_\perp^4}\int_{z_h}^1\frac{dz}{z}\left[\Delta\gamma(z, x_L)D_{q\to h}(\frac{z_h}{z}) + \cdots\right]$$

Guo & XNW'00

Modified splitting functions

$$\Delta\gamma(z, x_L) = \frac{1+z^2}{(1-z)_+}\frac{T_{qg}^A(x, x_L)}{f_a^A(x)}\frac{C_A 2\pi\alpha_s}{N_c} - \delta(1-z)v(\ell_\perp^2)$$

Two-parton correlation:

$$T_{qg}^A(x, x_L) = \int\frac{dy^-}{2\pi}dy_1^- dy_2^- e^{-ix_B p^+ y^-}\left\langle A\left|\bar\psi(0)\frac{\gamma^+}{2}F_\sigma^+(y_1^-)F^{+\sigma}(y_2^-)\psi(y^-)\right|A\right\rangle$$

$$\times\left(1 - e^{-ix_L p^+ y_2^-}\right)\left(1 - e^{ix_L p^+(y_1^- - y^-)}\right)$$

$$\frac{2\pi\alpha_s}{N_c}\frac{T_{qg}^A(x,x_L)}{f_q^A(x)} \approx \int d\xi^-[\hat{q}(\xi,x_T)+\hat{q}(\xi,x_L)][1-\cos(x_L p^+\xi^-)]$$

$$\hat{q}(\xi,x_L) = \frac{4\pi\alpha_s C_F}{N_c^2-1}\rho_A(\xi)x_L G(x_L)$$

$$x_L \leq 1$$

Comparison with GLV

$$\frac{dN_{\rm HT}}{dz} = \frac{N_c\alpha_s}{\pi}\frac{1+(1-z)^2}{z}\int\frac{d\ell_T^2}{\ell_T^4}\int d\xi[c(x_L)\hat{q}(\xi,0)+\hat{q}(\xi,x_L)]$$
$$\left[1-\cos\frac{\ell_T^2\xi}{2q^-z(1-z)}\right].$$

$$\frac{dN_{\rm GLV}}{dz} = \frac{C_A\alpha_s}{\pi}\frac{1+(1-z)^2}{z}\int d\xi\rho_A(\xi)\sigma_{qN}\mu^2\int\frac{d\ell_T'^2}{\ell_T'^2(\ell_T'^2+\mu^2)}$$
$$\left[1-\cos\frac{\ell_T'^2\xi}{2q^-z(1-z)}\right].$$

$\hat{q} \leftrightarrow \rho_A\sigma_g\mu^2$ ρ_A quasi-particle density

Modified DGLAP Evolution

$$\frac{\partial \tilde{D}_q^h(z_h,\mu^2)}{\partial \ln \mu^2} = \frac{\alpha_s(\mu^2)}{2\pi}\int_{z_h}^1 \frac{dz}{z}\Big[\tilde{\gamma}_{q\to qg}(z,\mu^2)\tilde{D}_q^h(\frac{z_h}{z},\mu^2) + \tilde{\gamma}_{q\to gq}(z,\mu^2)\tilde{D}_g^h(\frac{z_h}{z},\mu^2)\Big]$$

$$\frac{\partial \tilde{D}_g^h(z_h,\mu^2)}{\partial \ln \mu^2} = \frac{\alpha_s(\mu^2)}{2\pi}\int_{z_h}^1 \frac{dz}{z}\Big[\sum_{q=1}^{2n_f}\tilde{\gamma}_{g\to q\bar{q}}(z,\mu^2)\tilde{D}_q^h(\frac{z_h}{z},\mu^2) + \tilde{\gamma}_{g\to gg}(z,\mu^2)\tilde{D}_g^h(\frac{z_h}{z},\mu^2)\Big]$$

Modified splitting functions

$$\tilde{\gamma}_{a\to bc}(z,l_T^2) = \gamma_{a\to bc}(z) + \Delta\gamma_{a\to bc}(z,l_T^2)$$

Parton shower distr. in a Brick

Initial conditions:

$$\tilde{D}_a(Q_0^2) = D_a(Q_0^2) + \Delta D_a(Q_0^2)\,, \quad a = g, q, \bar{q}.$$

$$\begin{aligned} D_a^a(z, Q_0^2) &= \delta(1-z); \\ D_a^{b\neq a}(z, Q_0^2) &= 0 \;\; (a, b = q, \bar{q}, g), \end{aligned}$$

$$\Delta D_a(Q_0^2) = 0 \;\; ?$$

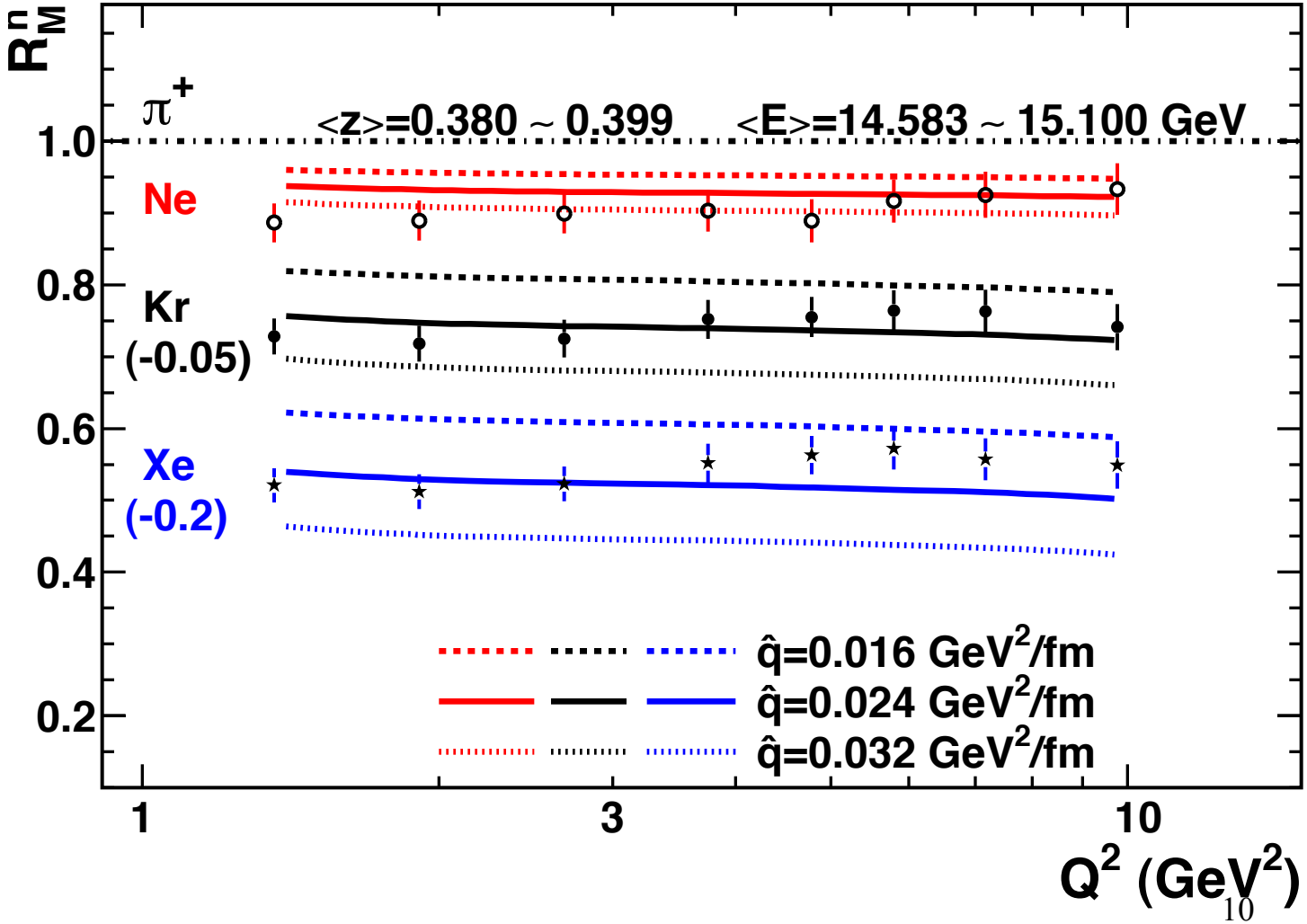
Initial Condition in medium

DGLAP scheme:

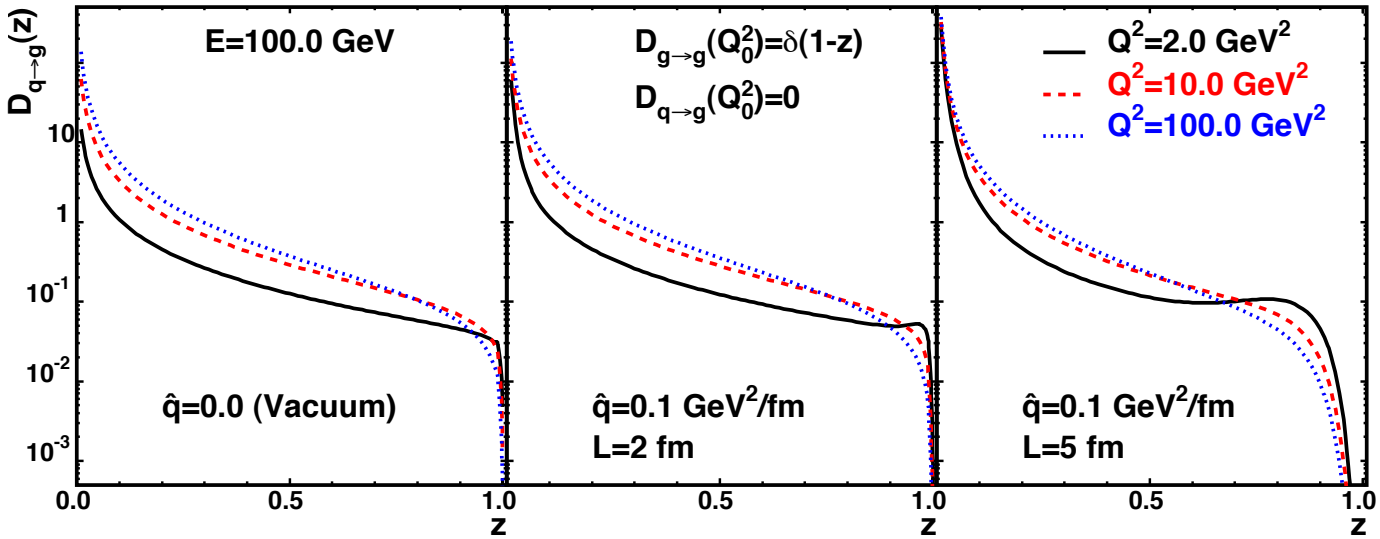
$$\tilde{\gamma}_{a\to bc}(z, l_T^2) = \Delta\gamma_{a\to bc}(z, l_T^2)$$

Freeze: $$\alpha_s(Q^2) = \alpha_s(Q_0^2), \;\; Q^2 < Q_0^2$$

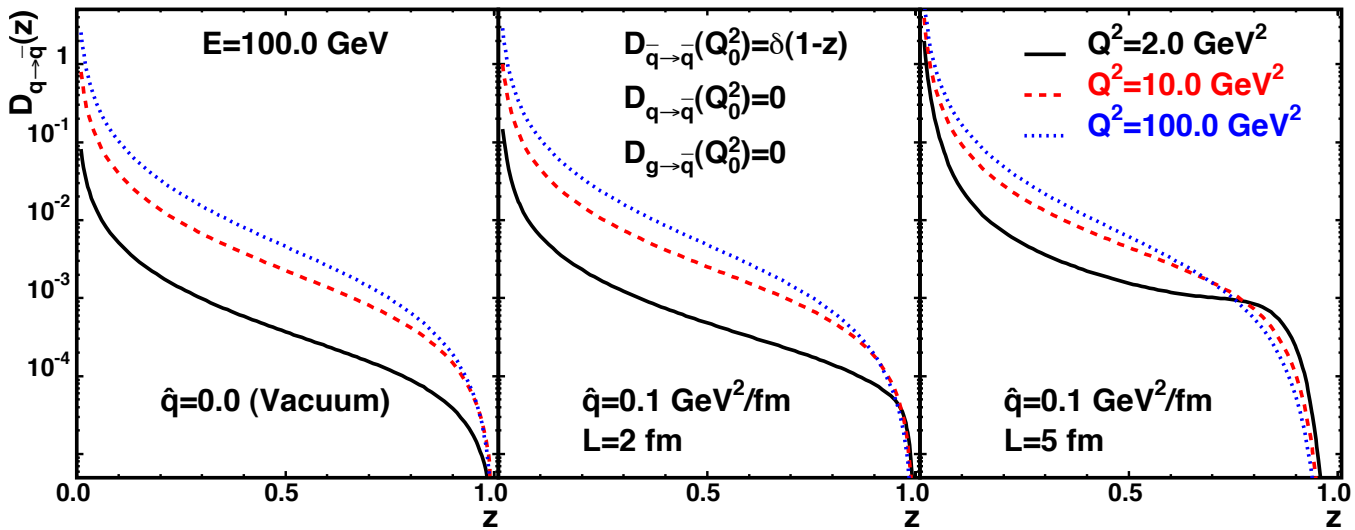
$$D_a(Q_0^2) \to \tilde{D}_a(Q_0^2)$$


R_M^n
π^+
<z>=0.380 ~ 0.399
<E>=14.583 ~ 15.100 GeV
Ne
Kr
(-0.05)
Xe
(-0.2)
$\hat{q}$=0.016 GeV^2/fm
$\hat{q}$=0.024 GeV^2/fm
$\hat{q}$=0.032 GeV^2/fm
1.0
0.8
0.6
0.4
0.2
1
3
10
Q^2 (GeV^2)

Shower parton distr. in a quark jet

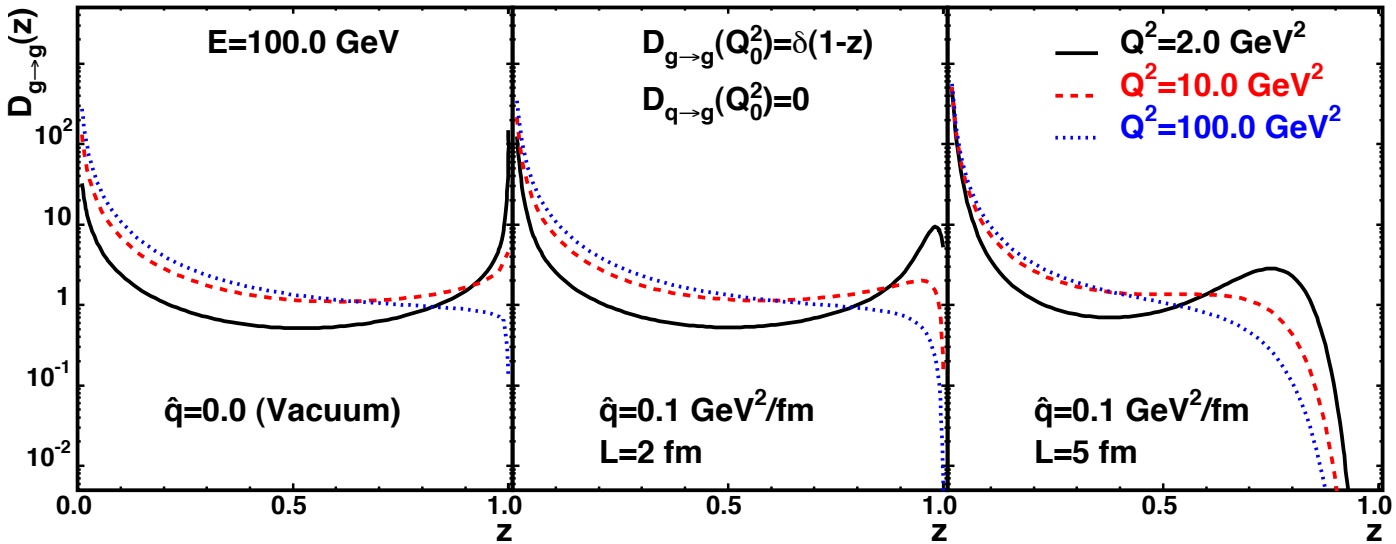


q --> g

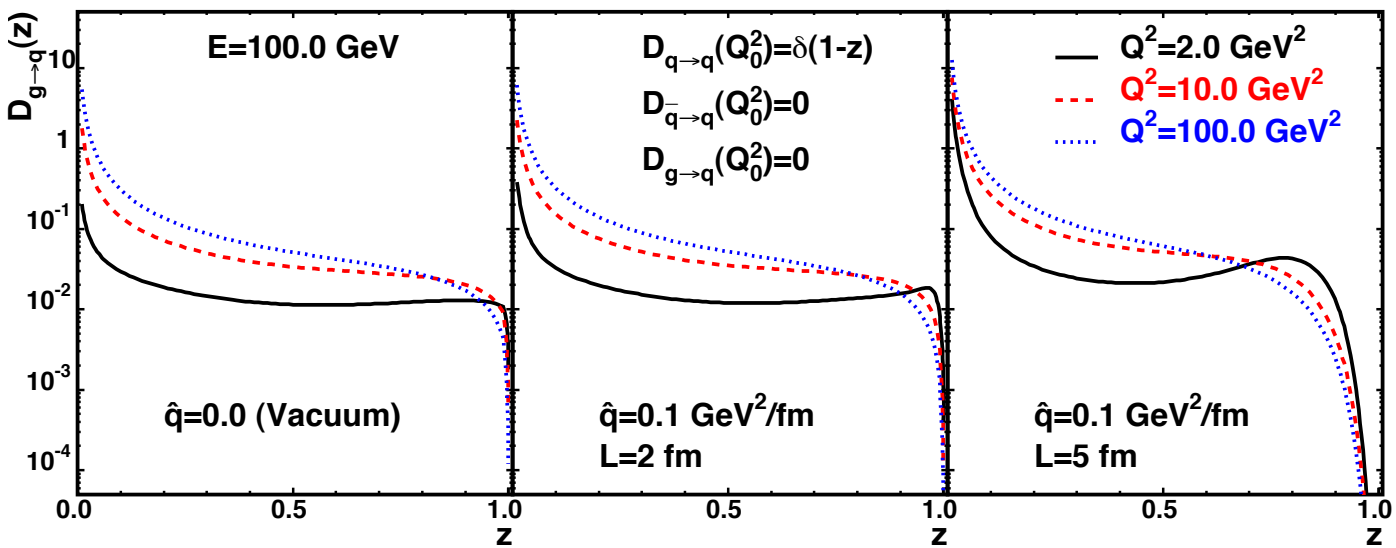


q--> qbar

Shower parton distr. in a gluon jet

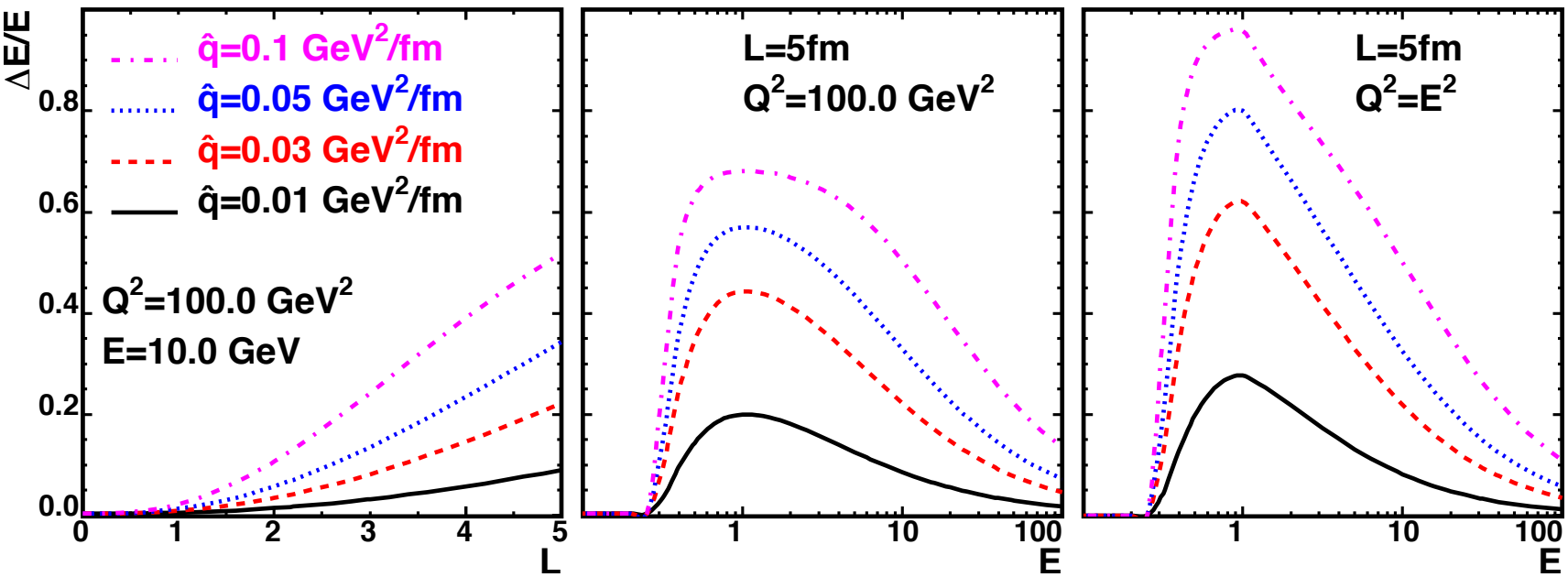


g--> g

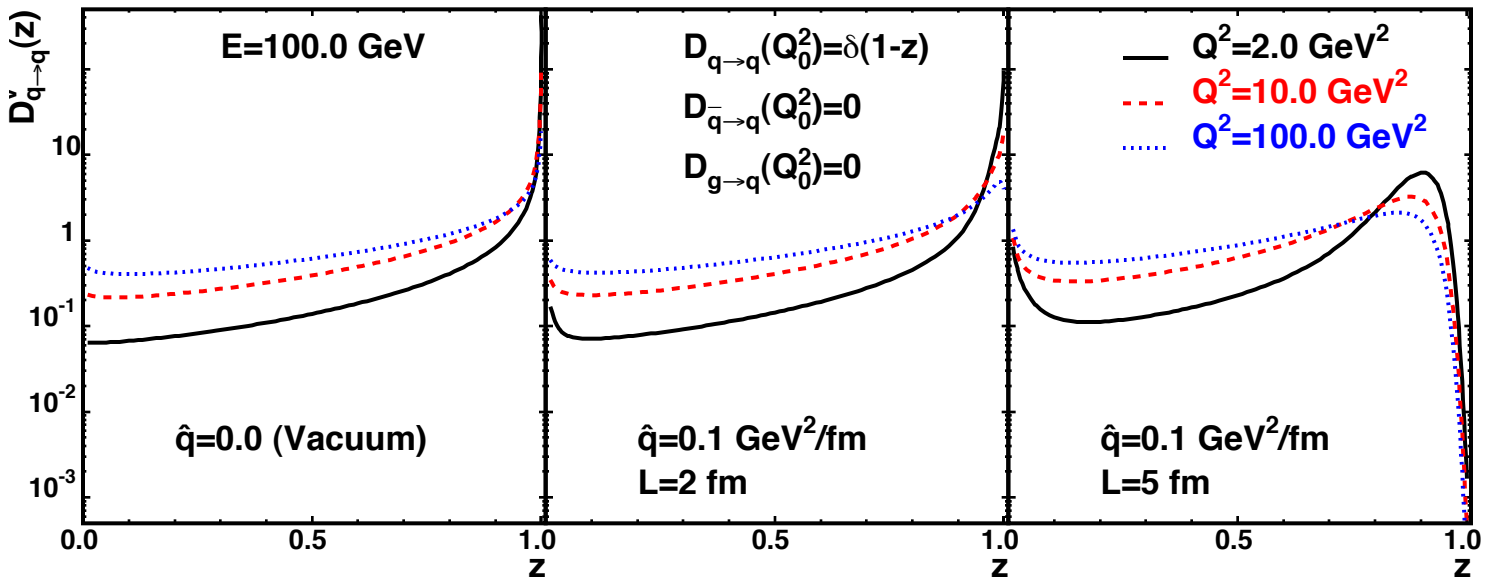


g --> q

Valence quark energy loss



Valence quark distribution



$$\frac{\Delta E}{E} = \frac{\Delta E_m - \Delta E_v}{E} = \int_0^1 dz\, z \left[D_q^v(z, Q^2) - \tilde{D}_q^v(z, Q^2) \right]$$

Beyond the Brick Problem

Jet transport parameter & phases of dense matter

$$\hat{q}(\xi_N) \equiv \frac{4\pi^2 \alpha_s C_F}{N_c^2 - 1} \rho_A(\xi_N) x G_N(x)|_{x\approx 0}$$

Jet transport parameter

$$x G_N(x) = -\int \frac{d\xi^-}{2\pi p^+} e^{ixp^+\xi^-} \langle N | F_{+\sigma}(0) L_{\parallel}^A(0,\xi^-) F_+^{\sigma}(\xi^-) | N \rangle$$

Constrain "implementations" with DIS data (talk by Wei-tian Deng)

$$\langle \delta q_\perp^2 \rangle = 0.016 A^{1/3} GeV^2/fm \longleftrightarrow R_A(z)$$

qhat during bulk evolution

$$\hat{q}(\tau, r) = \hat{q}_0 \frac{\rho^{QGP}(\tau, r)}{\rho^{QGP}(\tau_0, 0)}(1 - f) + \hat{q}_h(\tau, r) f \,,$$

$$f \,:\, \text{hadronic fraction}$$

$$\hat{q}_h = \frac{\hat{q}_N}{\rho_N}\left[\frac{2}{3}\sum_M \rho_M(T) + \sum_B \rho_B(T)\right]$$