Comparison of Jet Quenching Formalisms for a Quark-Gluon Plasma “Brick” (Outline Version II)

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This is the second draft of the outline of a report describing the comparison of various pQCD based formalisms treating the energy loss of hard partons in a thermal quark-gluon plasma for a simplified geometry. Specifically, we compare the predictions of the WHDG and ASW, and Higher Twist (HT) formalisms in the opacity expansion, and of the BDMPS-Z and AMY formalisms in the multiple soft scattering approximation.

I. DETAILED COMPARISONS OF MODELS

A. ASW

The 'ASW-formalism' calculates parton energy loss based on a path-integral formalism [1-3]. The path-integral can be evaluated in two different approximations:

a. Multiple soft scattering limit: Technically, this is a saddle-point approximation of the path integral. For the case of infinite in-medium pathlength, the result coincides with the BDMPS expression for parton energy loss [2]. For this reason, we refer to this limit sometimes as "BDMPS-limit".

b. Opacity expansion: Technically, this is an expansion of the integrand of the path integral in powers of (density times path-length). The GLV $N = 1$ opacity result reproduces our expression [1] on the level of the Feynman diagrams and the analytic expression for the ω- and k_T-differential gluon energy distribution.

1. Probability distribution of energy loss

Figs. 1 and 2 show results from the ASW multiple soft scattering limit. (All results have been computed with $\alpha_s = 0.3$.) First, we plot the probability $P(\Delta E/E)$ - the so-called *quenching weight* - that a light quark loses a fraction of its energy. The information about $P(\Delta E/E)$ is contained in three pieces:

1. "Untouched survival": This is the discrete probability that a parton does not interact with a medium of length L and that it loses no energy. This probability is represented by a color-coded dot at $\Delta E/E = -0.05$.
2. "Survival with finite energy loss": This is the continuous probability that a parton makes it through a medium of length L but loses a finite fraction $\Delta E/E$ during its passage. This is denoted by the color-coded curve at finite $0 \leq \Delta E/E \leq 1$.
3. "Death before arrival": In general, if one shoots a particle into a wall of thickness L, it can get stopped on its journey before reaching the length L. This probability is denoted by the color-coded dot at $\Delta E/E = 1.05$.

In general, as one increases the average energy loss (i.e. as one increases $\hat{q}$),

- the probability of untouched survival decreases,
- the probability of survival with finite energy loss shifts to larger values of $\Delta E/E$,
- the probability of death before arrival increases.

This is seen clearly in all the plots shown for $P(\Delta E/E)$. The most extreme curve is that for $E = 10$ GeV and $L = 2$ fm. Requiring an energy loss of $\Delta E = 4$ GeV in this case amounts to > 40 percent probability of untouched survival but 30 percent probability of death before arrival. This comes close to an all-or-nothing scenario, where a particle either goes through without medium-modification or gets stuck, but emerges with relatively small probability as an object with reduced energy.

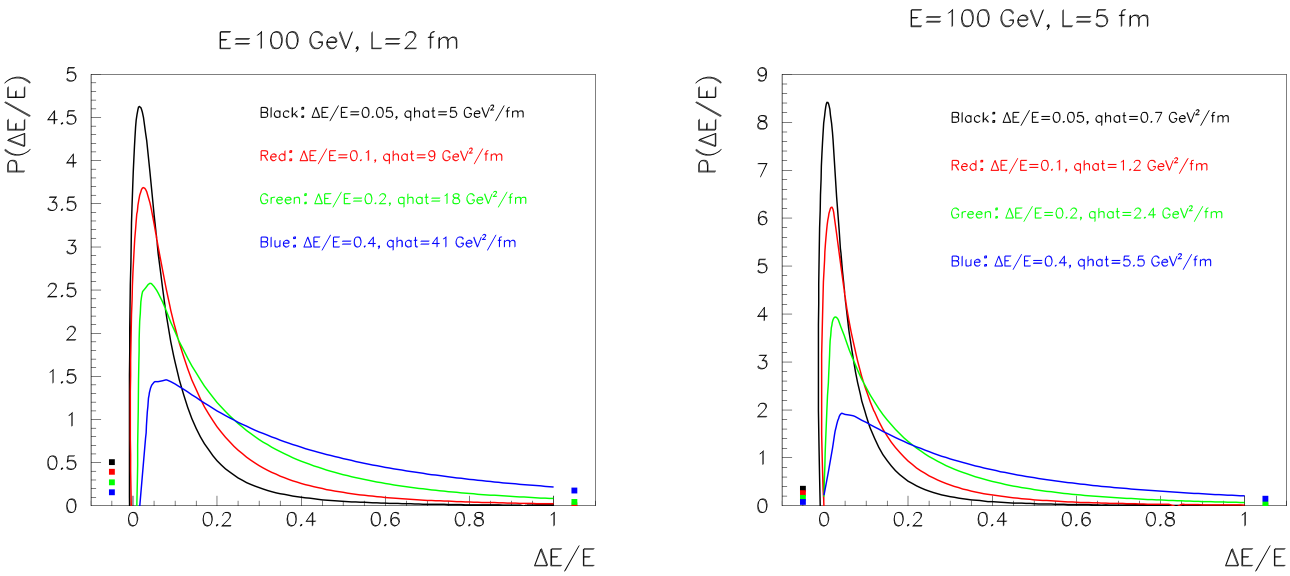

FIG. 1: Probability of energy loss, see the text for explanations, for a light quark of $E = 100$ GeV with $L = 2$ fm (plot on the left) and $L = 5$ fm (plot on the right). The legends on the plots indicate the average energy loss and the corresponding value of the transport coefficient $\hat{q}$.

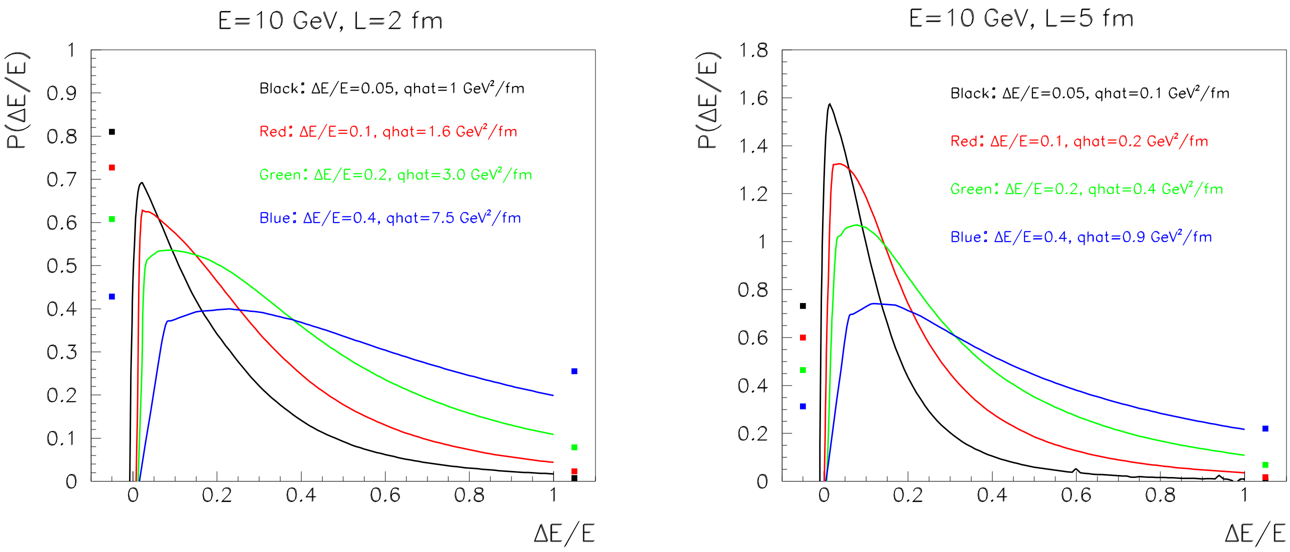

FIG. 2: Id. to Fig. 1 but for a light quark of $E = 10$ GeV.

2. *Energy spectra of radiated gluons*

We now turn to the corresponding spectra: The multiple soft scattering limit suppresses the production of infrared gluons by a destructive interference effect. As a consequence, all spectra are peaked at finite gluon energies. In general, the radiated gluons become harder as one increases the average energy loss (i.e. as one increases $\hat{q}$). If the projectile energy is sufficiently large and the in-medium pathlength is sufficiently small, then the radiated gluons carry small fractions of the projectile energy. This is the case for a projectile quark energy $E = 100$ GeV shown in Fig. 3.

However, if the projectile energy is too small, see Fig. 4, one faces a particular problem: One calculates the radiated gluon spectrum as if the parton would propagate through a medium of path-length L, though with finite probability the parton does not have sufficient initial energy to make it through L, and gets stuck before. In the present calculational framework, finding yield in the spectrum for $\omega > E$, i.e., above the kinematical boundary, signals that one has assumed that the particle propagates through a length L though its probability of "death before arrival"

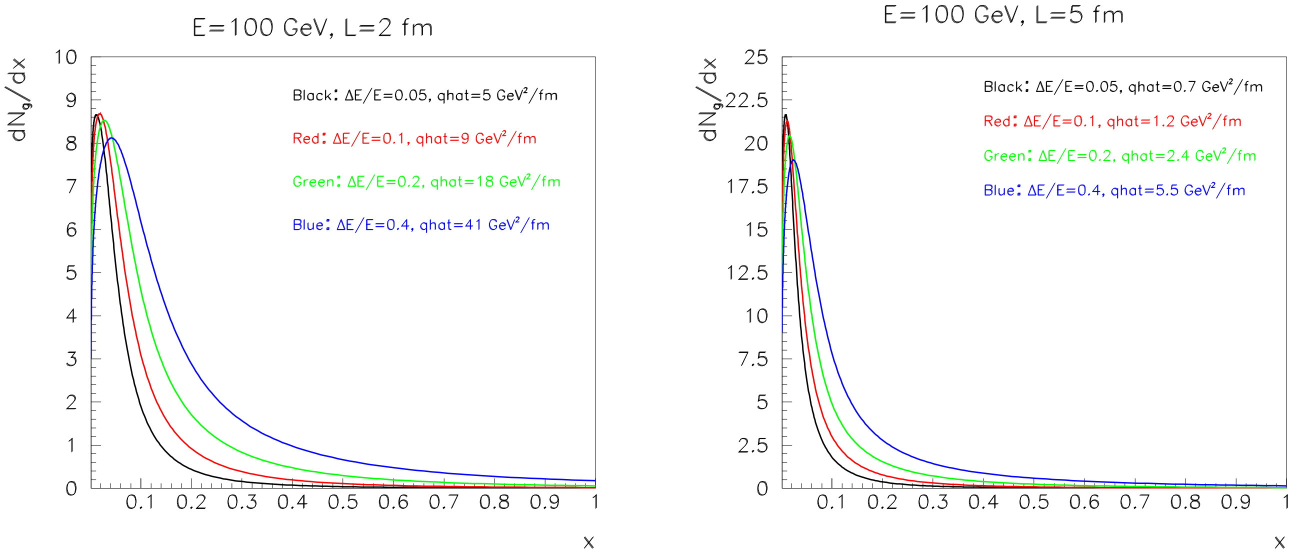

FIG. 3: Energy spectrum of radiated gluons, for a light quark of $E = 100$ GeV with $L = 2$ fm (plot on the left) and $L = 5$ fm (plot on the right). The legends on the plots indicate the average energy loss and the corresponding value of the transport coefficient $\hat{q}$.

is finite.

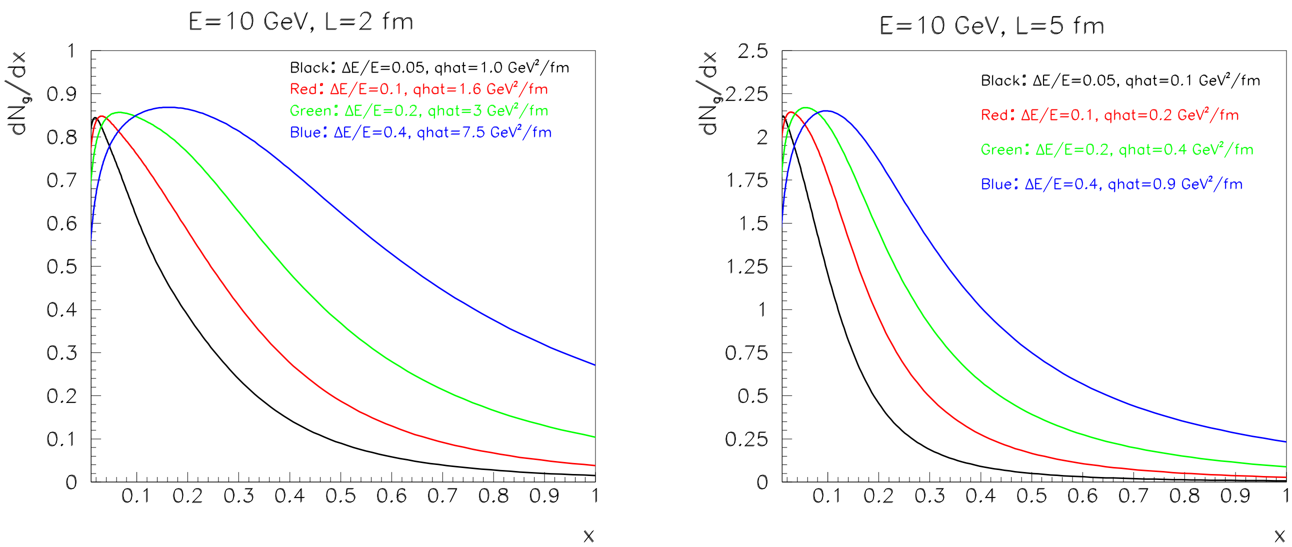

FIG. 4: Id. to Fig. 3 but for a light quark of $E = 10$ GeV.

The limitations of the high-energy eikonal approximations used in the derivation of the path-integral formalism from which both the multiple soft scattering limit and the opacity expansion stem, were discussed in the original papers [1-3], together with a comparison of both limits. These limitations and the comparison between both approximations have been recently re-analyzed in much detail in [4]. At this point, let us mention that they were, together with the need of a reliable framework for computing more differential observables like particle correlations or jet shapes, the motivation to include finite energy corrections in the formalism, both at the level of the DGLAP evolution equations [5] or in the form of Monte Carlo algorithms for final state radiation [6-9]. In this respect, it should be noted that large values of the transport coefficient were required to reproduce RHIC data on single particle suppression[10-15] and back-to-back correlations [12,15], extracted in analysis using the ASW multiple soft scattering limit through the quenching weights for different models for the medium produced in the collisions. But similar values have been obtained using the mentioned recent developments [5,8] which do conserve, by construction, energy-momentum both at the one-splitting level and in the parton shower as a whole. This finding confirms the validity of the ASW formalism for phenomenological studies of jet quenching.

3. Treatment of kinematic uncertainties

Here we expand on some of the statements made in the last paragraph: It had been stated as early as 2000 that [16] " ... the BDMPS-Z formalism is based on the assumption of small transverse gluon momentum $|k_T \ll \omega$ while we find the main contribution to radiative energy loss for $|k_T| = O(\omega)$. Both features question the validity of the BDMPS-Z formalism ..." The basic observation in this work and several other early papers [1-3] is that the calculations of quenching weights $P(\Delta E/E)$ involve integrals over transverse gluon momenta $\int dk_T\, f_{\rm integrand}(k_T)$ [17]. If the integrand $f_{\rm integrand}(k_T)$ were known without kinematic approximations, then this integrand would vanish in the kinematically forbidden region $k_T > \omega$, and it would approach this forbidden region in a physically reasonable way. In such a case without approximation, one would not need to worry about the upper bound of the k_T-integral and one could take it to infinity, since it is the physics of the integrand, which cuts off the integral.

In the high energy approximation $|k_T| \ll \omega$, however, the integrand $f_{\rm integrand}(k_T)$ does not vanish for $|k_T| > \omega$. The k_T-integral must then be cut off "by hand". Technically, the k_T cut off can be varied in the ASW formalism by varying the parameter R on the level of the quenching weights. The early ASW works [1-3] knew about and commented on the uncertainties arising from this k_T-integration. Several of these works also varied kinematical cuts to quantify these uncertainties. These limitations and the comparison between different small-x approximations have been re-analyzed recently and expanded in much detail in [4]. This is discussed in other sections of this document (point to section).

We note, however, that strictly speaking, varying the upper cut-off of the k_T-integral does not allow one to fully quantify the theoretical uncertainties associated to the approximation $|k_T \ll \omega$. This is so, since the problem with the approximation $|k_T| \ll \omega$ is not solved by cutting off the integral for $k_T > \omega$. Rather, the problem remains that the approximate evaluation of the integrand is also unreliable in the entire physical region $|k_T| = O(\omega)$. The proper solution to this problem is hence not an ad hoc modification of the upper bound of the integral, but an improved calculation of the integrand, which does not rely on $|k_T| \ll \omega$. Such an improvement is a rather automatic by-product if one models parton energy loss with Monte Carlo algorithms for final state radiation [6-9]. Aside from several physics motivations, this was one of the main technical reason for turning towards the formulation of parton energy loss in event generators, which do not require any approximation of the form $|k_T \ll \omega$. We refer to this fact often as exact energy-momentum conservation at each splitting.

4. References

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[17] For the following argument, it is not necessary to specify the form of the integrand $f_{\mathrm{integrand}}(k_T)$, which is what is calculated in a radiative energy loss formalism. We believe that the observations made here were known to BDMPS since 1995, but we cannot point to a published paper where they were put down in writing before 2000.