Overview of initial conditions and early non-equilibrium dynamics

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François Gelis
CERN, PH-TH

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Initial time (in)dependence

Criterion for gluon recombination

Gribov, Levin, Ryskin (1983)

Number of gluons per unit area :

$$\rho \sim \frac{xG_A(x, Q^2)}{\pi R_A^2}$$

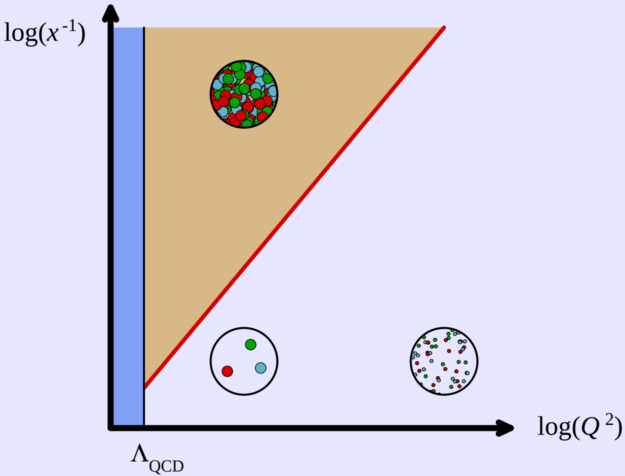
Recombination cross-section :

$$\sigma_{gg \to g} \sim \frac{\alpha_s}{Q^2}$$

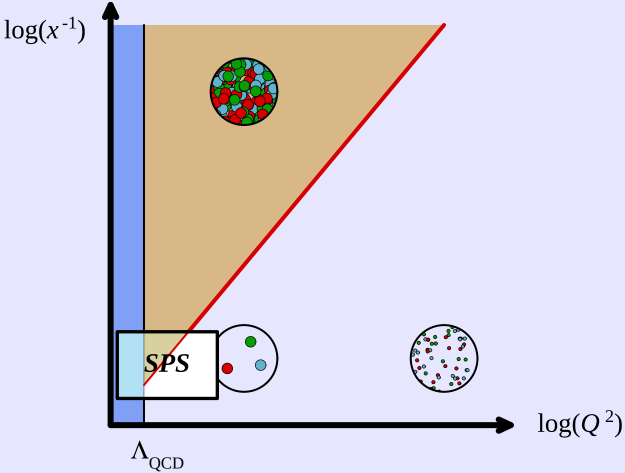
Recombination happens if $\rho\sigma_{gg\to g} \gtrsim 1$, i.e. $Q^2 \lesssim Q_s^2$, with :

$$Q_s^2 \sim \frac{\alpha_s xG_A(x, Q_s^2)}{\pi R_A^2} \sim A^{1/3} \frac{1}{x^{0.3}}$$

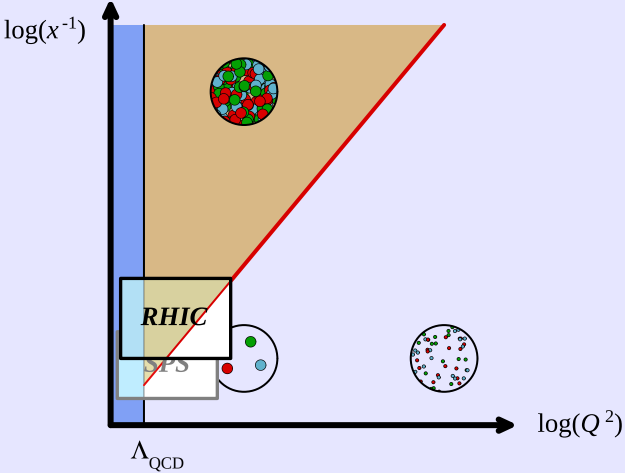
Saturation domain



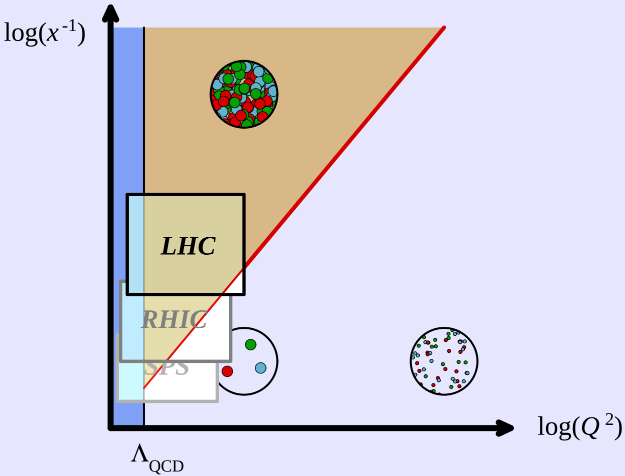
Saturation domain



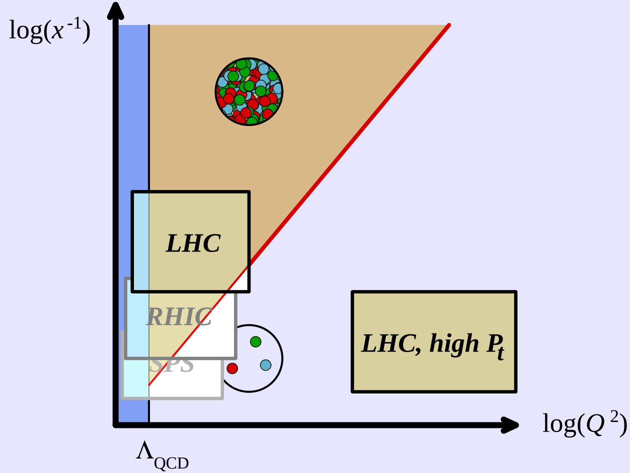
Saturation domain



Saturation domain

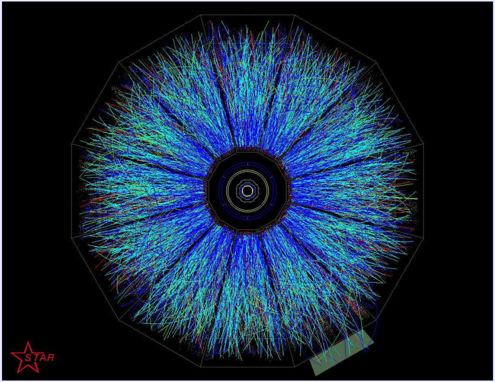


Saturation domain



Heavy Ion Collisions

Nucleus-Nucleus collision at RHIC



- 99% of the multiplicity below $p_\perp \sim 2$ GeV
- Q_s^2 might be as large as 10 GeV^2 at the LHC ($\sqrt{s} = 5.5$ TeV)

  ▷ saturation expected to play an important role

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Degrees of freedom

- The fast partons (large x) are frozen by time dilation
  ▷ described as static color sources on the light-cone :

$$J^\mu = \delta^{\mu +}\delta(x^-)\rho(\vec{\boldsymbol{x}}_\perp) \qquad (x^- \equiv (t-z)/\sqrt{2})$$

- Slow partons (small x) cannot be considered static over the time-scales of the collision process ▷ they must be treated as the usual gauge fields

  Since they are radiated by the fast partons, they must be coupled to the current J^μ by a term : $A_\mu J^\mu$

- The color sources ρ are random, and described by a distribution functional $W_Y[\rho]$, with Y the rapidity that separates "soft" and "hard"

Color Glass Condensate

Evolution equation (JIMWLK) :

$$\frac{\partial W_Y}{\partial Y} = \mathcal{H}\ W_Y$$

$$\mathcal{H} = \frac{1}{2} \int_{\vec{\boldsymbol{x}}_\perp, \vec{\boldsymbol{y}}_\perp} \frac{\delta}{\delta \widetilde{\mathcal{A}}^+(\epsilon, \vec{\boldsymbol{y}}_\perp)} \eta(\vec{\boldsymbol{x}}_\perp, \vec{\boldsymbol{y}}_\perp) \frac{\delta}{\delta \widetilde{\mathcal{A}}^+(\epsilon, \vec{\boldsymbol{x}}_\perp)}$$

where $-\partial_\perp^2 \widetilde{\mathcal{A}}^+(\epsilon, \vec{\boldsymbol{x}}_\perp) = \rho(\epsilon, \vec{\boldsymbol{x}}_\perp)$

- $\eta(\vec{\boldsymbol{x}}_\perp, \vec{\boldsymbol{y}}_\perp)$ is a non-linear functional of ρ
- This evolution equation resums all the powers of $\alpha_s \ln(1/x)$ and of $Q_s/p_\perp$ that arise in loop corrections
- This equation simplifies into the BFKL equation when the source ρ is small (one can expand η in powers of ρ)

CGC and Nucleus-Nucleus collisions

- For nucleus-nucleus collisions, there are two strong sources that contribute to the color current :

$$J^\mu \equiv \delta^{\mu+}\delta(x^-)\,\rho_1(\vec{\boldsymbol{x}}_\perp) + \delta^{\mu-}\delta(x^+)\,\rho_2(\vec{\boldsymbol{x}}_\perp)$$

Average over the sources ρ_1, ρ_2 :

$$\langle\mathcal{O}\rangle_Y = \int [D\rho_1]\,[D\rho_2]\,W_{Y_{\rm beam}-Y}\,[\rho_1]\,W_{Y+Y_{\rm beam}}\,[\rho_2]\,\mathcal{O}[\rho_1,\rho_2]$$

Reminder on hydro

Equations of hydrodynamics :

$$\partial_\mu T^{\mu\nu} = 0$$
$$\partial_\mu J^\mu_{\rm B} = 0$$

Additional inputs :

$$p = f(\epsilon) \quad , \quad \eta, \zeta, \cdots$$

- These equations contain only first order time derivatives
- Required initial conditions :

$T^{\mu\nu}(\tau = \tau_0, \eta, \vec{\boldsymbol{x}}_\perp), J^\mu_{\rm B}(\tau = \tau_0, \eta, \vec{\boldsymbol{x}}_\perp)$

CGC initial conditions (Leading Order)

- In the CGC framework, $J^\mu_{\rm B} = 0$
- In the saturation regime, $\rho_{1,2} \sim g^{-1}$, and we have the following expansion for $T^{\mu\nu}$:

$$T^{\mu\nu} = \frac{Q_s^4}{g^2}\left[c_0 + c_1\,g^2 + c_2\,g^4 + \cdots\right]$$

- The Leading Order contribution is given by classical fields :

$$T^{\mu\nu}_{\rm LO} \equiv c_0\frac{Q_s^4}{g^2} = \frac{1}{4}g^{\mu\nu}\,\mathcal{F}^{\lambda\sigma}\mathcal{F}_{\lambda\sigma} - \mathcal{F}^{\mu\lambda}\mathcal{F}^{\nu}{}_{\lambda}$$

$$\text{with}\quad \underbrace{\left[\mathcal{D}_\mu, \mathcal{F}^{\mu\nu}\right] = J^\nu}_{\rm Yang-Mills\ \ equation}\quad,\quad \lim_{t\to-\infty}\mathcal{A}^\mu(t,\vec{\boldsymbol{x}}) = 0$$

- Note : $T^{\mu\nu}_{\rm LO}$ is boost invariant and $P_{\rm L} = 0$

CGC initial conditions (Leading Log)

- The previous power counting implicitly assumes that the coefficients c_n are numbers of order one. However, they contain (possibly large) logarithms of $1/x_{1,2}$:

$$c_1 = d_{10} + d_{11} \ln\left(\frac{1}{x_{1,2}}\right)$$

$$c_2 = d_{20} + d_{21} \ln\left(\frac{1}{x_{1,2}}\right) + \underbrace{d_{22} \ln^2\left(\frac{1}{x_{1,2}}\right)}_{\text{Leading Log terms}}$$

$$\left\langle T^{\mu\nu}(\tau,\eta,\vec{\boldsymbol{x}}_\perp)\right\rangle_{\rm LLog} = \int \left[D\rho_1\, D\rho_2\right]\, W_{Y_1}\left[\rho_1\right]\, W_{Y_2}\left[\rho_2\right]\, \underbrace{T^{\mu\nu}_{\rm LO}(\tau,\vec{\boldsymbol{x}}_\perp)}_{\text{for fixed }\rho_{1,2}}$$

with $\partial_Y W = \mathcal{H} W$, $Y_1 = Y_{\rm beam} - \eta$, $Y_2 = Y_{\rm beam} + \eta$

(FG, Lappi, Venugopalan (2008))

- Note : $T^{\mu\nu}$ is now rapidity dependent, but still $P_{\rm L} = 0$

Importance of factorization

- A factorization formula divides an observable into a perturbatively calculable part and a non-perturbative part describing the partonic content of nuclei :

$$\langle \mathcal{O} \rangle = \int F_1 \otimes F_2 \otimes \mathcal{O}_{\text{partonic}}$$

- QCD has no predictive power, unless :

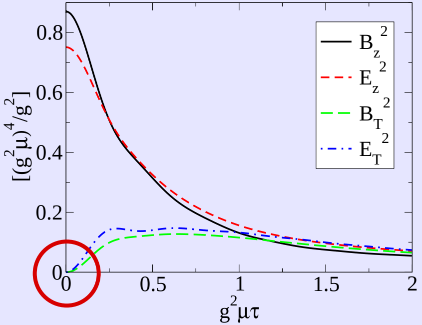
- F does not depend on the observable
- F of one projectile does not depend on the second projectile

- Factorization goes along with resummations :
  - Loop corrections generate corrections like $[g^2 \log(x^{-1})]^n$
  - Universality requires that these logs are resummed by the same evolution equation (JIMWLK in the saturation regime)
  - The summation of these logs drives the x dependence of F

Initial classical fields, Glasma

Lappi, McLerran (2006)

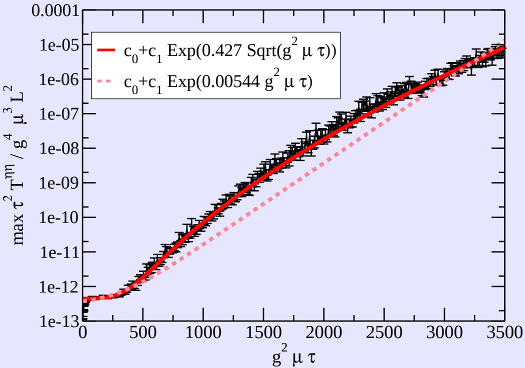
- Immediately after the collision, the chromo-$\vec{E}$ and $\vec{B}$ fields are purely longitudinal and boost invariant :



Unstable fluctuations

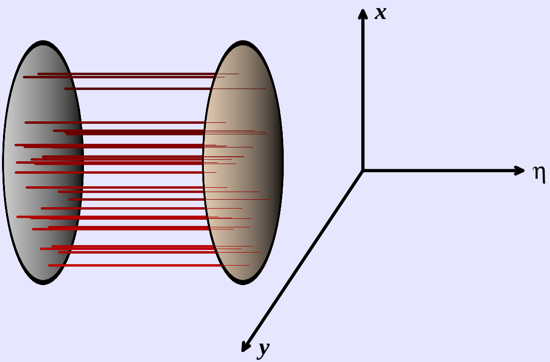
Romatschke, Venugopalan (2005)

- Rapidity dependent perturbations to the classical fields grow like $\exp(\sqrt{\mu\tau})$ $(\mu \sim Q_s)$ until the non-linearities become important :



Unstable fluctuations

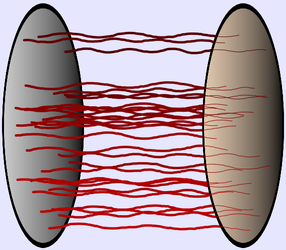
- Leading order magnetic fields at $\tau = 0^+$:



- At $\tau = 0^+$, the classical chromo-electric and chromo-magnetic fields are longitudinal
- They are also boost invariant (independent of η)

Unstable fluctuations

- Leading order + quantum fluctuations at $\tau = 0^+$:



- Loop corrections bring quantum fluctuations in this picture
- In the weak coupling regime, they are small corrections

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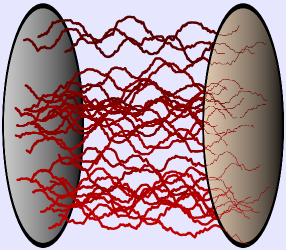
Unstable fluctuations

- Effect of the instability :



- η-dependent perturbations grow quickly in time
- Breakdown of the CGC approach at $\tau_{\rm max} \sim Q_s^{-1}\ln^2(g^{-2})$?
- Outcome : disordered configurations of color fields

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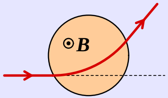
Anomalous transport

Asakawa, Bass, Muller (2006)

- Assume that $\alpha_s = \dfrac{g^2}{4\pi} \ll 1$
- Consider a domain of size Q_s^{-1}, in which the magnetic field is uniform and large, of order $B \sim Q_s^2/g$
- Let a particle of energy $E \sim Q_s$ go through this domain

$$\frac{d\vec{\boldsymbol{p}}}{dt} = g\,\vec{\boldsymbol{v}} \times \vec{\boldsymbol{B}} \quad \Rightarrow \quad \dot{\theta} = \frac{gB}{E} \sim Q_s$$

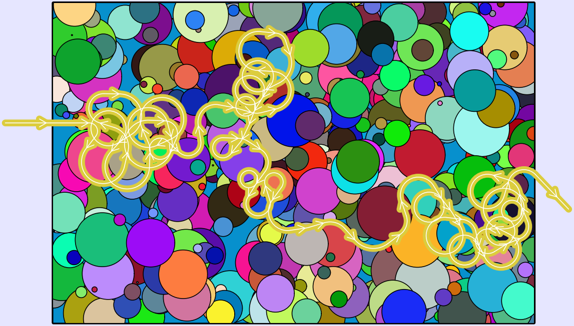
time spent in the domain : $\delta\tau \sim Q_s^{-1}$



▷ The Lorenz force deflects its trajectory by an angle of order unity

Anomalous transport

- Consider now a region filled with such domains, with random orientations for the magnetic field in each domain



▷ In such a medium, the mean free path of a particle of energy Q_s is of order Q_s^{-1}, i.e. as low as permitted by the uncertainty principle ▷ fast thermalization?

Resummation of the unstable terms

- To go beyond the time $\tau_{\rm max}$, one must resum all the fastest growing instabilities $\sim [g^2 e^{\sqrt{\mu\tau}}]^n$

- Summing both the large logs of $1/x_{1,2}$ and these unstable terms, we get (FG, Lappi, Venugopalan (2008)) :

$$\big\langle T^{\mu\nu}(\tau,\eta,\vec{\boldsymbol{x}}_\perp)\big\rangle \underset{\substack{\rm LLog\\ \rm resummed}}{=} \int \big[D\rho_1\,D\rho_2\big]\; W_{Y_1}[\rho_1]\,W_{Y_2}[\rho_2] \times \int \big[Da\big]\; F[a]\;\; T^{\mu\nu}_{\rm LO}[\underbrace{\mathcal{A}+a}_{\text{initial field}}]$$

- $F[a]$ is the spectrum of the initial color field fluctuations. It can be calculated analytically (in some approximation) (Fukushima, FG, McLerran (2006))

Initial correlations matter for hydro

- The equations of hydrodynamics are non-linear. Therefore, running hydro for event averaged initial conditions is not the same as running hydro event-by-event, and averaging observables at the end :

$$\mathrm{HYDRO}\Big[\Big\langle T^{\mu\nu}_{\rm init}\Big\rangle\Big] \neq \Big\langle \mathrm{HYDRO}\Big[T^{\mu\nu}_{\rm init}\Big]\Big\rangle$$

- To do hydro event by event, one needs an event generator for $T^{\mu\nu}(\tau_0,\eta,\vec{\boldsymbol{x}}_\perp)$

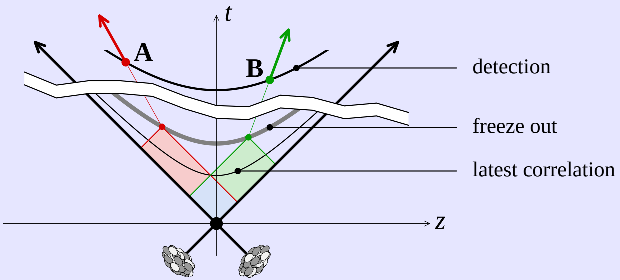
- To achieve this, it is not sufficient to know the average $\Big\langle T^{\mu\nu}(\tau_0,\eta,\vec{\boldsymbol{x}}_\perp)\Big\rangle$. We also need :

$$\Big\langle T^{\mu_1\nu_1}(\tau_0,\eta_1,\vec{\boldsymbol{x}}_{1\perp})T^{\mu_2\nu_2}(\tau_0,\eta_2,\vec{\boldsymbol{x}}_{2\perp})\Big\rangle$$

$$\Big\langle T^{\mu_1\nu_1}(\tau_0,\eta_1,\vec{\boldsymbol{x}}_{1\perp})T^{\mu_2\nu_2}(\tau_0,\eta_2,\vec{\boldsymbol{x}}_{2\perp})T^{\mu_3\nu_3}(\tau_0,\eta_3,\vec{\boldsymbol{x}}_{3\perp})\Big\rangle$$

$$\cdots$$

Initial rapidity correlations matter by themselves



Long range rapidity correlations are created early

From causality, the latest time at which a correlation between two particles can be created is :

$$t_{\text{correlation}} \le t_{\text{freeze out}} \;\; e^{-\frac{1}{2}|y_A - y_B|}$$

With $t_{\text{freeze out}} = 10$ fm/c, $|y_A - y_B| = 6$: $\;\; t_{\text{correlation}} \le 0.5$ fm/c

Sources of correlations

- Nucleons inside a nucleus are correlated by the binding nuclear forces

  ▷ Monte-Carlo generator for the positions of the nucleons inside the nuclei (this information enters in the initial condition for the JIMWLK equation)

- The color sources inside the nucleons acquire correlations when they evolve to smaller values of x

  ▷ two color sources that resulted from the splitting of a common ancestor are correlated. These correlations are included in the solution $W_Y[\rho]$ of the JIMWLK equation

Correlations in $\vec{x}_\perp$

- The factorization valid for $\langle T^{\mu\nu}\rangle$ can be extended to correlations between points in the transverse plane :

$$\left\langle T^{\mu_1\nu_1}(\tau,\eta,\vec{x}_{1\perp})\cdots T^{\mu_n\nu_n}(\tau,\eta,\vec{x}_{n\perp})\right\rangle_{\rm LLog} =$$
$$= \int \big[D\rho_1\, D\rho_2\big]\; W_{Y_1}\,[\rho_1]\; W_{Y_2}\,[\rho_2]$$
$$\times\; T^{\mu_1\nu_1}_{\rm LO}(\tau,\vec{x}_{1\perp})\cdots T^{\mu_n\nu_n}_{\rm LO}(\tau,\vec{x}_{n\perp})$$

with $Y_1 = Y_{\rm beam} - \eta$ and $Y_2 = Y_{\rm beam} + \eta$

- Numerically, this is not more difficult than calculating $\langle T^{\mu\nu}\rangle$ itself
- This formula cannot be applied to correlations between points separated by $\alpha_s^{-1} \lesssim \Delta\eta$ (new large logarithmic corrections exist for large $\Delta\eta$'s)

Numerical state of the art

- Solution of the Yang-Mills equations :
  Krasnitz, Nara, Venugopalan (1999-2003)
  Lappi (2003)
  Romatschke, Venugopalan (2005) (in 3-dimensions)

- Solution of the JIMWLK equation for $W_Y[\rho]$:
  Rummukainen, Weigert (2004) (not reproduced since then, very heavy computationnally)

- Solution of JIMWLK in the mean field approximation :
  ▷ Balitsky-Kovchegov equation
  Many independent codes exist for solving the BK equation

- Merging the solutions of BK and of Yang-Mills equations in order to obtain the rapidity dependence of $\langle T^{\mu\nu}\rangle$ is fairly straightforward (Lappi, Venugopalan (in progress))

- Average over the spectrum of initial fluctuations : work in progress, complicated by strong ultraviolet divergences

Limitations to keep in mind

- All this is based so far on Leading Log calculations

  ▷ as usual, the K-factor for going from LLog to NLLog can be rather large. Expect $K \sim 2$

  ▷ some efforts are ongoing to extend the JIMWLK evolution equation to NLLog (Kovchegov, Weigert, Balitsky,...), but that's only half of the story

- As with any evolution equation, the initial condition (for $W[\rho]$) is not predicted by pQCD

  ▷ it must be constrained by data. *eA* collisions at an EIC would be the best. *pA* collisions is the next best thing

- The calculation of long range rapidity correlations is not yet on a par with the rest of the framework

  ▷ Missing : resummation of the large corrections in $(\alpha_s \Delta Y)^n$ when the rapidity difference is $\Delta Y \geq \alpha_s^{-1}$

Instabilities, fluctuations and thermalization

- Why is it difficult to resum the initial fluctuations ?
  - The spectrum of these fluctuations contains arbitrarily high momentum modes (origin : these fluctuations amount to loop corrections, in which the momentum that circulates has no upper bound)
  - The energy momentum tensor has dimension $(\mathrm{momentum})^4$. It diverges like the power four of the hardest momentum in the fluctuation spectrum

- Strategy 1 : put a cutoff $\Lambda \sim Q_s$ by hand

- Strategy 2 : do all the calculations with a cutoff Λ. Try to perform the cancellation of the UV divergences numerically
  - ▷ delicate cancellations between large numbers

- Strategy 3 : work out the structure of the UV divergences analytically, in order to get finite analytical expressions, that are then evaluated numerically

Initial time (in)dependence

- The CGC provides the value of $T^{\mu\nu}$ at some initial time τ_0
- Hydrodynamics is used to describe the system at $\tau \geq \tau_0$
- Some observable $\mathcal{O}$ is measured at a late time τ_f
- The value of $\mathcal{O}(\tau_f)$ should be independent of the time τ_0 at which one switches from the CGC description to the hydro description (at least in a reasonable window)
- Analogy : τ_0 is similar to the factorization scale used in fragmentation functions

CGC	$\longleftrightarrow$	hard cross section
Hydro	$\longleftrightarrow$	fragmentation function

▷ Varying the factorization scale (i.e. τ_0) and checking the effect of this variation on the final result is a good way of assessing the uncertainty of the whole chain

Summary

Things that work right now: Leading Log framework

- Evolution equations for the distribution of sources (JIMWLK, BK)
- Initial condition (some constraints from DIS and RHIC)
- $\big\langle T^{\mu\nu}(\tau,\eta,\vec{\boldsymbol{x}}_\perp)\big\rangle$
- $\big\langle T^{\mu\nu}(\tau,\eta,\vec{\boldsymbol{x}}_\perp)T^{\rho\sigma}(\tau,\eta,\vec{\boldsymbol{y}}_\perp)\cdots\big\rangle$ at the same η

Things that need immediate work

- Resummation of the unstable modes (to get longitudinal P)

Things that need to be looked at eventually

- Independence w.r.t. the time at which one starts hydro
- Long range rapidity correlations
- Next to Leading Log