

1 Light Cone Coordinates: Definitions, Identities

A four-vector is not bold-faced (e.g. p, k), a three-vector is bold-faced with a vector symbol (e.g. $\vec{\mathbf{p}}, \vec{\mathbf{k}}$), and a transverse two-vector is bold-faced without a vector symbol (e.g. \mathbf{p}, \mathbf{k}). Minkowski four-vectors are written with parentheses, $()$; light-cone four-vectors with brackets, $[\]$.

$$p = (p^0, p^z, \mathbf{p}) = [p^+, p^-, \mathbf{p}]. \quad (1)$$

We will use non-symmetrized lightcone coordinates:

$$p^+ = p^0 + p^z \quad (2)$$

$$p^- = p^0 - p^z \quad (3)$$

$$\mathbf{p} = \mathbf{p}. \quad (4)$$

The inverse transformation is then

$$p^0 = \frac{1}{2}(p^+ + p^-) \quad (5)$$

$$p^z = \frac{1}{2}(p^+ - p^-) \quad (6)$$

$$\mathbf{p} = \mathbf{p}. \quad (7)$$

The Minkowski dot product in lightcone coordinates is:

$$p \cdot k = p^0 k^0 - p^z k^z - \mathbf{p} \cdot \mathbf{k} = \frac{1}{2}(p^+ k^- + p^- k^+) - \mathbf{p} \cdot \mathbf{k}. \quad (8)$$

The length of a vector using lightcone coordinates is then:

$$p \cdot p = p^+ p^- - \mathbf{p} \cdot \mathbf{p}. \quad (9)$$

2 Derivation of WHDG Kinematic Limits

First, see Fig. 1 for notation. For a massless parent parton of momentum P , a massless radiated gluon of momentum k , and a final massless parent parton momentum of p we have that

$$P = (E, E, 0, 0) = [E^+, 0, 0], \quad (10)$$

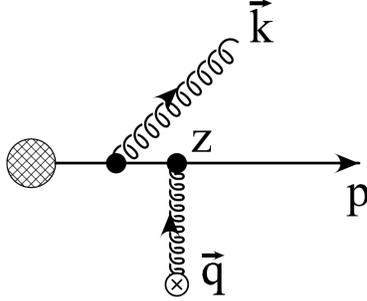


Figure 1: One of the diagrams contributing to the first order in opacity matrix element. \mathbf{q} is the momentum transfer between the parent parton and the in-medium scattering center. \mathbf{k} is the momentum carried off by the radiated gluon. z is the distance from the hard production vertex of the parent parton and the scattering center. Note that the parent parton emerges with momentum P from the blob on the left. Figure adapted from Djordjevic and Gyulassy, Nucl.Phys.A733:265-298, 2004.

where $(,)$ denote the usual 4-momenta, $[,]$ denote light-cone momenta, and we choose the normalization between the two as $E^+ = 2E$. Taking x to be the fraction of plus momentum carried away by the radiated gluon then

$$k = [xE^+, \frac{\mathbf{k}_\perp^2}{xE^+}, \mathbf{k}_\perp] \quad (11)$$

$$p = [(1-x)E^+, \frac{(\mathbf{q}_\perp - \mathbf{k}_\perp)^2}{(1-x)E^+}, \mathbf{q}_\perp - \mathbf{k}_\perp]. \quad (12)$$

The assumption of eikonality requires that the parent parton continues essentially along its original path. This clearly implies $p^+ \gg p^-$; i.e. that radiating a gluon doesn't make the parent parton go backwards. Similarly we require that the radiated gluon go in the forward direction, $k^+ \gg k^-$; i.e. radiative energy loss does not lead to an energy gain. The first of these conditions implies that ($k_\perp = |\mathbf{k}_\perp|$)

$$(1-x)E^+ \gg |\mathbf{q}_\perp - \mathbf{k}_\perp| \approx k_\perp, \quad (13)$$

where we note that $q_{max} = \sqrt{6ET} \ll k$ for $E \gg T$; the second condition that

$$xE^+ \gg k_\perp. \quad (14)$$

Taking the k_\perp integral cutoff to occur precisely at equality for these

conditions leads to

$$k_{max} = \min(x, 1 - x)E^+ \quad (15)$$

$$= 2 \min(x, 1 - x)E \quad (16)$$

$$\approx 2x(1 - x)E, \quad (17)$$

where the last line is used in the WHDG implementation for convergence reasons and makes little difference in the final result (see the last TECHQM meeting).

3 On Mass Shell Gluon

First note that Eq. (11) implies that the massless gluon is always on shell,

$$k \cdot k = xE^+ \frac{\mathbf{k}_\perp^2}{xE^+} - \mathbf{k}_\perp \cdot \mathbf{k}_\perp = 0. \quad (18)$$

Specifically, if we invert back to 4-momenta the inner product $k \cdot k = 0$ is preserved for all x , \mathbf{k}_\perp , and E (as it must be):

$$k = \left(\frac{1}{2} \left(xE^+ + \frac{\mathbf{k}_\perp^2}{xE^+} \right), \frac{1}{2} \left(xE^+ - \frac{\mathbf{k}_\perp^2}{xE^+} \right), \mathbf{k}_\perp \right) \quad (19)$$

$$\begin{aligned} \therefore k \cdot k &= \frac{1}{4} \left((xE^+)^2 + 2k_\perp^2 + \frac{k_\perp^2}{(xE^+)^2} \right) \\ &\quad - \frac{1}{4} \left((xE^+)^2 - 2k_\perp^2 + \frac{k_\perp^2}{(xE^+)^2} \right) - k_\perp^2 = 0. \end{aligned} \quad (20)$$

We are of course done. Nevertheless Urs specifically claimed that when $k_\perp = k_{max}$ then $\omega = 2xE = 2\omega$. This apparent contradiction (that the gluon **appears** to be off shell) is due to naïvely taking $\omega = k^+/2$. While this is valid when $E^+ \rightarrow \infty$ keeping k_\perp fixed, this assumption is not valid near the cutoff $k_\perp \sim xE^+$. Plugging $k_{max} = xE^+$ into Eq. (19) yields

$$k \xrightarrow{k_\perp \rightarrow k_{max}} \left(xE^+, 0, xE^+ \frac{\mathbf{k}_\perp}{k_\perp} \right). \quad (21)$$

Clearly we have $\omega = k_{max}$ and $k \cdot k = 0$ when $k_\perp = k_{max}$.

Ultimately the matrix element calculation assumes eikonality; i.e. $k_\perp \ll xE^+$. Unfortunately the derived integrand doesn't know about this assumption, and it has nonzero support for $k_\perp \gtrsim xE^+$. WHDG imposes this assumption onto the k_\perp integration by cutting off when equality between the

quantities is reached, i.e. $k_{max} = xE^+ = 2xE$. There is clearly at least an $\mathcal{O}(1)$ multiplicative uncertainty in the determination of k_{max} that will result, at the current level of approximation and as described in the note, in a large systematic theoretical uncertainty in the calculation of any physical observable.