In the Hard Thermal Loop formalism, the following useful relations can be derived [1, 2].

1 Plasma properties

For a gluon gas:

$$\mu = m_D = gT = \sqrt{4\pi\alpha_s}T. \tag{1}$$

Density N, ρ:

$$N = \frac{6}{\pi^2}\zeta(3)T^3 \tag{2}$$
$$\rho = \frac{8}{9}C_A N \tag{3}$$

One way to calculate

$$\hat{q} = \int d^q_\perp \frac{d\Gamma_{el}}{d^2q_\perp} q_\perp^2, \tag{4}$$

is using the high-energy limit:

$$\frac{d\Gamma_{el}}{d^2q_\perp} = \frac{C_R}{(2\pi)^2}\frac{g^4 N}{q_\perp^4}, \tag{5}$$

and cutting the integral off at m_D

$$\hat{q} = \int_{m_D}^{q_{max}} d^2q_\perp \frac{C_R}{(2\pi)^2}\frac{g^4N}{q_\perp^4} q_\perp^2 = 4\pi C_R\alpha^2 N \ln(q_{max}^2/m_D^2). \tag{6}$$

For GLV, we also need to calculate $1/\lambda$. Taking

$$\frac{1}{\lambda} = \int d^q_\perp \frac{d\Gamma_{el}}{d^2q_\perp} \tag{7}$$

gives a 'nice' result if we take the integration to infinity

$$\frac{1}{\lambda} = \int_{m_D}^{\infty} d^2q_\perp \frac{C_R}{(2\pi)^2}\frac{g^4N}{q_\perp^4} = 4\pi C_R\alpha^2 N\frac{1}{m_D^2} = \frac{9\pi\alpha^2}{2m_D^2}\rho. \tag{8}$$

Note that the value now depends entirely on the IR cut-off. That's probably not a good situation. What value of $1/\lambda$ should really enter in the GLV formalism? Note, by the way, that the above result leads to the same transport (?) cross section $\sigma = 1/\lambda\rho$ as used by Baier [3].

Comparing Eqs 6 and 8, we see the following relation between $\hat{q}$, $\mu = m_D$ and λ

$$\hat{q} = \frac{m_D^2}{\lambda}\ln(q_{max}^2/m_D^2). \tag{9}$$

This looks reasonable, but it should be noted that **different upper integration limits** were used for λ and $\hat{q}$.

2 Further questions

If we decide to use the equations above, what is a good value for q_{max}? Common choices seem to be $q_{max} = 2\sqrt{ET}$ and $q_{max} = 3\sqrt{ET}$.

Various refinements can be made to Eq. 6, as detailed in [1], which may change the relation between $\hat{q}$ and T by a factor 1.5. Should we pursue any of those and/or introduce an uncertainty band related to a variation from the simple ansatz to more complete results?

References

[1] P. Arnold and W. Xiao, Phys. Rev. D **78**, 125008 (2008) [arXiv:0810.1026 [hep-ph]].

[2] P. Arnold, private communication

[3] R. Baier and D. Schiff, JHEP **0609**, 059 (2006) [arXiv:hep-ph/0605183].