Top quark cross sections and differential distributions

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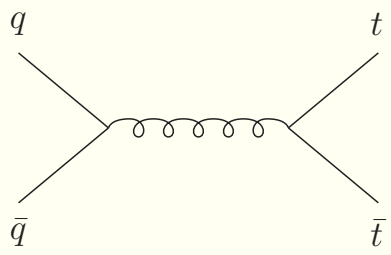
- $t\bar{t}$ and single top production channels
- Higher-order two-loop corrections
- $t\bar{t}$ cross section at Tevatron and LHC
- Top quark p_T and Y distribution at Tevatron and LHC
- t-channel production at Tevatron and LHC
- s-channel production at Tevatron and LHC
- Associated production of a top with a W^- or H^-

Partonic processes at LO

Top-antitop pair production

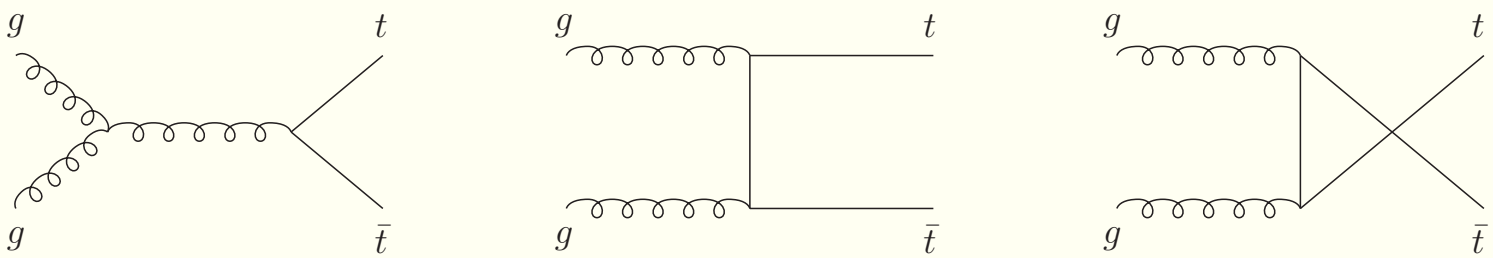
- $q\bar{q} \longrightarrow t\bar{t}$

dominant at Tevatron



- $gg \longrightarrow t\bar{t}$

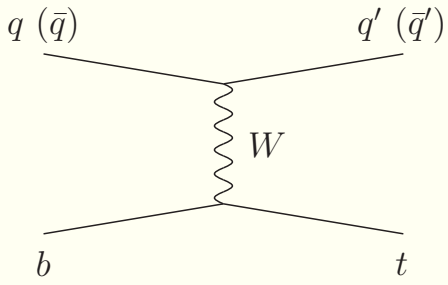
dominant at LHC



Single top quark production

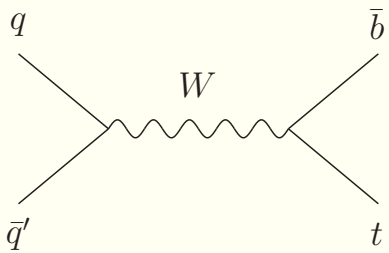
- t **channel:** $qb \longrightarrow q't$ **and** $\bar{q}b \longrightarrow \bar{q}'t$

dominant at Tevatron and LHC



- s **channel:** $q\bar{q}' \longrightarrow \bar{b}t$

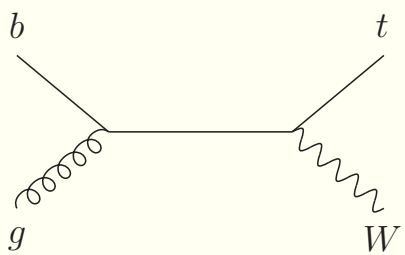
small at Tevatron and LHC

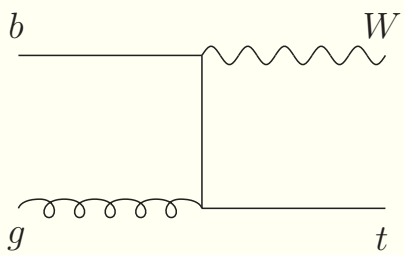


- **associated** tW **production:** $bg \longrightarrow tW^-$

very small at Tevatron, significant at LHC

Related process: $bg \longrightarrow tH^-$





Higher-order corrections

QCD corrections significant for top pair and single top quark production

NLO corrections fully known

Soft-gluon corrections from incomplete cancellations of infrared divergences between virtual diagrams and real diagrams with soft (low-energy) gluons

Soft corrections $\left[\frac{\ln^k(s_4/m^2)}{s_4}\right]_+$ **with** $k \leq 2n-1$ **and** s_4 **distance from threshold**

Soft-gluon corrections are dominant near threshold

Resum (exponentiate) these soft corrections

At NLL accuracy requires one-loop calculations in the eikonal approximation

Recent results at NNLL–two-loop calculations completed

Approximate NNLO cross section from expansion of resummed cross section

Essential ingredient: two-loop soft anomalous dimension

Allows NNLL resummation

Resummed cross section

Resummation follows from factorization properties of the cross section - performed in moment space

Use RGE to evolve function associated with soft-gluon emission

H: **hard-scattering function** S: **soft-gluon function**

$$\hat{\sigma}^{res}(N) = \exp\left[\sum_i E_i(N)\right] \exp\left[\sum_j E'_j(N)\right] \mathrm{tr}\left\{H\left(\alpha_s\right)\right.$$
$$\left.\times \exp\left[\int_{\sqrt{s}}^{\sqrt{s}/\tilde{N}} \frac{d\mu}{\mu} \Gamma_S^\dagger\left(\alpha_s(\mu)\right)\right] S\left(\alpha_s\left(\frac{\sqrt{s}}{\tilde{N}}\right)\right) \exp\left[\int_{\sqrt{s}}^{\sqrt{s}/\tilde{N}} \frac{d\mu}{\mu} \Gamma_S\left(\alpha_s(\mu)\right)\right]\right\}$$

where

Γ_S **is the soft anomalous dimension - a matrix in color space**

and a function of kinematical invariants s, t, u

Calculate Γ_S **in eikonal approximation**

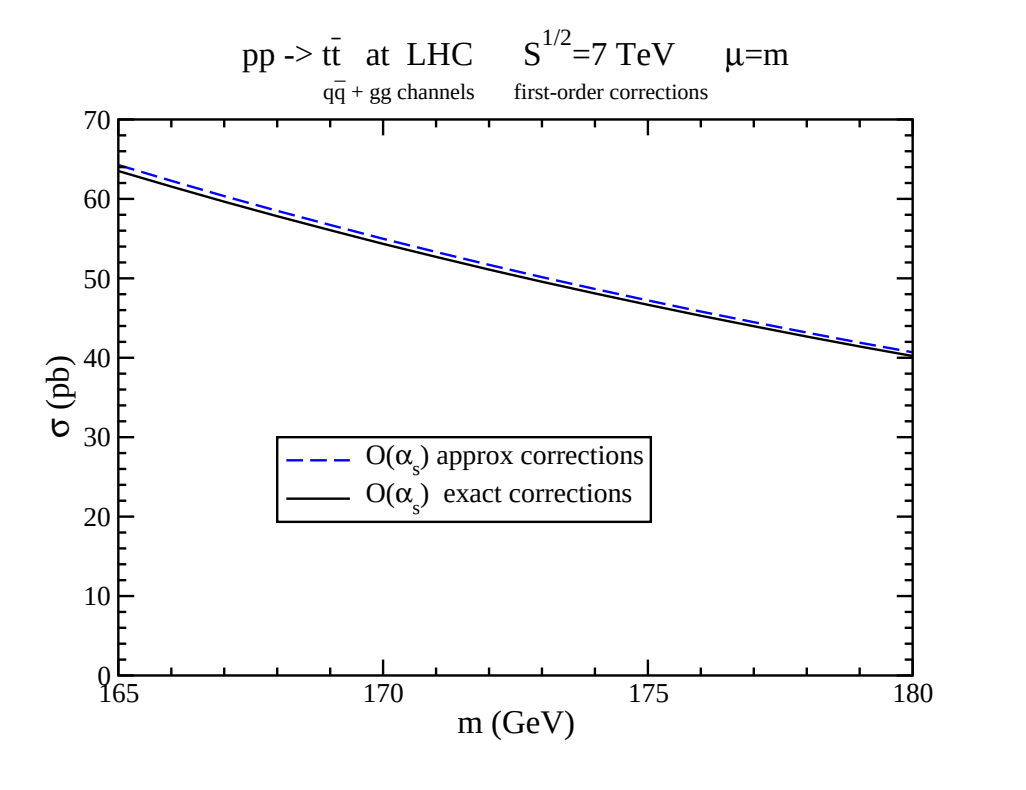
Calculation is at differential cross section level

kinematics refer to partonic threshold (not just absolute threshold)

Threshold approximation

approximate cross sections using differential-level 1PI kinematics for partonic threshold

approximation works very well not only for Tevatron but also for LHC energies because partonic threshold is still important



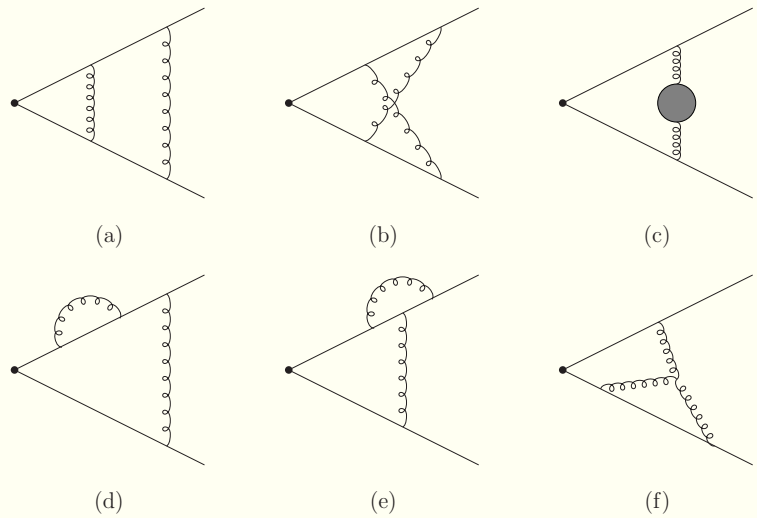
only 1% difference between first-order approximate and exact corrections

→ less than 1% difference between NLO approximate and exact cross sections

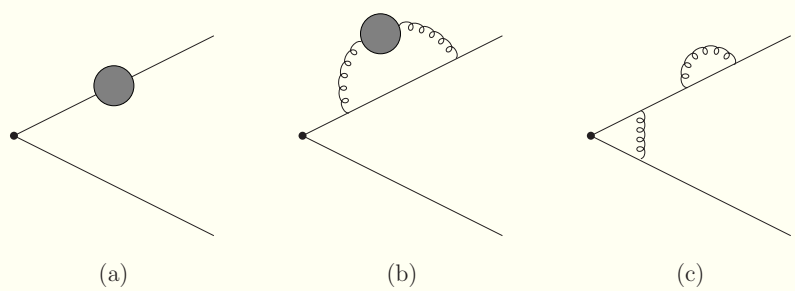
For best prediction add NNLO approximate corrections to exact NLO cross section

Two-loop soft (cusp) anomalous dimension

Vertex correction graphs



Heavy-quark self-energy graphs



N. Kidonakis, Phys. Rev. Lett. 102, 232003 (2009), arXiv:0903.2561 [hep-ph]

Top-antitop production in hadron colliders

The soft anomalous dimension matrix for $q\bar{q} \to t\bar{t}$ is

$$\Gamma_{S\,q\bar{q}} = \begin{bmatrix} \Gamma_{q\bar{q}\,11} & \Gamma_{q\bar{q}\,12} \\ \Gamma_{q\bar{q}\,21} & \Gamma_{q\bar{q}\,22} \end{bmatrix}$$

At one loop

$$\Gamma^{(1)}_{q\bar{q}\,11} = -C_F\left[L_\beta + 1\right] \qquad \Gamma^{(1)}_{q\bar{q}\,21} = 2\ln\left(\frac{u_1}{t_1}\right) \qquad \Gamma^{(1)}_{q\bar{q}\,12} = \frac{C_F}{C_A}\ln\left(\frac{u_1}{t_1}\right)$$

$$\Gamma^{(1)}_{q\bar{q}\,22} = C_F\left[4\ln\left(\frac{u_1}{t_1}\right) - L_\beta - 1\right] + \frac{C_A}{2}\left[-3\ln\left(\frac{u_1}{t_1}\right) + \ln\left(\frac{t_1 u_1}{s m^2}\right) + L_\beta\right]$$

where $L_\beta = \frac{1+\beta^2}{2\beta}\ln\left(\frac{1-\beta}{1+\beta}\right)$ with $\beta = \sqrt{1-4m^2/s}$

Write the two-loop cusp anomalous dimension as $\Gamma^{(2)}_S = \frac{K}{2}\Gamma^{(1)}_S + C_F C_A M_\beta$. Then at two loops

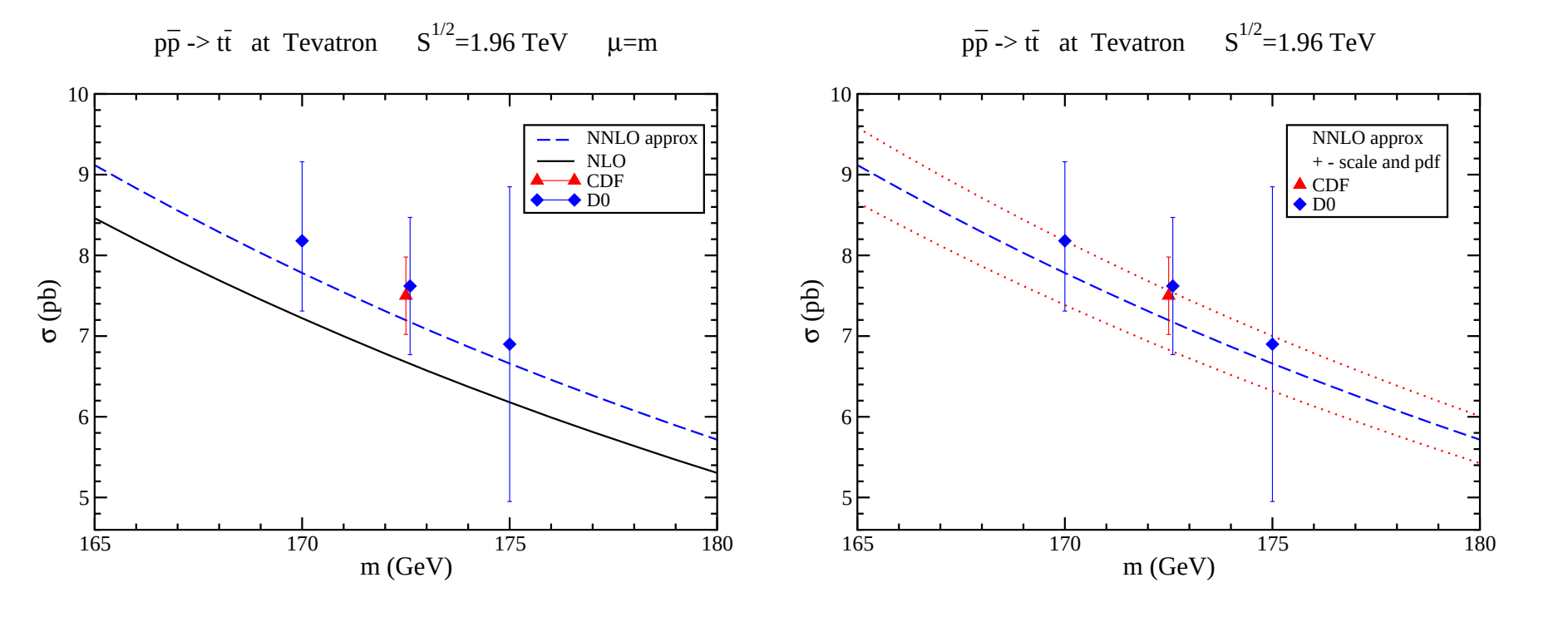
$$\Gamma^{(2)}_{q\bar{q}\,11} = \frac{K}{2}\Gamma^{(1)}_{q\bar{q}\,11} + C_F C_A\, M_\beta \qquad \Gamma^{(2)}_{q\bar{q}\,22} = \frac{K}{2}\Gamma^{(1)}_{q\bar{q}\,22} + C_A\left(C_F - \frac{C_A}{2}\right) M_\beta$$

$$\Gamma^{(2)}_{q\bar{q}\,21} = \frac{K}{2}\Gamma^{(1)}_{q\bar{q}\,21} + C_A N_\beta \ln\left(\frac{u_1}{t_1}\right) \qquad \Gamma^{(2)}_{q\bar{q}\,12} = \frac{K}{2}\Gamma^{(1)}_{q\bar{q}\,12} - \frac{C_F}{2} N_\beta \ln\left(\frac{u_1}{t_1}\right)$$

with N_β a subset of terms of M_β

Similar results for $gg \to t\bar{t}$ channel **N. Kidonakis, Phys. Rev. D 82, 114030 (2010), arXiv:1009.4935 [hep-ph]**

$t\bar{t}$ cross section at the Tevatron



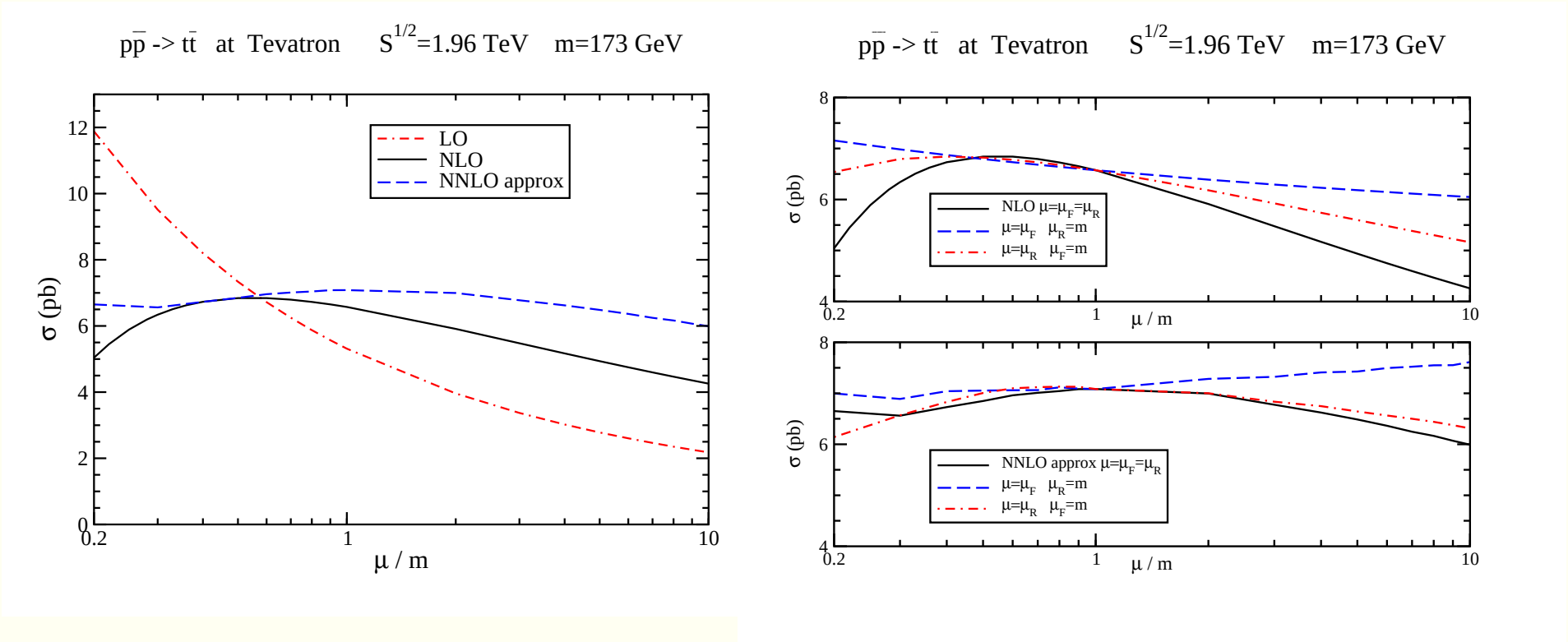
$$\sigma_{t\bar{t}}^{\mathrm{NNLOapprox}}(m_t = 173\,\mathrm{GeV},\ 1.96\,\mathrm{TeV}) \quad = \quad 7.08^{+0.00\,+0.36}_{-0.24\,-0.27}\ \mathrm{pb}$$

scale pdf

NNLO approx: 7.8% enhancement over NLO

used MSTW 2008 NNLO pdf

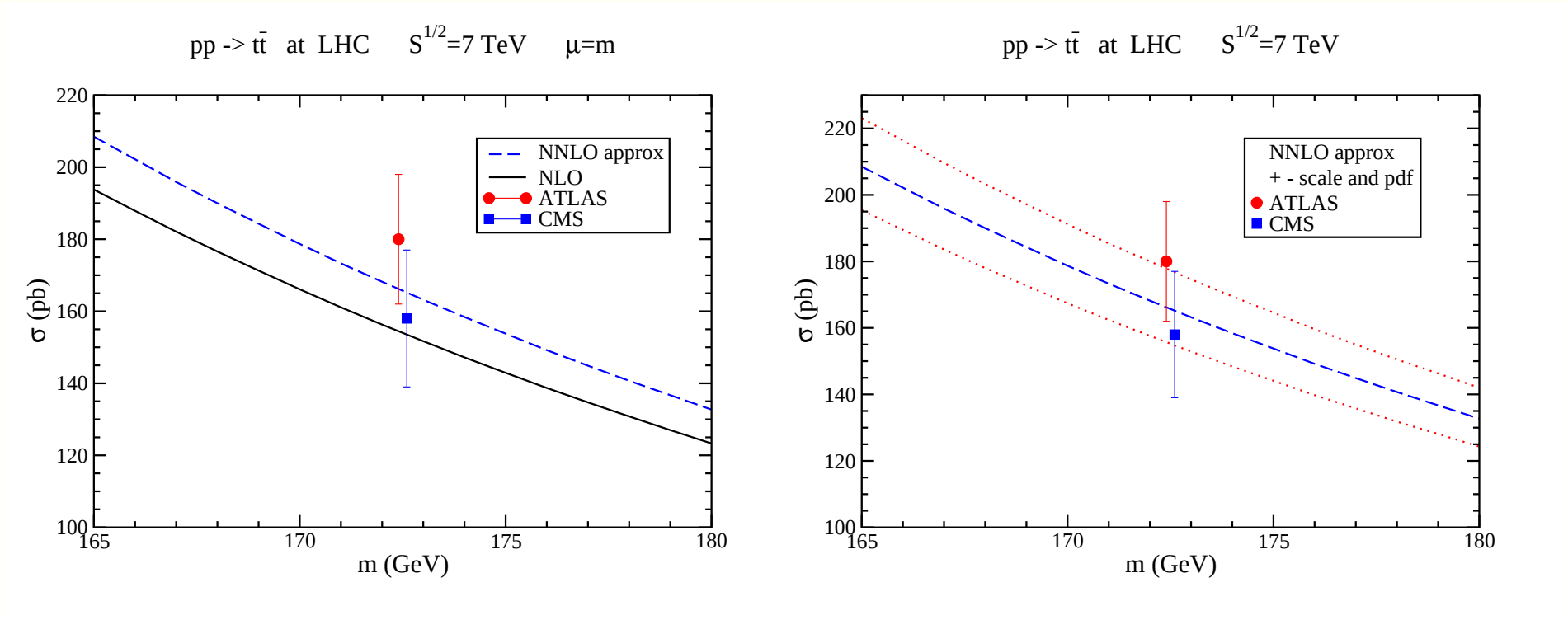
$t\bar{t}$ cross section at the Tevatron



scale dependence greatly reduced over a large range

separate factorization and renormalization scale dependence calculated

$t\bar{t}$ cross section at the LHC

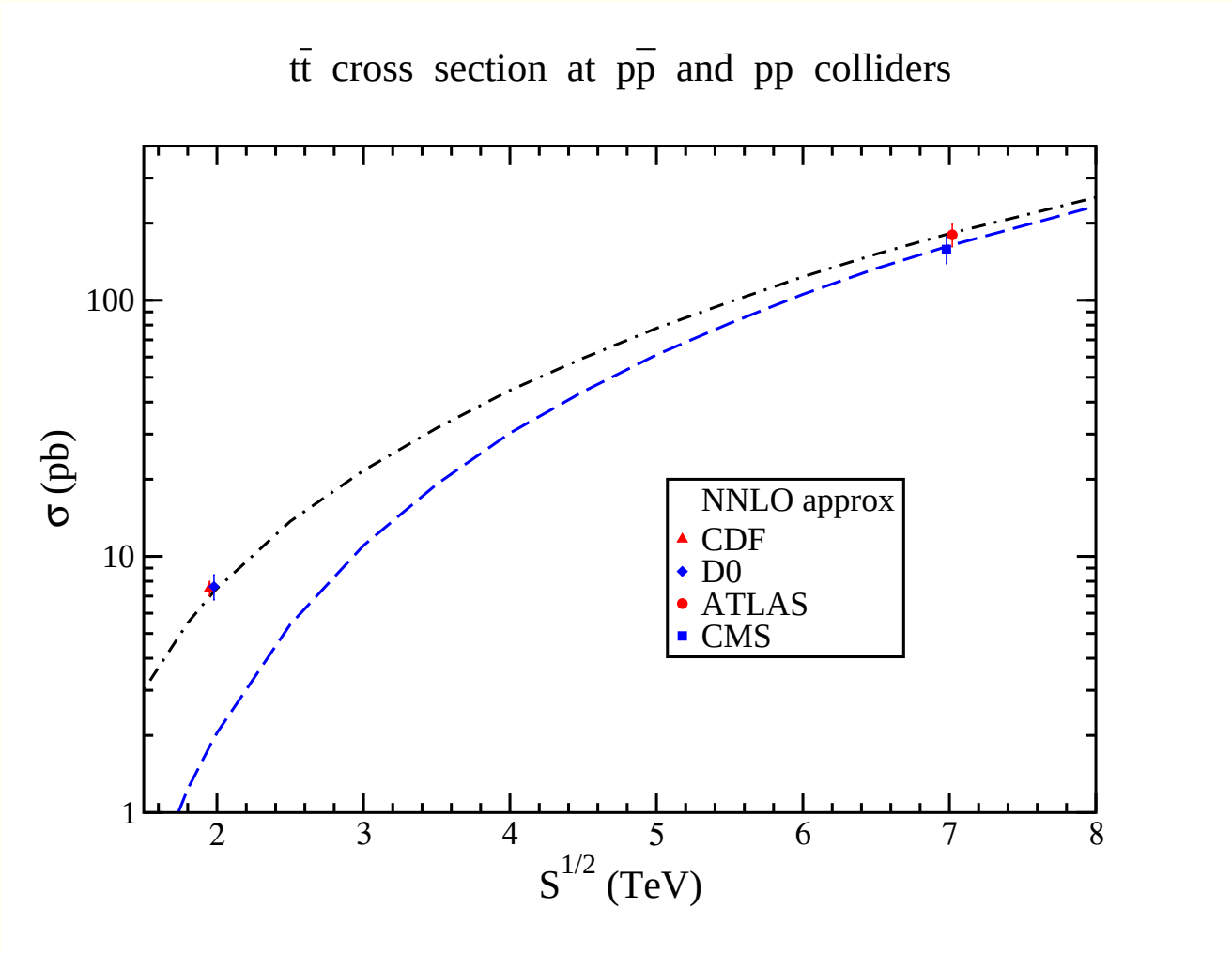


$$\sigma_{t\bar{t}}^{\mathrm{NNLOapprox}}(m_t = 173\,\mathrm{GeV},\, 7\,\mathrm{TeV}) \quad = \quad 163^{+7+9}_{-5-9}\ \mathrm{pb}$$

$$\sigma_{t\bar{t}}^{\mathrm{NNLOapprox}}(m_t = 173\,\mathrm{GeV}, 14\,\mathrm{TeV}) \quad = \quad 920^{+50+33}_{-39-35}\ \mathrm{pb}$$

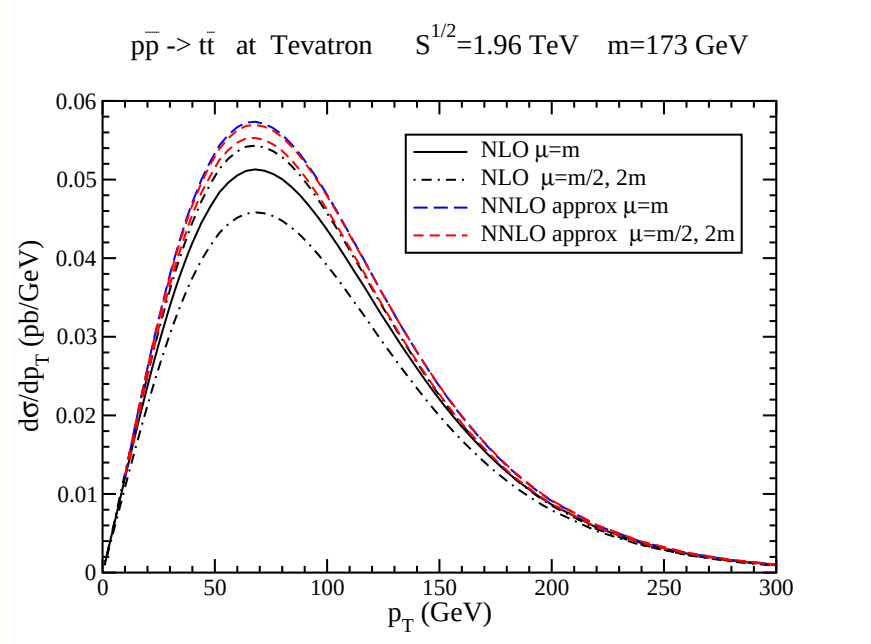
NNLO approx: enhancement over NLO is 7.6% at 7 TeV; 8.0% at 14 TeV

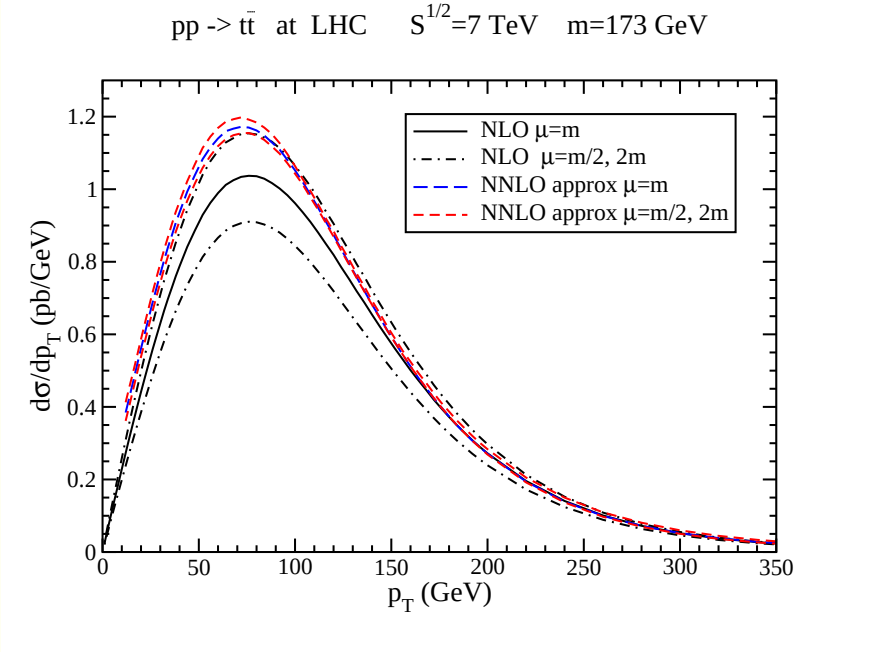
$t\bar{t}$ cross section versus energy



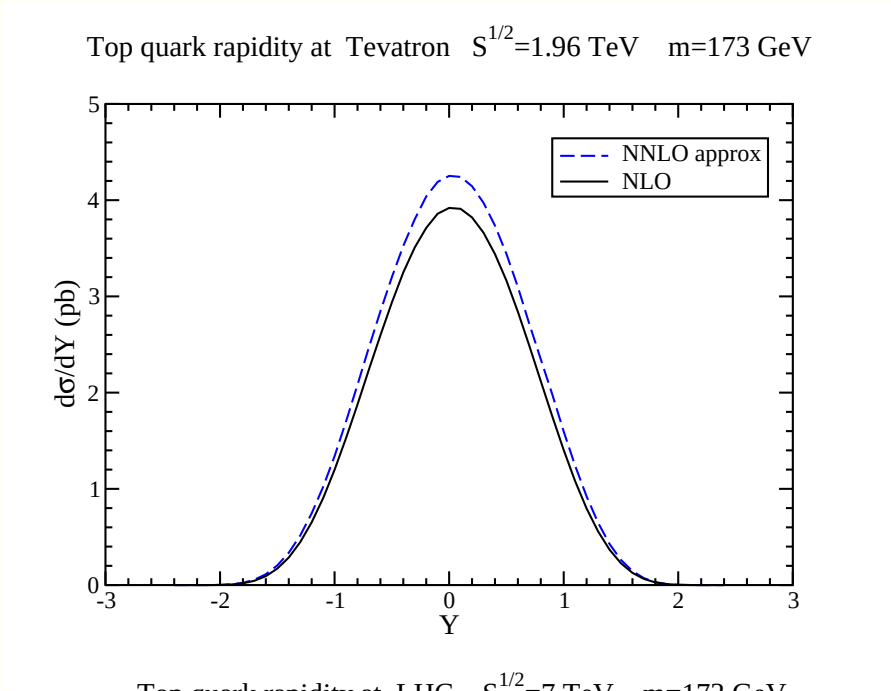
Tevatron and LHC data are consistent with theory

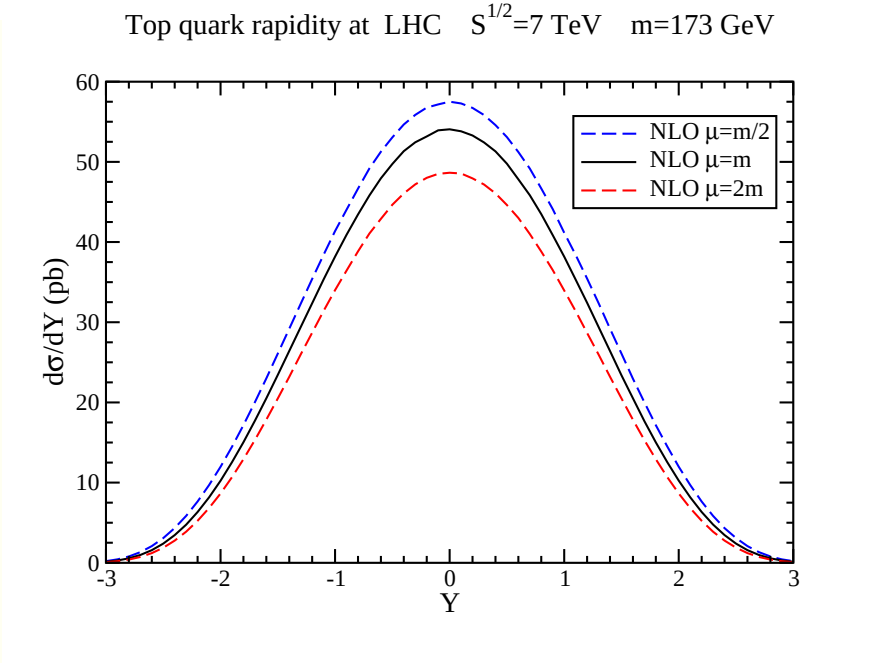
Top quark p_T distribution at Tevatron and LHC





Top quark rapidity distribution at Tevatron and LHC





Single top quark production - t channel

Dominant single top production channel at both Tevatron and LHC energies

Soft anomalous dimension for t-channel single top production

One loop

$$\Gamma^{(1)}_{S\,11} = C_F \left[\ln\left(\frac{-t}{s}\right) + \ln\left(\frac{m_t^2 - t}{m_t\sqrt{s}}\right) - \frac{1}{2} \right]$$

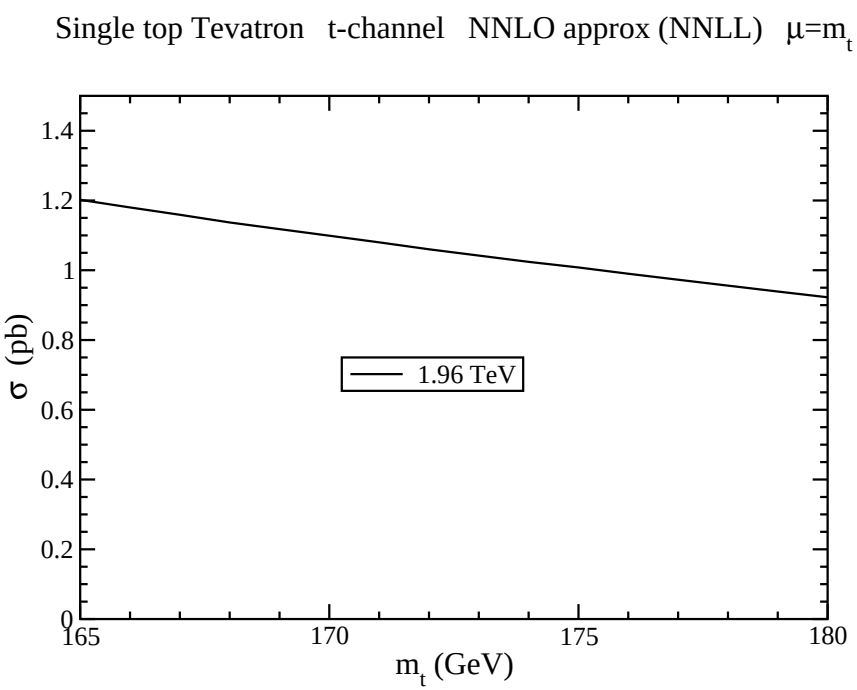
$$\Gamma^{(1)}_{S\,21} = \ln\left(\frac{u(u-m_t^2)}{s(s-m_t^2)}\right) \qquad \Gamma^{(1)}_{S\,12} = \frac{C_F}{2N_c}\,\Gamma^{(1)}_{S\,21}$$

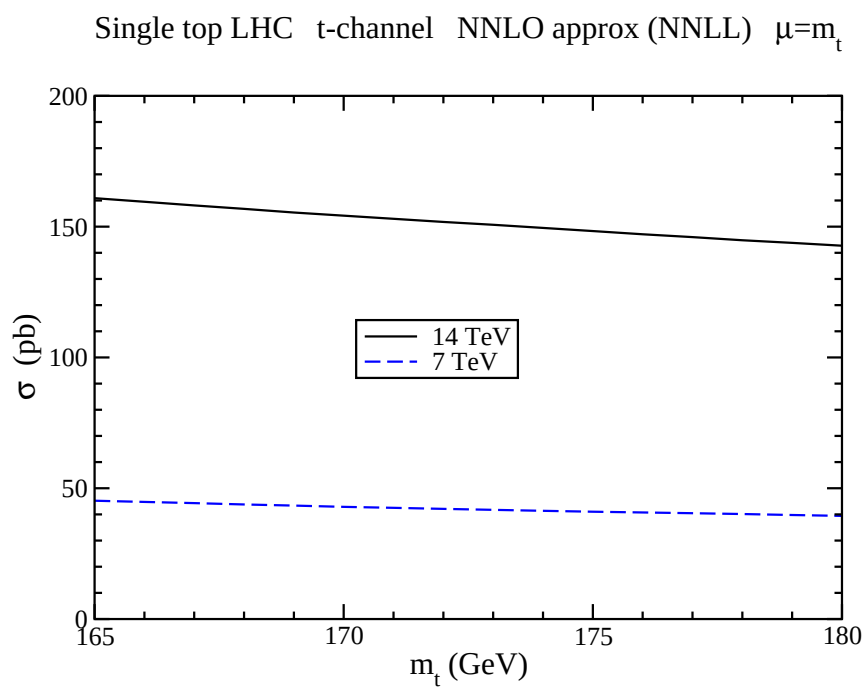
Two loops

$$\Gamma^{(2)}_{S\,11} = \frac{K}{2}\Gamma^{(1)}_{S\,11} + C_F C_A \frac{(1-\zeta_3)}{4}$$

N. Kidonakis, Phys. Rev. D (in press), arXiv:1103.2792 [hep-ph]

Single top quark production at Tevatron and LHC - t channel





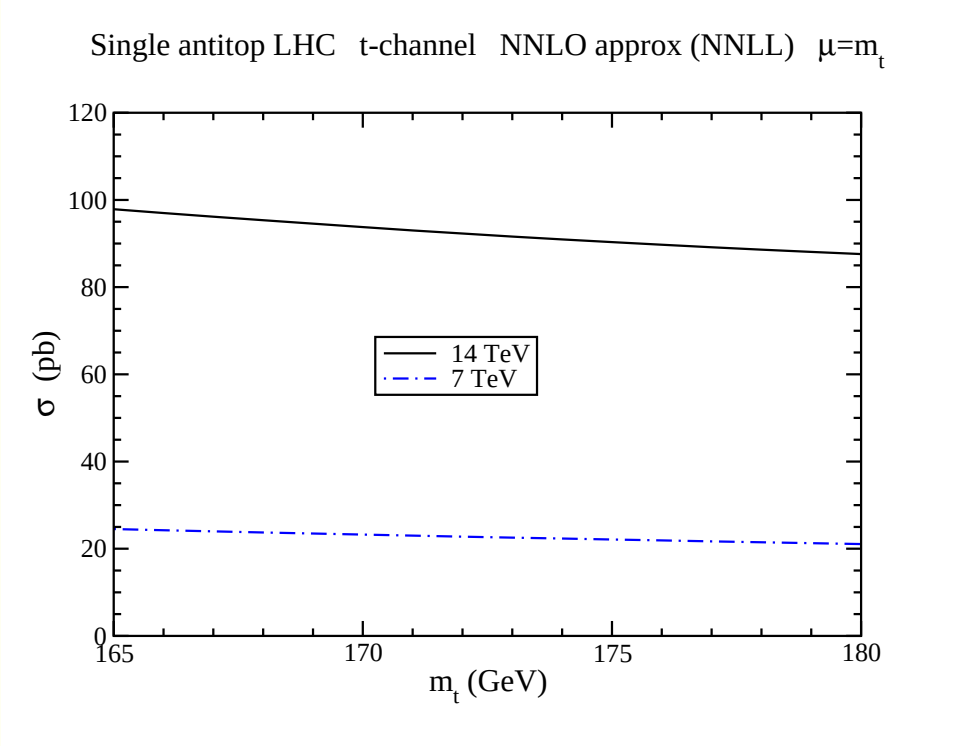
$$\sigma^{\text{NNLOapprox, top}}_{t-\text{channel}}(m_t = 173\,\text{GeV}, 1.96\,\text{TeV}) = 1.04^{+0.00}_{-0.02} \pm 0.06\ \text{pb}$$

$$\sigma^{\text{NNLOapprox, top}}_{t-\text{channel}}(m_t = 173\,\text{GeV}, 7\,\text{TeV}) = 41.7^{+1.6}_{-0.2} \pm 0.8\ \text{pb}$$

$$\sigma^{\text{NNLOapprox, top}}_{t-\text{channel}}(m_t = 173\,\text{GeV}, 14\,\text{TeV}) = 151^{+4}_{-1} \pm 3\ \text{pb}$$

NNLO approx: 4% increase at Tevatron; 1% decrease at 7 TeV; 3% decrease at 14 TeV relative to NLO

Single antitop production at LHC - t channel

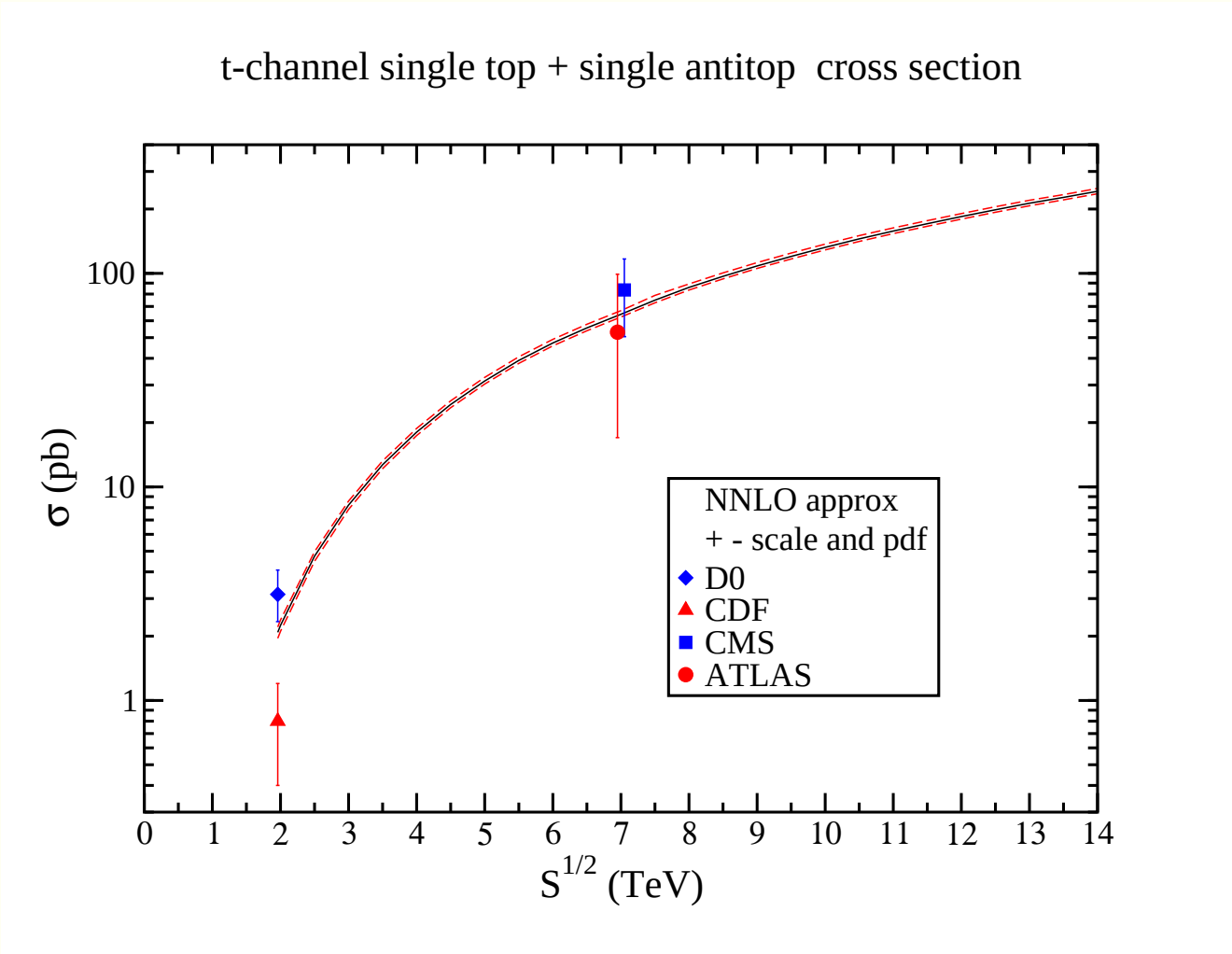


$$\sigma^{\text{NNLOapprox, antitop}}_{t-\text{channel}}(m_t = 173\,\text{GeV}, 7\,\text{TeV}) \quad = \quad 22.5 \pm 0.5^{+0.7}_{-0.9}\ \text{pb}$$

$$\sigma^{\text{NNLOapprox, antitop}}_{t-\text{channel}}(m_t = 173\,\text{GeV}, 14\,\text{TeV}) \quad = \quad 92^{+2+2}_{-1-3}\ \text{pb}$$

NNLO approx: 1% decrease at 7 TeV; 3% decrease at 14 TeV relative to NLO

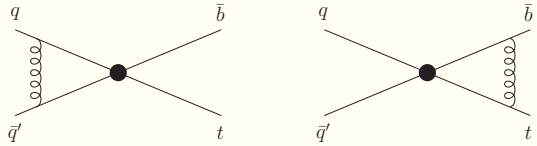
t-channel combined cross section versus energy



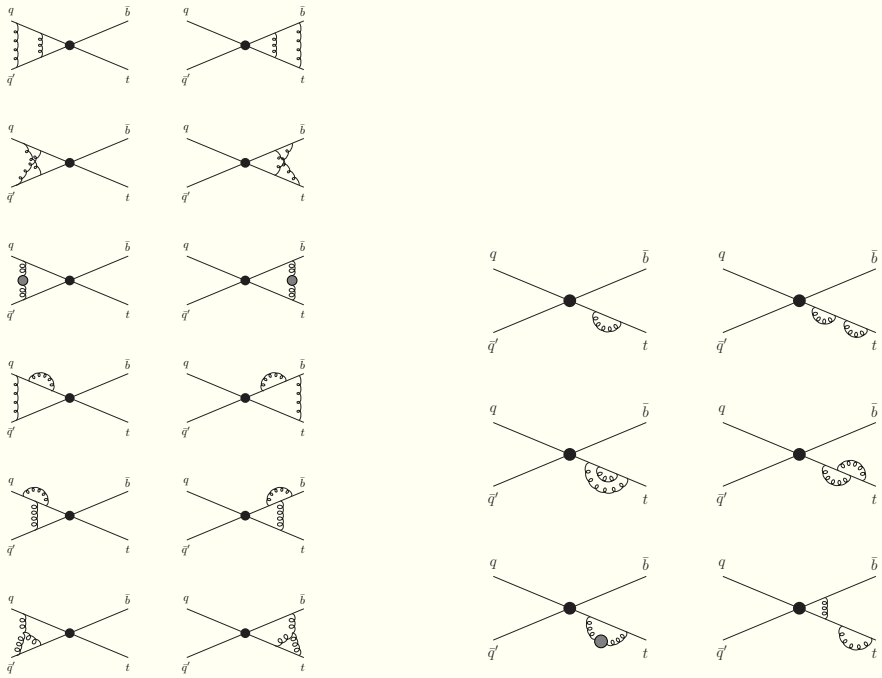
Tevatron and LHC results are consistent with theory

Single top quark production - s channel

One-loop eikonal diagrams



Two-loop eikonal diagrams

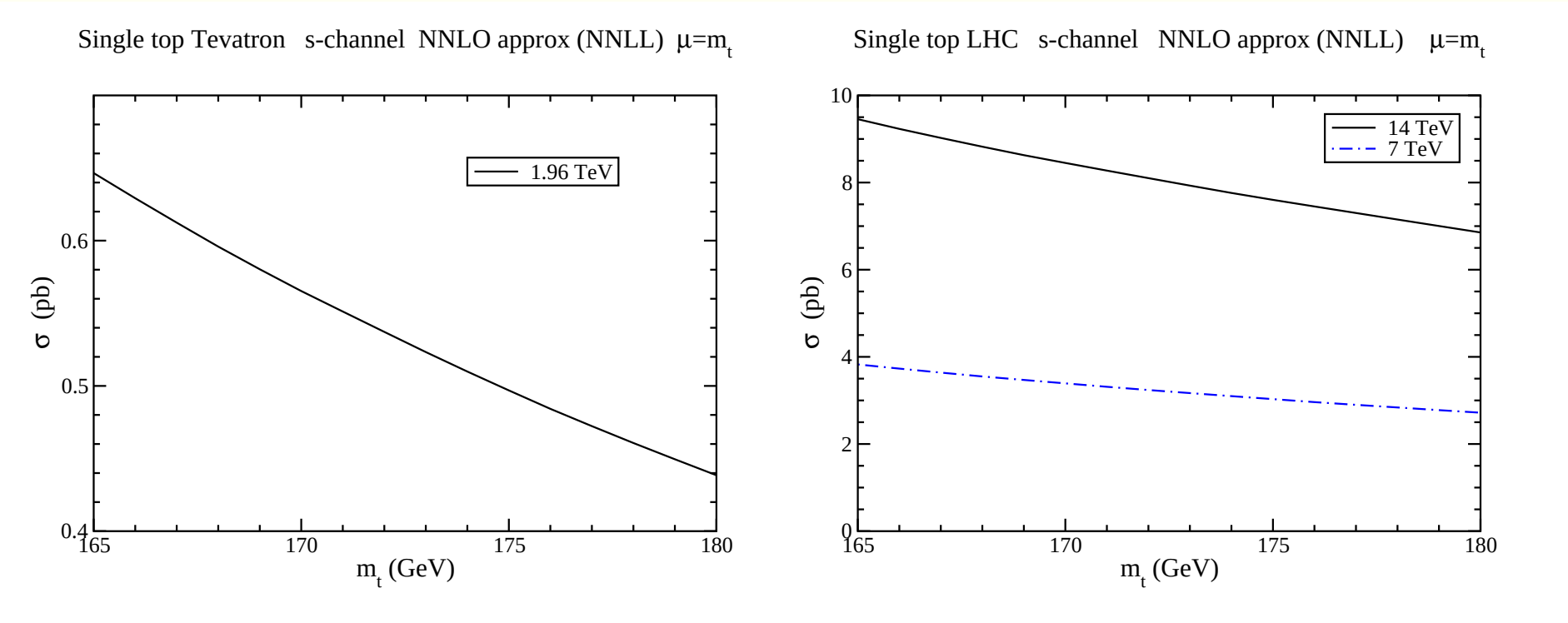


Soft anomalous dimension for s-channel single top production

$$\Gamma^{(1)}_{S\,11} = C_F \left[\ln\left(\frac{s - m_t^2}{m_t \sqrt{s}} \right) - \frac{1}{2} \right], \qquad \Gamma^{(2)}_{S\,11} = \frac{K}{2} \Gamma^{(1)}_{S\,11} + C_F C_A \frac{(1 - \zeta_3)}{4}$$

N. Kidonakis, Phys. Rev. D 81, 054028 (2010), arXiv:1001.5034 [hep-ph]

Single top quark production at Tevatron and LHC - s channel



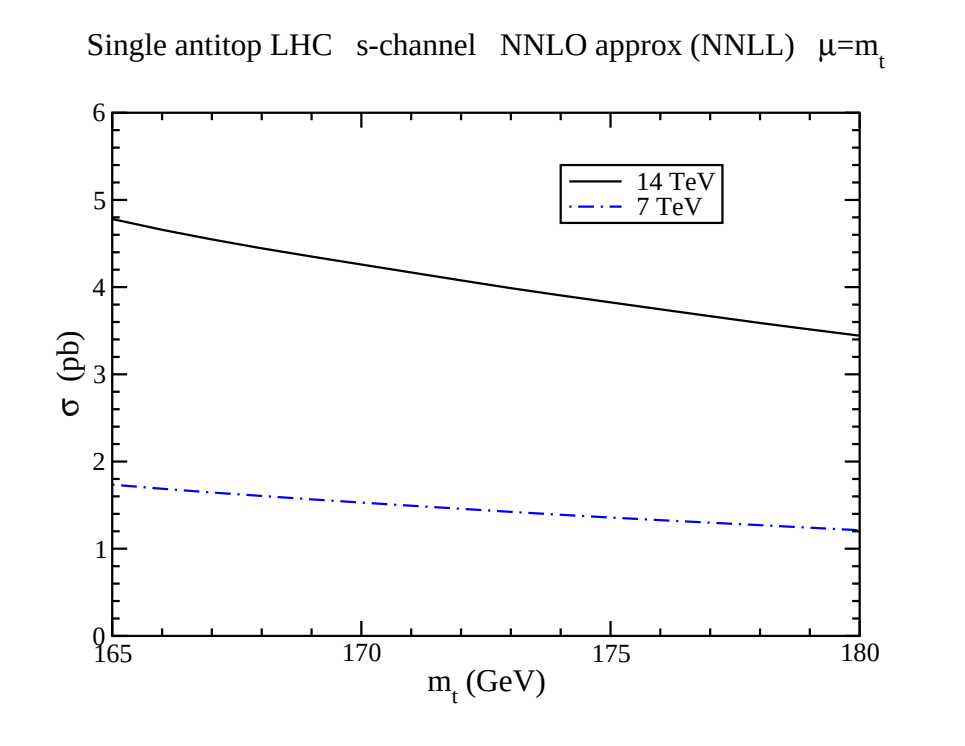
$$\sigma^{\mathrm{NNLOapprox,\,top}}_{s-\mathrm{channel}}(m_t = 173\,\mathrm{GeV}\,1.96\,\mathrm{TeV}) = 0.523^{+0.001+0.030}_{-0.005-0.028}\ \mathrm{pb}$$

$$\sigma^{\mathrm{NNLOapprox,\,top}}_{s-\mathrm{channel}}(m_t = 173\,\mathrm{GeV},\,7\,\mathrm{TeV}) = 3.17 \pm 0.06^{+0.13}_{-0.10}\ \mathrm{pb}$$

$$\sigma^{\mathrm{NNLOapprox,\,top}}_{s-\mathrm{channel}}(m_t = 173\,\mathrm{GeV},14\,\mathrm{TeV}) = 7.93 \pm 0.14^{+0.31}_{-0.28}\ \mathrm{pb}$$

NNLO approx: enhancement over NLO is 15% at Tevatron; 13% at LHC

Single antitop production at LHC - s channel

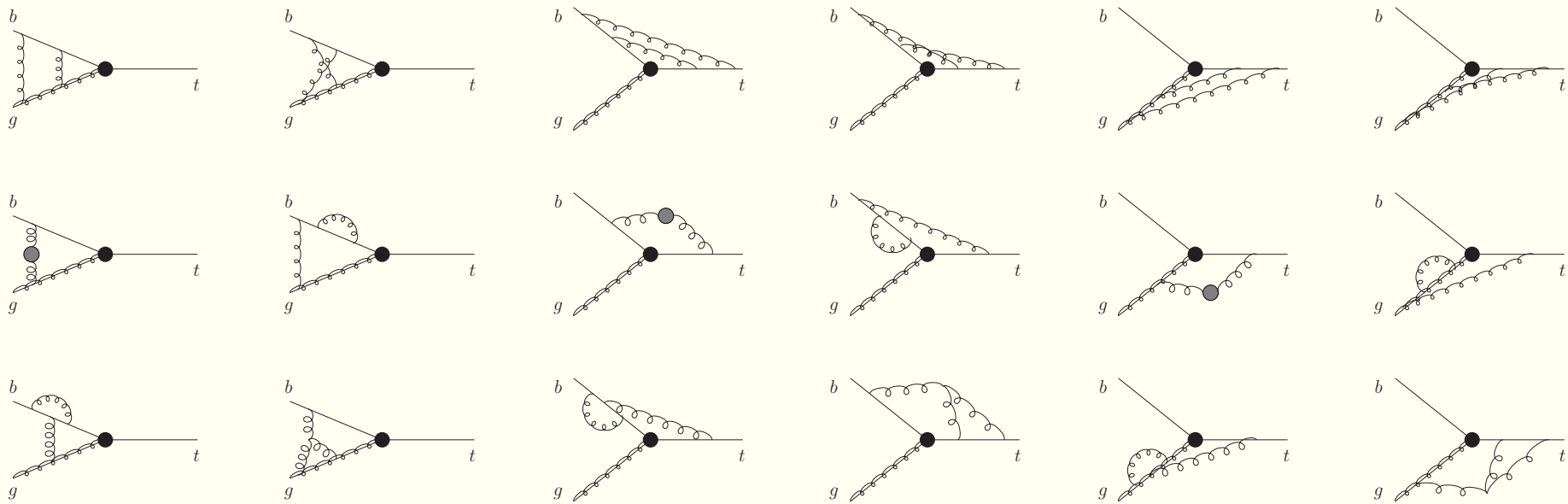


$$\sigma_{s-\text{channel}}^{\text{NNLOapprox, antitop}}(m_t = 173\,\text{GeV}, 7\,\text{TeV}) = 1.42 \pm 0.01^{+0.06}_{-0.07}\ \text{pb}$$

$$\sigma_{s-\text{channel}}^{\text{NNLOapprox, antitop}}(m_t = 173\,\text{GeV}, 14\,\text{TeV}) = 3.99 \pm 0.05^{+0.14}_{-0.21}\ \text{pb}$$

Associated production of a top quark with a W^-

Two-loop eikonal diagrams (+ extra top-quark self-energy graphs)

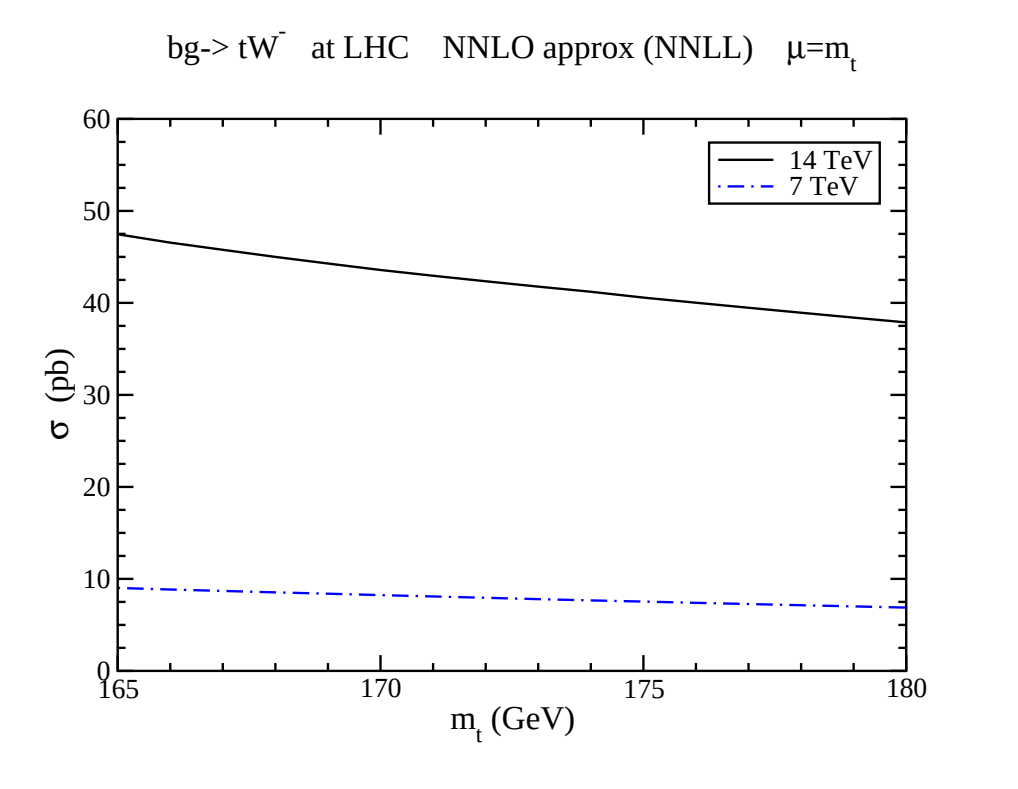


Soft anomalous dimension for $bg \to tW^-$ (and for $bg \to tH^-$)

$$\Gamma^{(1)}_{S,tW^-} = C_F \left[\ln\left(\frac{m_t^2 - t}{m_t\sqrt{s}} \right) - \frac{1}{2} \right] + \frac{C_A}{2} \ln\left(\frac{m_t^2 - u}{m_t^2 - t} \right)$$

$$\Gamma^{(2)}_{S,tW^-} = \frac{K}{2}\Gamma^{(1)}_{S,tW^-} + C_F C_A \frac{(1-\zeta_3)}{4}$$

N. Kidonakis, Phys. Rev. D 82, 054018 (2010), arXiv:1005.4451 [hep-ph]



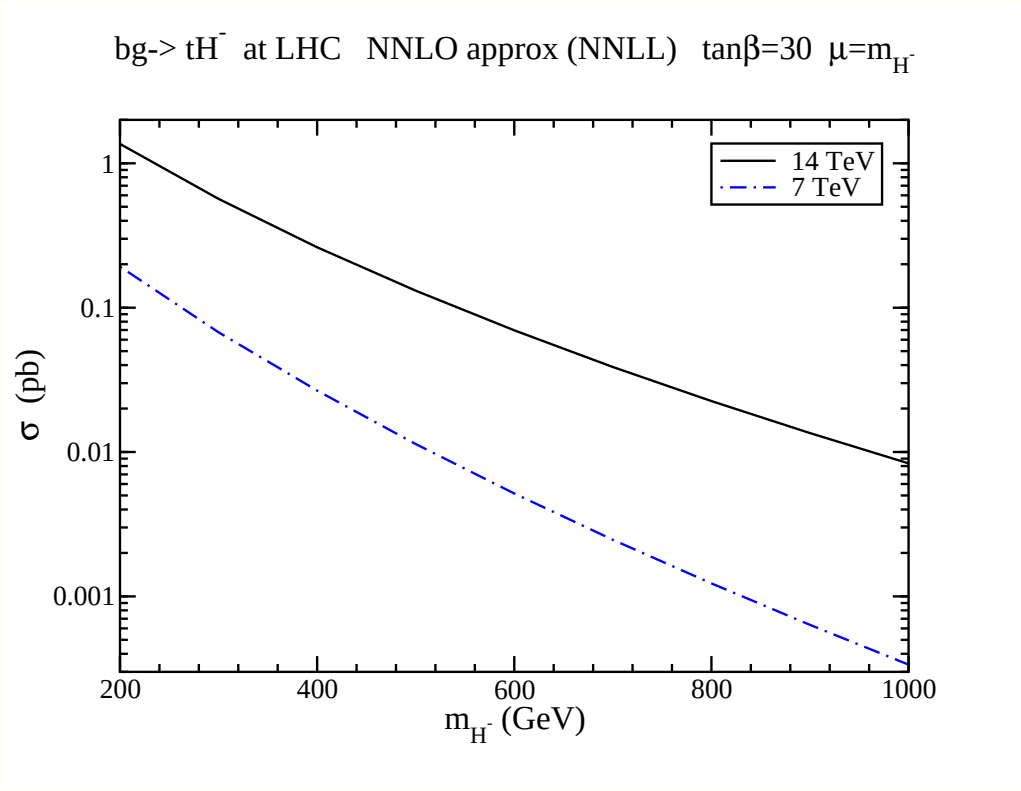
$$\sigma_{tW}^{\mathrm{NNLOapprox}}(m_t = 173\,\mathrm{GeV},\, 7\,\mathrm{TeV}) \quad = \quad 7.8 \pm 0.2^{+0.5}_{-0.6}\ \mathrm{pb}$$

$$\sigma_{tW}^{\mathrm{NNLOapprox}}(m_t = 173\,\mathrm{GeV}, 14\,\mathrm{TeV}) \quad = \quad 41.8 \pm 1.0^{+1.5}_{-2.4}\ \mathrm{pb}$$

NNLO approx corrections increase NLO cross section by $\sim 8\%$

Cross section for $\bar{t}W$ production is identical

Associated production of a top quark with a charged Higgs



NNLO approx corrections increase NLO cross section by $\sim$ 15 to $\sim$ 20%

Summary

- **NNLL resummation for top quark pair and single top production**
- **$t\bar{t}$ production cross section**
- **top quark p_T and rapidity distributions**
- **t-channel and s-channel single top production cross section**
- **$bg \longrightarrow tW^-$ and $bg \longrightarrow tH^-$ at LHC**
- **NNLO approx corrections for top pair and single top production are significant at Tevatron and LHC**