

# Impact of the triple-gluon correlation functions on the single spin asymmetries in $pp$ collisions

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**Abstract.** We calculate the single-spin-dependent cross section formula for the  $D$ -meson production and the direct-photon production in the  $pp$  collision induced by the twist-3 triple-gluon correlation functions in the transversely polarized nucleon. We also present a model calculation for the asymmetries in comparison with the preliminary data given by RHIC, showing the impact of the correlation functions on the asymmetries.

**Keywords:** single spin asymmetry, twist-3, triple-gluon correlation function

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## 1. INTRODUCTION

Understanding the origin of the large single spin asymmetries (SSAs) observed in various high-energy semi-inclusive processes have been a big challenge during the past decades. The SSA can be generated as a consequence of the multiparton correlations inside the hadrons in the collinear factorization approach which is valid when the transverse momentum of final state hadron can be regarded as hard. Recently the measurement of SSA for heavy meson production by the PHENIX collaboration [1] have motivated theoretical works for multigluon correlation inside the transversely polarized proton which is represented by the triple-gluon correlation functions [2, 3] because heavy quarks fragmenting into final state meson are mainly produced by the gluon fusion mechanism.

In this work, we study the contribution of the triple-gluon correlation functions to SSA for the  $D$ -meson and the direct photon productions in the  $pp$  collision [4, 5]. We will derive the corresponding single-spin dependent cross sections by applying the formalism developed for the semi-inclusive deep inelastic scattering [3]. We will also present a model estimate for the triple-gluon correlation functions by comparing our result with the RHIC preliminary data for the  $D$ -meson production [1]. Finally we perform numerical calculation of the asymmetry for the direct photon production by using the models obtained from  $p^\uparrow p \rightarrow DX$  to see its impact on the SSA for this process.

## 2. TRIPLE-GLUON CORRELATION FUNCTIONS

Triple-gluon correlation functions for the transversely polarized nucleon are defined as the color-singlet nucleon matrix element composed of the three gluon's field strength

tensors  $F^{\alpha\beta}$ . Corresponding to the two structure constants for the color SU(3) group,  $d_{bca}$  and  $f_{bca}$ , one obtains two independent triple-gluon correlation functions  $O(x_1, x_2)$  and  $N(x_1, x_2)$  as [3]

$$\begin{aligned} O^{\alpha\beta\gamma}(x_1, x_2) &= -g(i)^3 \int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \langle pS | d_{bca} F_b^{\beta n}(0) F_c^{\gamma n}(\mu n) F_a^{\alpha n}(\lambda n) | pS \rangle \\ &= 2iM_N \left[ O(x_1, x_2) g^{\alpha\beta} \varepsilon^{\gamma p n S} + O(x_2, x_2 - x_1) g^{\beta\gamma} \varepsilon^{\alpha p n S} + O(x_1, x_1 - x_2) g^{\gamma\alpha} \varepsilon^{\beta p n S} \right], \quad (1) \end{aligned}$$

$$\begin{aligned} N^{\alpha\beta\gamma}(x_1, x_2) &= -g(i)^3 \int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \langle pS | i f_{bca} F_b^{\beta n}(0) F_c^{\gamma n}(\mu n) F_a^{\alpha n}(\lambda n) | pS \rangle \\ &= 2iM_N \left[ N(x_1, x_2) g^{\alpha\beta} \varepsilon^{\gamma p n S} - N(x_2, x_2 - x_1) g^{\beta\gamma} \varepsilon^{\alpha p n S} - N(x_1, x_1 - x_2) g^{\gamma\alpha} \varepsilon^{\beta p n S} \right], \quad (2) \end{aligned}$$

where  $M_N$  is the nucleon mass,  $S$  is the transverse-spin vector for the nucleon,  $n$  is the light-like vector satisfying  $p \cdot n = 1$  and we used the shorthand notation as  $F^{\beta n} \equiv F^{\beta\rho} n_\rho$  etc. The gauge-link operators which restore gauge invariance of the correlation functions are suppressed in (1) and (2) for simplicity.

### 3. D-MESON PRODUCTION IN $pp$ COLLISION

Applying the formalism for the contribution of the triple-gluon correlation functions to SSA developed in [3], the twist-3 cross section for  $p^\uparrow(p, S_\perp) + p(p') \rightarrow D(P_h) + X$  (center-of-mass energy  $\sqrt{S}$ ) can be obtained in the following form [4]:

$$\begin{aligned} P_h^0 \frac{d\Delta\sigma}{d^3P_h} &= \frac{\alpha_s^2 M_N \pi}{S} \varepsilon^{P_h p n S_\perp} \sum_{f=c\bar{c}} \int \frac{dx'}{x'} G(x') \int \frac{dz}{z^2} D_f(z) \int \frac{dx}{x} \delta(\tilde{s} + \tilde{t} + \tilde{u}) \frac{1}{z\tilde{u}} \\ &\times \left[ \delta_f \left\{ \left( \frac{d}{dx} O(x, x) - \frac{2O(x, x)}{x} \right) \hat{\sigma}^{O1} + \left( \frac{d}{dx} O(x, 0) - \frac{2O(x, 0)}{x} \right) \hat{\sigma}^{O2} + \frac{O(x, x)}{x} \hat{\sigma}^{O3} + \frac{O(x, 0)}{x} \hat{\sigma}^{O4} \right\} \right. \\ &\left. + \left\{ \left( \frac{d}{dx} N(x, x) - \frac{2N(x, x)}{x} \right) \hat{\sigma}^{N1} + \left( \frac{d}{dx} N(x, 0) - \frac{2N(x, 0)}{x} \right) \hat{\sigma}^{N2} + \frac{N(x, x)}{x} \hat{\sigma}^{N3} + \frac{N(x, 0)}{x} \hat{\sigma}^{N4} \right\} \right], \quad (3) \end{aligned}$$

where  $\delta_c = 1$  and  $\delta_{\bar{c}} = -1$ ,  $D_f(z)$  represents the  $c \rightarrow D$  or  $\bar{c} \rightarrow \bar{D}$  fragmentation functions,  $G(x')$  is the unpolarized gluon density,  $p_c$  is the four-momentum of the  $c$  (or  $\bar{c}$ ) quark (mass  $m_c$ ) fragmenting into the final  $D$  (or  $\bar{D}$ ) meson and  $\tilde{s}$ ,  $\tilde{t}$ ,  $\tilde{u}$  are defined as  $\tilde{s} = (xp + x'p')^2$ ,  $\tilde{t} = (xp - p_c)^2 - m_c^2$ ,  $\tilde{u} = (x'p' - p_c)^2 - m_c^2$ . The hard cross sections  $\hat{\sigma}^{O1, O2, O3, O4}$  and  $\hat{\sigma}^{N1, N2, N3, N4}$  are listed in [4]. The cross section (3) receives contributions from  $O(x, x)$ ,  $O(x, 0)$ ,  $N(x, x)$  and  $N(x, 0)$  separately, which differs from the previous result [2].

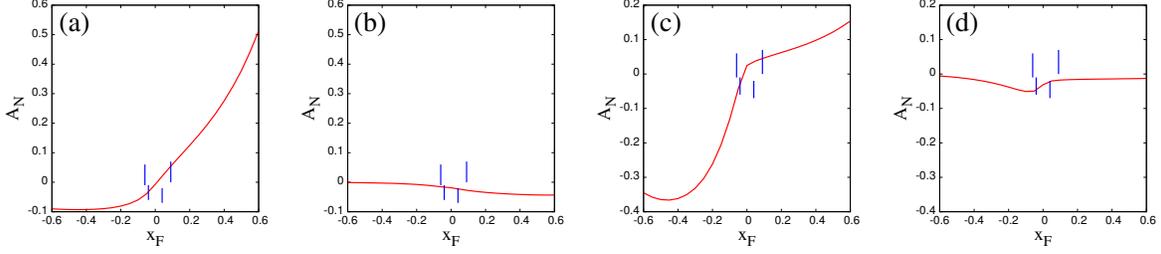
We perform numerical estimate for  $A_N$  based on (3). Since  $|\hat{\sigma}^{O3, O4, N3, N4}| \ll |\hat{\sigma}^{O1, O2, N1, N2}|$  and  $\hat{\sigma}^{O1} \simeq \hat{\sigma}^{O2} \simeq \hat{\sigma}^{N1} \simeq -\hat{\sigma}^{N2}$ , we assume the relation for the four functions as  $O(x, x) = O(x, 0) = N(x, x) = -N(x, 0)$  for simplicity. For the functional form of each functions, we employ the following two models:

$$\text{Model 1 :} \quad O(x, x) = K_G x G(x), \quad (4)$$

$$\text{Model 2 :} \quad O(x, x) = K'_G \sqrt{x} G(x), \quad (5)$$

where  $K_G$  and  $K'_G$  are the constants to be determined so that the calculated asymmetry is consistent with the RHIC data [1].

For the numerical calculation, we use GJR08 [6] for  $G(x)$  and KKKS08 [7] for  $D_f(z)$ . We calculate  $A_N$  for the  $D$  and  $\bar{D}$  mesons at the RHIC energy at  $\sqrt{S} = 200$  GeV and the transverse momentum of the  $D$ -meson  $P_T = 2$  GeV. We set the scale of all the distribution and fragmentation functions at  $\mu = \sqrt{P_T^2 + m_c^2}$  with the charm quark mass  $m_c = 1.3$  GeV.



**FIGURE 1.** Results of  $A_N^D$  for  $D^0$  (a) and  $\bar{D}^0$  (b) for Model 1 in (4) with  $K_G = 0.002$ , and  $A_N^D$  for  $D^0$  (c) and  $\bar{D}^0$  (d) for Model 2 in (5) with  $K'_G = 0.0005$ . Short bars denote the RHIC preliminary data taken from [1].

Fig. 1 shows the result of  $A_N$  for the  $D^0$  and  $\bar{D}^0$  mesons together with the preliminary data [1] denoted by the short bars. The sign of the contribution from  $\{O(x, x), O(x, 0)\}$  changes between  $D^0$  and  $\bar{D}^0$  as shown in (3), which causes the large difference between  $A_N$  for the  $D^0$  and  $\bar{D}^0$ . If one reverses the relative sign between  $O$  and  $N$ , the result for the  $D^0$  and  $\bar{D}^0$  mesons will be interchanged. The values  $K_G = 0.002$  and  $K'_G = 0.0005$  have been determined so that  $A_N$  does not overshoot the RHIC data. By comparing the results for the models 1 and 2 in Fig. 1, one sees that the behavior of  $A_N$  at  $x_F < 0$  depends strongly on the small- $x$  behavior of the triple-gluon correlation functions. Therefore  $A_N$  at  $x_F < 0$  is useful to get constraint on the small- $x$  behavior of the three-gluon correlation functions.

#### 4. DIRECT PHOTON PRODUCTION IN $pp$ COLLISION

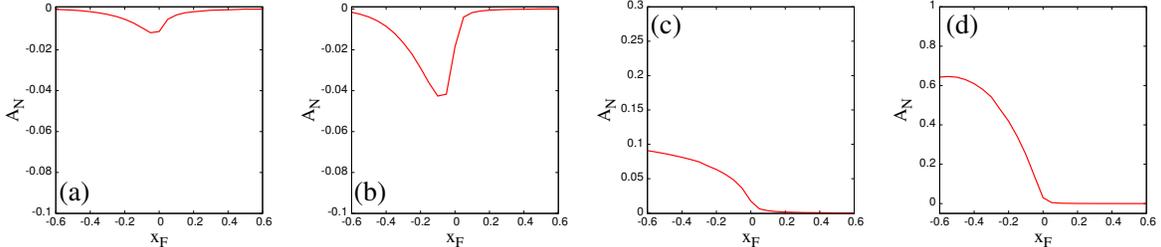
Applying the same formalism, the twist-3 cross section for the direct photon production,  $p^\uparrow(p, S_\perp) + p(p') \rightarrow \gamma(q) + X$ , induced by the triple-gluon correlation functions can be obtained as [5]

$$\begin{aligned}
E_\gamma \frac{d\sigma}{d^3q} &= \frac{4\alpha_{em}\alpha_s M_N \pi}{S} \sum_a \int \frac{dx'}{x'} f_a(x') \int \frac{dx}{x} \delta(\hat{s} + \hat{t} + \hat{u}) \epsilon^{qpmS_\perp} \frac{1}{\hat{u}} \\
&\times \left[ \delta_a \left( \frac{d}{dx} O(x, x) - \frac{2O(x, x)}{x} + \frac{d}{dx} O(x, 0) - \frac{2O(x, 0)}{x} \right) \right. \\
&\left. - \frac{d}{dx} N(x, x) + \frac{2N(x, x)}{x} + \frac{d}{dx} N(x, 0) - \frac{2N(x, 0)}{x} \right] \left( \frac{1}{N} \frac{\hat{s}^2 + \hat{u}^2}{\hat{s}\hat{u}} \right), \quad (6)
\end{aligned}$$

where  $f_a(x')$  is the twist-2 unpolarized quark density,  $\delta_a = 1(-1)$  for quark (antiquark) and  $\hat{s}, \hat{t}, \hat{u}$  are defined as  $\hat{s} = (xp + x'p')^2$ ,  $\hat{t} = (xp - q)^2$ ,  $\hat{u} = (x'p' - q)^2$ . As shown in (6), the combinations  $O(x, x) + O(x, 0)$  and  $N(x, x) - N(x, 0)$  appear in the cross section accompanying the common partonic hard cross section which is the same as the twist-2

hard cross section for the  $qg \rightarrow q\gamma$  scattering. This result differs from the previous study in [8].

We performed a numerical calculation for  $A_N^\gamma$  for the following two cases: Case 1;  $O(x,x) = O(x,0) = N(x,x) = -N(x,0)$  and Case 2;  $O(x,x) = O(x,0) = -N(x,x) = N(x,0)$ . We use GJR08 [6] for  $f_q(x')$  and the models (4) and (5) with  $K_G = 0.002$  and  $K'_G = 0.0005$  which are consistent with RHIC  $A_N^D$  data. We calculate  $A_N^\gamma$  at the RHIC energy at  $\sqrt{S} = 200$  GeV and the transverse momentum of the photon  $q_T = 2$  GeV, setting the scale of all the distribution at  $\mu = q_T$ .



**FIGURE 2.** (a)  $A_N$  for Case 1 with Model 1. (b)  $A_N$  for Case 1 with Model 2. (c)  $A_N$  for Case 2 with Model 1. (d)  $A_N$  for Case 2 with Model 2.

Fig. 2 shows the result for  $A_N^\gamma$  for each case. One can see  $A_N$  at  $x_F > 0$  become almost zero regardless of the magnitude of the triple-gluon correlation functions, while  $A_N$  at  $x_F < 0$  depends strongly on the small- $x$  behavior of the triple-gluon correlation functions as in the case of  $p^\uparrow p \rightarrow DX$ . At negative  $x_F$ , large- $x'$  region of the unpolarized quark distributions and the small- $x$  region of the triple-gluon distributions are relevant. For the above case 1, only antiquarks in the unpolarized nucleon are active and thus lead to small  $A_N^\gamma$  as shown in Figs. 2(a) and (b). On the other hand, for the case 2, quarks in the unpolarized nucleon are active and thus lead to large  $A_N^\gamma$  as shown in Figs. 2(c) and (d). Therefore  $A_N^\gamma$  at  $x_F < 0$  for the direct photon production could provides us with an important information on the relative sign between  $O$  and  $N$ .

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