Beauty quark, $b$-jet and quarkonium production at LHC: $k_T$—factorization and CASCADE versus data

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1. Motivation

Beauty production at h.e. is subject of intense studies from both theor. and exp. points of view. Firstly, in order to test QCD predictions. Secondly, $b$-jets represent an important source of background to many of the searches at LHC as the Higgs boson and SUSY extentions of SM.

Our study is motivated by very recent measurement of open beauty quark and $b$-jet production performed by the CMS Collaboration. It was observed that the data tends to be higher than the MC@NLO predictions and that the shape of the pseudo-rapidity distribution is not well described by MC@NLO. The $p_T$-spectra of $b$-jets are not well discribed too.
Recently we have demonstrated reasonable agreement between the $k_T$-factorization predictions and the Tevatron data on the $b$-quarks, $b\bar{b}$ di-jets, $B^+$- and $D$-mesons:

H. Jung, M. Krämer, A.V. Lipatov, N.Z., JHEP 1101 (2011) 085, and also agreement with total set of HERA data for $J/\psi$-mesons:


A. Bertolin, talk at this Workshop.

Based on these results, here we give first analysis of the CMS data in the framework of the $k_T$-factorization approach.

We produce the relevant numerical calculations in two ways:

- We will perform analytical parton-level calculations (which are labeled as LZ).

- The measured cross sections of heavy quark production will be compared also with the predictions of full hadron level Monte Carlo event generator CASCADE:


2. Ingredients of the $k_T$-factorization

- The basic dynamical quantity of the $k_T$-factorization approach is the unintegrated ($k_T$-dependent) gluon distribution (UGD) $A(x, k_T^2, \mu^2)$ obtained from the analytical or numerical solution of the BFKL or CCFM evolution equations. The cross section of any physical process is calculated as a convolution of the partonic cross section $\hat{\sigma}$ and the u.g.d. $A_g(x, k_T^2, \mu^2)$, which depend on both the longitudinal momentum fraction $x$ and transverse momentum $k_T$:

$$\sigma_{pp} = \int A_g(x_1, k_{1T}^2, \mu^2) A_g(x_2, k_{2T}^2, \mu^2) \hat{\sigma}_{gg}(x_1, x_2, k_{1T}^2, k_{2T}^2, ...) \, dx_1 \, dx_2 \, dk_{1T}^2 \, dk_{2T}^2.$$

- The partonic cross section $\hat{\sigma}$ has to be taken off mass shell ($k_T$-dependent).
• It also assumes a modification of their polarization density matrix. It has to be taken in BFKL form:

$$\sum \epsilon^\mu \epsilon^{\ast \nu} = \frac{k^\mu_T k^\nu_T}{k^2_T}.$$ 


Concerning the uPDF in a proton, we used two different sets.

First of them is the KMR one. The KMR approach represent an approximate treatment of the parton evolution mainly based on the DGLAP equation and incorporating the BFKL effects at the last step of the parton ladder only, in the form of the properly defined Sudakov formfactors $T_q(k^2_T, \mu^2)$ and $T_g(k^2_T, \mu^2)$, including logarithmic loop corrections.


We use the version of KMRW UPD obtained from DGLAP eqs.:

The CCFM ev. eq. have been solved numerically using a Monte-Carlo method:

\[ \mu^2 = \hat{s} + Q_T^2 (= \mu_f^2). \]

According to the CCFM ev. eq., the emission of gluons during the initial cascade is only allowed in an angular-ordered region of phase space. The maximum allowed angle \( \Xi \) related to the hard quark box sets the scale \( \mu \):

\[ \mu^2 = \hat{s} + Q_T^2 (= \mu_f^2). \]

UGD are determined by a convolution of the non-perturbative starting distribution \( A_0(x) \) and CCFM evolution denoted by \( \bar{A}(x, k_T^2, \mu^2) \):

\[ xA(x, k_T^2, \mu^2) = \int dz A_0(z) \frac{x}{z} \bar{A}\left(\frac{x}{z}, k_T^2, \mu^2\right), \]

where

\[ xA_0(x) = N x^{p_0}(1 - x)^{p_1} \exp(-k_T^2/k_0^2). \]

The parameters were determined in the fit to \( F_2 \) data.
4. Numerical results

HEAVY QUARK PRODUCTION IN PP-INTERACTION.

The hard partonic subprocess $g^*g^* \rightarrow Q\bar{Q}$ amplitude is described by three Feynman’s diagrams (Fig. 1).
The cross section of the process $pp \rightarrow Q\bar{Q}X$ is

$$
\sigma(p\bar{p} \rightarrow Q\bar{Q} X) = \frac{1}{16\pi(x_1 x_2 s)^2} \int \mathcal{A}(x_1, k_{1T}^2, \mu^2) \mathcal{A}(x_2, k_{2T}^2, \mu^2) |\tilde{M}(g^* g^* \rightarrow Q\bar{Q})|^2 \times
$$

$$
\times dp_{1T}^2 dk_{1T}^2 dk_{2T}^2 dy_1^* dy_2^* \frac{d\phi_1}{2\pi} \frac{d\phi_2}{2\pi}.
$$

In the numerical calculations in the case CCFM u.g.d. we have used two different sets, namely $A0$ and $B0$. The difference between these sets is connected with the different values of soft cut and width of the intrinsic $k_T$ distribution. A reasonable description of the $F_2$ data can be achieved by both these sets. For KMR we have used the standard GRV 94 (LO) (in LZ calculations) and MRST 99 (in CASCADE) sets.

The unintegrated gluon distributions depend on the renormalization and factorization scales $\mu_R$ and $\mu_F$. We set $\mu_R^2 = m_Q^2 + (p_{1T}^2 + p_{2T}^2)/2$, $\mu_F^2 = \hat{s} + Q_T^2$, where $Q_T$ is the transverse momentum of the initial off-shell gluon pair, $m_c = 1.4 \pm 0.1$ GeV, $m_b = 4.75 \pm 0.25$ GeV. We use the LO formula for the coupling $\alpha_s(\mu_R^2)$ with $n_f = 4$ active quark flavors at $\Lambda_{QCD} = 200$ MeV, such that $\alpha_s(M_Z^2) = 0.1232$. 
We begin the discussion by presenting our results for the muons originating from the semileptonic decays of the $b$ quarks.

To produce muons from $b$-quarks, we first convert $b$-quarks into $B$ mesons using the Peterson fragmentation function with default value $\epsilon_b = 0.006$ and then simulate their semileptonic decay according to the standard electroweak theory taking into account the decays $b \rightarrow \mu$ as well as the cascade decay $b \rightarrow c \rightarrow \mu$. In CASCADE calculations also Peterson f. f. is used but with full PYTHIA fragmentation.
Figure 1: The pseudo-rapidity distributions of muons arising from the semileptonic decays of beauty quarks. The first column shows the LZ numerical results while the second one depicts the CASCADE predictions. The solid, dashed and dash-dotted, dotted histograms correspond to the results obtained with the CCFM A0, B0 and KMR unintegrated gluon densities. The experimental data are from CMS.
Figure 2: The transverse momentum distributions of muons arising from the semileptonic decays of beauty quarks. The first column shows the LZ numerical results while the second one depicts the CASCADE predictions. Notation of all histograms is the same as in Fig. 2. The experimental data are from CMS.
Figure 3: The dependence of our predictions on the fragmentation scheme. The solid, dashed and dash-dotted histograms correspond to the results obtained using the Peterson fragmentation function with $\epsilon_b = 0.006$, $\epsilon_b = 0.003$ and the non-perturbative fragmentation functions respectively. We use CCFM (A0) gluon density for illustration. The experimental data are from CMS.
Figure 4: Parton shower effects in the pseudo-rapidity and transverse momentum distributions of the muons. The four lines represent full parton shower (solid line), no parton shower (dashed line), initial state parton shower (dashed dotted line) and final state parton shower (dotted line).
Figure 5: The double differential cross sections $d\sigma/dy dp_T$ of inclusive $b$-jet production as a function of $p_T$ in different $y$ regions calculated at $\sqrt{s} = 7$ TeV (LZ predictions). Notation of all histograms is the same as in Fig. 2. The experimental data are from CMS.
In the case quarkonium production we used Color-Singlet (CS) gluon-gluon fusion in the framework of the $k_T$-factorization approach.

Figure 6: Differential cross sections $J/\psi$ mesons at HERA. The solid, dashed and dash-dotted histograms correspond to the results obtained using the CCFM A0, BO and KMR gluon densities. The upper and lower dashed histograms represent the scale variations.
Comparison with LHC data on the $J/\psi$ production

![Graphs showing comparison with LHC data on the $J/\psi$ production.](image-url)
Conclusions

- We have analysed the first data on the beauty and $J/\psi$ production in $pp$ collisions at LHC taken by the CMS collaboration.

- Our study is based on a semi-analitical parton level calculations and a full hadron level MC generator CASCADE.

- The overall description of the data is reasonable. In most of the distributions it is similar to MC@NLO except in some particular distributions where the $k_T$-factorization approach does describe the data better, like in $b$-jet.

- $J/\psi$ production in the $k_T$-factorization approach with CS model comes much closer to the data than the collinear calculations. The reason is the off-shell ME, which includes even higher order contributions than the NLO collinear calculations.
KMR UPDFs are given by

\[ A_q(x, k_T^2, \mu^2) = T_q(k_T^2, \mu^2) \frac{\alpha_s(k_T^2)}{2\pi} \times \]

\[ \times \int_x^1 dz \left[ P_{qq}(z) \frac{x}{z} q \left( \frac{x}{z}, k_T^2 \right) \Theta (\Delta - z) + P_{qg}(z) \frac{x}{z} g \left( \frac{x}{z}, k_T^2 \right) \right], \]

(1)

\[ A_g(x, k_T^2, \mu^2) = T_g(k_T^2, \mu^2) \frac{\alpha_s(k_T^2)}{2\pi} \times \]

\[ \times \int_x^1 dz \left[ \sum_q P_{gq}(z) \frac{x}{z} q \left( \frac{x}{z}, k_T^2 \right) + P_{gg}(z) \frac{x}{z} g \left( \frac{x}{z}, k_T^2 \right) \Theta (\Delta - z) \right]. \]

(2)

\[ \Theta \text{-functions imply the angular-ordering constraint } \Delta = \frac{\mu}{\mu + k_T} \text{ specifically to the last evaluation step (to regulate the soft gluon singularities). For other evolution steps the strong ordering in transverse momentum within DGLAP eq. automatically ensures angular ordering.} \]
$T_a(k^2_T, \mu^2)$ - the probability of evolving from $k^2_T$ to $\mu^2$ without parton emission. $T_a(k^2_T, \mu^2) = 1$ at $k^2_T > \mu^2$.

Such definition of the $A_a(x, k^2_T, \mu^2)$ is correct for $k^2_T > \mu^2_0$ only, where $\mu_0 \sim 1$ GeV is the minimum scale for which DGLAP evolution of the collinear parton densities is valid.

In this case ($a(x, \mu^2) = xG$ or $a(x, \mu^2) = xq$) the normalization condition

$$a(x, \mu^2) = \int_0^{\mu^2} A_a(x, k^2_T, \mu^2) dk^2_T,$$

is satisfied, if

$$A_a(x, k^2_T, \mu^2)|_{k^2_T<\mu^2_0} = a(x, \mu^2_0)T_a(\mu^2_0, \mu^2),$$

where $T_a(\mu^2_0, \mu^2)$ are the quark and gluon Sudakov form factors.

The UPD $A_a(x, k^2_T, \mu^2)$ is defined in all $k^2_T$ region.
Qurakonium production.

Spin projection operators to guarantee the proper quantum numbers:

for Spin-triplet states \[ P(3S_1) = \ell_V(p_Q + m_Q)/(2m_Q) \]
for Spin-singlet states \[ P(1S_0) = \gamma_5(p_Q + m_Q)/(2m_Q) \]

Probability to form a bound state is determined by the wave function:

for S-wave states \( |R_S(0)|^2 \) is known from leptonic decay widths;
for P-wave states \( |R'_P(0)|^2 \) is taken from potential models.


If \( L \neq 0 \) and \( S \neq 0 \) we use the Clebsch-Gordan coefficients to express the \( |L, S\rangle \) states in terms of \( |J, J_z\rangle \) states, namely, the \( \chi_0, \chi_1, \chi_2 \) mesons.
Feed-down from P-wave states.

Assuming the dominance of electric dipole transitions, we have angular distributions in the polarized $\chi_J$ decays:

$$d\Gamma(\chi_1 \rightarrow V\gamma)/d\cos\theta \propto \left[ \left(1 + \frac{1}{2}\rho\right) + \left(1 - \frac{3}{2}\rho\right)\cos^2\theta \right],$$

$$d\Gamma(\chi_2 \rightarrow V\gamma)/d\cos\theta \propto \left[ \left(\frac{5}{6} - \frac{1}{12}\xi - \frac{1}{3}\tau\right) - \left(\frac{1}{2} - \frac{1}{4}\xi - \tau\right)\cos^2\theta \right],$$

where $\rho = d\sigma_{\chi_1(|h|=1)}/d\sigma_{\chi_1}$, $\xi = d\sigma_{\chi_2(|h|=1)}/d\sigma_{\chi_2}$, $\tau = d\sigma_{\chi_2(|h|=2)}/d\sigma_{\chi_2}$ (all known from the $\chi_J$ production matrix elements).

Polarization of the decay products

$$\sigma_{V(|h|=0)} = B(\chi_1 \rightarrow V\gamma) \left[ (1/2)\sigma_{\chi_1(|h|=1)} \right]$$
$$\quad + B(\chi_2 \rightarrow V\gamma) \left[ (2/3)\sigma_{\chi_2(|h|=0)} + (1/2)\sigma_{\chi_2(|h|=1)} \right]$$

$$\sigma_{V(|h|=1)} = B(\chi_1 \rightarrow V\gamma) \left[ \sigma_{\chi_1(|h|=0)} + (1/2)\sigma_{\chi_1(|h|=1)} \right]$$
$$\quad + B(\chi_2 \rightarrow V\gamma) \left[ (1/3)\sigma_{\chi_2(|h|=0)} + (1/2)\sigma_{\chi_2(|h|=1)} + \sigma_{\chi_2(|h|=2)} \right].$$

More on theoretical uncertainties

Effect of the scale in the $\alpha_s(\mu^2)$:
Upper (dashed) lines $\mu^2 = k_T^2$;
lower (solid) lines $\mu^2 = p^2_t + m^2$
Upper panel $\Upsilon$, lower panel $\chi_b$

Effect of the flux definition:
Solid lines $1/\lambda^{1/2}(\hat{s}, k_{t1}^2, k_{t2}^2)$
dashed lines $1/\hat{s}$
thick dash-dotted $1/(p^2_t + m^2)$
**Γ(1S) Spin alignment at the TEVATRON**

Dash-dotted lines – JB gluons; dashed – dGRV gluons; Thin lines – direct Γ only; thick lines – with χb decays added.