(Multi-) Jet Production and Determination of the Strong Coupling Constant

Markus Wobisch
Louisiana Tech University

XIX International Workshop on Deep-Inelastic Scattering and Related Subjects (DIS 2011)
April 12, 2011
Newport News, VA
\( \alpha_s \) and the RGE

- \( \alpha_s(\mu_R) \): depends on renormalization scale \( \rightarrow \) predicted by “RGE”
- Values \( \alpha_s(\mu_R) \) are not predicted
- if we know value at one scale, \( \alpha_s(\mu_0) \), we know the value at any scale \( \alpha_s(\mu_R) \)
- Agreement: compare \( \alpha_s(M_Z) \)

**QCD test:**
- Determine \( \alpha_s(M_Z) \) \( \rightarrow \) check process independence
- Test predicted running \( \alpha_s(\mu_R) \)

\[
Q^2 \frac{\partial \alpha_s(Q^2)}{\partial Q^2} = \beta \left( \alpha_s(Q^2) \right)
\]

\[
\beta(\alpha_s(Q^2)) = -\beta_0 \alpha_s^2(Q^2) - \beta_1 \alpha_s^3(Q^2) - \beta_2 \alpha_s^4(Q^2) - \beta_3 \alpha_s^5(Q^2) + O(\alpha_s^6)
\]
Knowledge of $\alpha_s$

Renormalization Group Equation has been tested for momenta up to 209 GeV

(LEP $e^+e^-$ data)

$\rightarrow$ But not yet for larger scales
Fermilab Tevatron and DoE

Most precise jet energy calibration at a hadron collider!
Inclusive Jet Cross Section

Very precise: benefit from hard work on jet energy calibration


\[ \sigma_{\text{theory}}(\alpha_s(M_Z)) = \sigma_{\text{pert}}(\alpha_s(M_Z)) \cdot c_{\text{non-pert}} \]
Basic principle

Perturbative cross section formula:

\[ \sigma_{\text{pert}}(\alpha_s) = \left( \sum_n \alpha_s^n c_n \right) \otimes f_1(\alpha_s) \otimes f_2(\alpha_s) \]

- pQCD matrix elements: explicit \( \alpha_s \) dependence
- \( f_1, f_2 \) (PDFs): implicit \( \alpha_s \) dependence

Determine \( \alpha_s \) from data:
- Vary \( \alpha_s \) until sigma-theory agrees with sigma-experiment
  \( \rightarrow \) chi2 minimization

\( \rightarrow \) Procedure requires PDFs as external input
Currently:
Main constraints on high-x gluon density come from Tevatron jet data

Goal:
Minimize correlations between data and PDF uncertainties

→ Restrict $\alpha_s$ analysis to kinematic regions where impact of Tevatron data for PDFs is small.

→ Tevatron jet data don’t affect gluon for $x < 0.2 - 0.3$
Jet cross section has access to x-values of: (in LO kinematics)

\[ x_a = x_T \frac{e^{y_1} + e^{y_2}}{2}, \quad x_b = x_T \frac{e^{-y_1} + e^{-y_2}}{2} \quad \text{with} \quad x_T = \frac{2p_T}{\sqrt{s}} \]

What is the x-value for a given incl. jet data point \( @(p_T, |y|) \)?

→ Not completely constrained – unknown kinematics since we integrate over other jet(s)

→ Construct “test-variable” (treat as if other jet was at \( y=0 \)):

\[ x = x_T \cdot (e^{|y|} + 1)/2 \]

→ Apply cut on this test-variable to restrict accessible x-range

→ Find: requirement \( x \)-test < 0.15
removes most of the contributions with \( x > 0.2 - 0.3 \)

→ 22 (of 110) data points remaining at \( 50 < p_T < 145 \) GeV
x_{\text{min}} / x_{\text{max}} \text{ distributions}

Every analysis bin $\rightarrow$ one plot
Each plot: $x$-min/$x$-max distributions

Cut on test-variable $x$-test $< 0.15$
$\rightarrow$ 22 (of 110) data points remain

These have small contributions from $x > 0.2 - 0.3$

$\leftarrow$ Only data points above green line are used
22(out of 110) inclusive jet cross section data points at $50 < p_T < 145$ GeV

→ Input in $\alpha_s$ analysis
→ Use best theory prediction: NLO + 2-loop threshold corrections (Kidonakis/Owens) with MSTW2008NNLO PDFs

\[
\alpha_s\left(M_Z\right) = 0.1161^{+0.0041}_{-0.0048}
\]

→ Most precise result from a hadron collider

→ Consistent with HERA results and world average

| All uncertainties are multiplied by a factor of 10^3 |
|---------------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| \(\alpha_s(M_Z) = 0.1161^{+0.0041}_{-0.0048}\) (DØ combined fit) |

<table>
<thead>
<tr>
<th>(p_T) (GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Total uncertainty</th>
<th>Experimental uncorrelated</th>
<th>Experimental correlated</th>
<th>Nonperturb. correction</th>
<th>PDF uncertainty</th>
<th>(\mu_{r,f}) variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1161 +4.1 -4.8</td>
<td>±0.1</td>
<td>+3.4 -3.3</td>
<td>+1.0 -1.6</td>
<td>+1.1 -1.2</td>
<td>+2.5 -2.9</td>
</tr>
</tbody>
</table>
Main result: use best theory predictions
NLO + 2-loop threshold corrections
(Kidonakis/Owens)
with MSTW2008NNLO PDFs
\[ \alpha_s \left( M_Z \right) = 0.1161^{+0.0041}_{-0.0048} \]

Use only NLO
with MSTW2008NLO PDFs
\[ 0.1202^{+0.0072}_{-0.0059} \]

- Larger value of “NLO-only” result:
  → due to missing \( O(\alpha_s^4) \) contributions
- Larger uncertainty of “NLO-only” result:
  → due to increased scale dependence (main effect)
  → and increased PDF uncertainty (minor effect)

All uncertainties are multiplied by a factor of \( 10^3 \)

<table>
<thead>
<tr>
<th>Total uncertainty</th>
<th>Experimental uncorrelated</th>
<th>Experimental correlated</th>
<th>Nonperturb. correction</th>
<th>PDF uncertainty</th>
<th>( \mu_{r,f} ) variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1161</td>
<td>±0.1</td>
<td>±0.1</td>
<td>±1.0</td>
<td>±1.1</td>
<td>±2.5</td>
</tr>
<tr>
<td></td>
<td>+4.1</td>
<td>+3.4</td>
<td>+1.0</td>
<td>+1.1</td>
<td>+2.5</td>
</tr>
<tr>
<td></td>
<td>−4.8</td>
<td>−3.3</td>
<td>−1.6</td>
<td>−1.2</td>
<td>−2.9</td>
</tr>
</tbody>
</table>

\( \alpha_s \) extraction at large \( p_T \) requires high (experimental & theory) precision
Running of $\alpha_s$ (?)

- so far tested up to $\mu_r = 209$ GeV (LEP)

Could be modified for scales $\mu_r > \mu_0$
e.g. by extra dimensions

here: $\mu_0 = 200$ GeV and $n=1,2,3$ extra dim.
(n=0 $\rightarrow$ Standard Model)

But: $\alpha_s$ extraction from inclusive jets uses PDFs which were derived assuming the RGE
$\rightarrow$ We cannot use the inclusive jets to test the RGE in yet untested region
... towards testing in the RGE in novel energy regimes

→ Cannot rely on PDF information
   (PDF parametrizations already assume RGE in DGLAP evolution)
Cancelling PDFs: Ratios

**Goal:** test pQCD (and $\alpha_s$) independent of PDFs

Conditional probability:

$$R_{3/2} = P(3\text{rd jet} | 2 \text{ jets}) = \frac{\sigma_{3\text{-jet}}}{\sigma_{2\text{-jet}}}$$

- Probability to find a third jet in an inclusive dijet event
- Sensitive to $\alpha_s$ (3-jets: $\alpha_s^3$ / 2-jets: $\alpha_s^2$)
- (almost) independent of PDFs
\[ R_{3/2} = \frac{\sigma_{3\text{-jet}}}{\sigma_{2\text{-jet}}} \]

**Measure as a function of two momentum scales:**

- \( p_{T\text{max}} \): common scale for both \( \sigma_{2\text{-jet}} \) and \( \sigma_{3\text{-jet}} \)
- \( p_{T\text{min}} \): scale at which 3\text{rd} jet is resolved (\( \sigma_{3\text{-jet}} \) only)

Sensitive to \( \alpha_s \) at the scale \( p_{T\text{max}} \) → probe running of \( \alpha_s(p_{T\text{max}}) \)

**Details:**

- inclusive \( n \)-jet samples (\( n=3,2 \)) with \( n \) (or more) jets above \( p_{T\text{min}} \)
- \(|y| < 2.4\) for all \( n \) leading \( p_T \) jets
- \( \Delta R_{\text{jet,jet}} > 1.4 \) (insensitive to overlapping jet cones)
- study \( p_{T\text{max}} \) dependence for different \( p_{T\text{min}} \) of 50, 70, 90 GeV
  → Measurement of \( R_{3/2}(p_{T\text{max}}; p_{T\text{min}}) \)
Using $R_{3/2}$ to test NLO matrix elements

For a given $\alpha_s(M_Z) = 0.118$:

$\rightarrow$ NLO results for MSTW2008NLO, NNPDF v2.1, ABKM09NLO agree

$\rightarrow$ CT10 slightly higher at high $p_T$
\( \alpha_s \) from inclusive jet cross section:
- detailed analysis to avoid inconsistencies (or circular arguments) related to
  - correlations between experimental and PDF uncertainties
  - assumptions of RGE in DGLAP evolution of PDFs
→ most precise result from a hadron collider → see consistency

\( \alpha_s \) determinations from cross section (assuming PDFs) can not be used to test running in novel energy regimes
→ Need observables which are insensitive to PDFs

\( R_{3/2} \) precision data:
- well described by NLO pQCD
→ basis to extend knowledge of \( \alpha_s \) to novel energy regime
SHERPA (out of the box) describes data
PYTHIA tune DW (tuned to D0 dijet azimuthal decorrelations) fails
→ “softer” tune BW describes the data
CDF Run I result

Claim:
“Test running over $40 < ET < 440$ GeV”

→ Not really!!
because analysis uses PDFs for which DGLAP evolution is already done under assumption of running according to RGE

→ RGE was already assumed

→ No independent test

→ Avoided in the D0 analyses