ASYMPTOTIC BEHAVIOR OF PION FORM FACTORS

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Outline

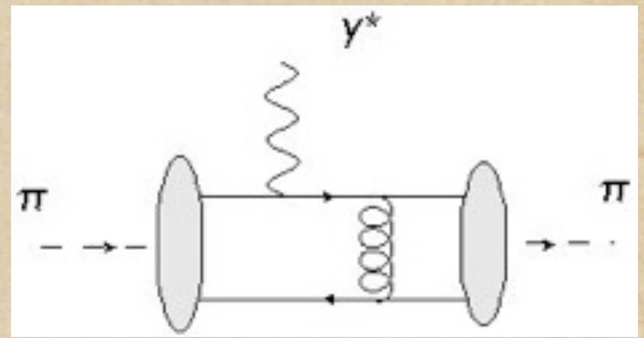
- Pion e.-m. and transition form factors and QCD
- Reggeization of gluons and quarks in QCD
- Phenomenology of reggeized quark exchanges: meson trajectories
- Application to triangle diagrams: pseudoscalar mesons form factors
- Conclusions & Outlook

Electromagnetic and transition form factors of the pion in pQCD

Pion FFs in pQCD

Pion e.-m. form factor $\langle \pi(p+q)|J^\mu(0)|\pi(p)\rangle = eF_\pi(q^2)(2p+q)^\mu$

$$J^\mu = e_u \bar{u}\gamma^\mu u + e_d \bar{d}\gamma^\mu d + \ldots$$



Pion distribution amplitude $\int_0^1 dz\phi_\pi(z) = f_\pi = 92.4\,\mathrm{MeV}$

$$\frac{1}{\sqrt{2}}\langle 0|\bar{d}(z)\gamma^+\gamma_5[z,0]u(0)|\pi^+(P)\rangle = iP^+\int_0^1 dx e^{ix(z\cdot P)}\phi_\pi(x)$$

Asymptotic QCD expectation for Q^2=-q^2»

$$\boxed{F_\pi^{as}(Q^2) = 16\pi\alpha_s(Q^2)\int_0^1 dxdy\frac{\phi_\pi(x)\phi_\pi(y)}{9xyQ^2}}$$

Chernyak, Zhitnitsky, JETP Lett.'77
Lepage, Brodsky, PLB'79
Efremov, Radyushkin, PLB'80; Th.Math.Ph.'80

Integral should converge - requirement on the form of DA

"Asymptotic" DA $\phi_\pi^{as}(z) = 6f_\pi z(1-z)$ $\qquad \int_0^1 dx\frac{\phi_\pi^{as}(x)}{x} = 3$

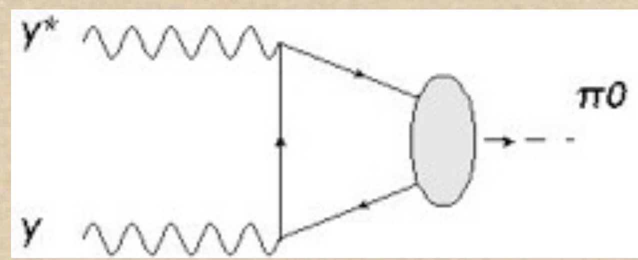
Chernyak, Zhitnitsky'84 $\phi_\pi^{CZ}(z) = 30f_\pi z(1-z)(1-2z)^2$ $\qquad \int_0^1 dx\frac{\phi_\pi^{CZ}(x)}{x} = 5$

By measuring pion FF - measure pion DA

Pion FFs in pQCD

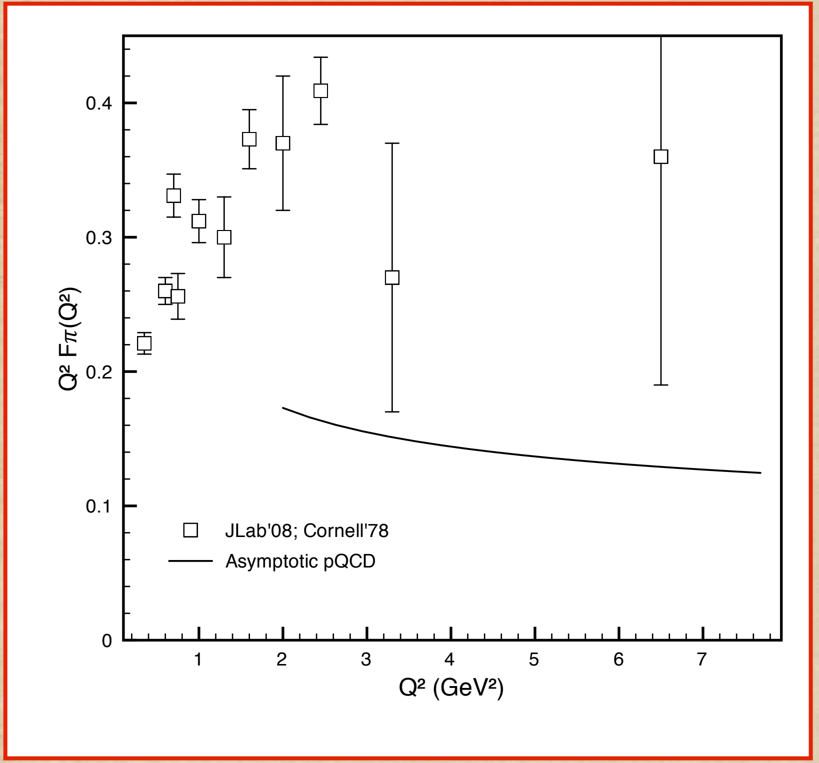
π^0 - γ transition form factor

$$\langle \pi^0(p')\gamma(\lambda,p)|J_\mu|0\rangle = ie^2\epsilon_{\mu\nu\alpha\beta}\epsilon^{*\nu}(\lambda)p'^\alpha p^\beta F_{\pi\gamma}(s)$$

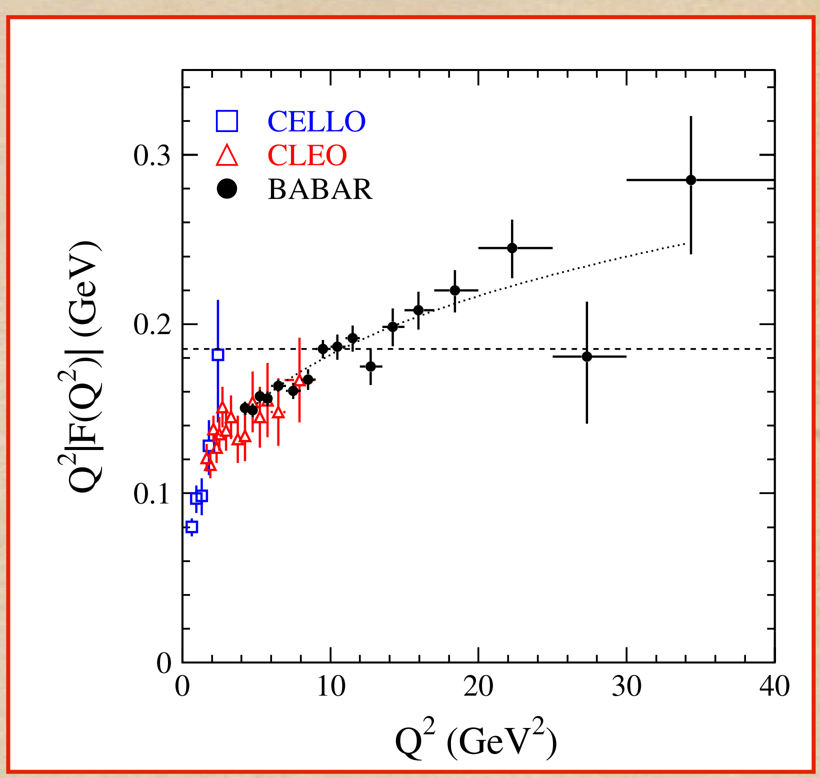


Chiral anomaly: $F_{\pi\gamma}(0) \approx \dfrac{1}{4\pi^2 f_\pi}$

Asymptotic QCD expectation: $Q^2 F_{\pi\gamma}(Q^2\to\infty) = \dfrac{2}{N_c}\displaystyle\int_0^1 dz \frac{\phi_\pi(z)}{z} = 2f_\pi$



Comparison to exp. data



Asymptotic QCD expectations fail badly: either the asymptotic regime sets in very late or an important mechanism is missing (non-perturbative effects, k-perp dependence etc.)

Pion FFs in pQCD

Much theoretical work is dedicated to understanding pion form factors

QCD Sum Rules	Bakulev, Radyushkin, PLB'91; Radyushkin, Acta Phys.Pol.'95; Bakulev, Pimikov, Stefanis, PRD'09
Flat DA + regularization of quark propagator	A.V. Radyushkin, PRD'09; M.V. Polyakov, JETP Lett.'09
Flat DA arise in NJL and chiral quark models	Arriola, Broniowsky, PRD'02, '03
BSE approach: flat pion DA ruled out	H.L.L. Roberts et al, PRC'10

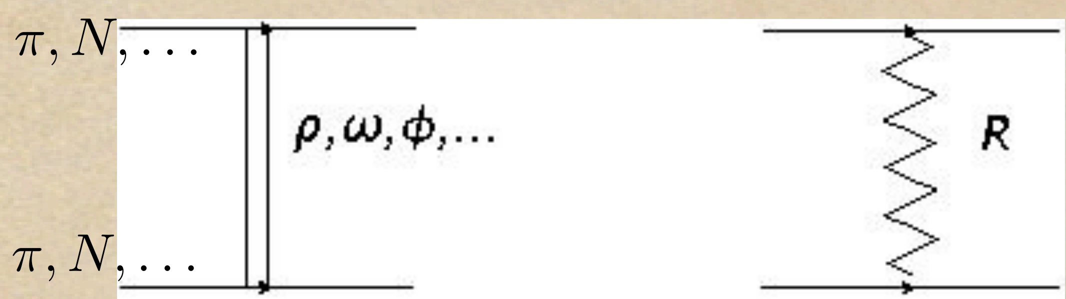
Common feature of models that describe BaBar data:
account for "soft" mechanisms (beyond leading twist) - corrections turn out to be very large!

Hadronic models (VDM, DR etc.)	Arriola, Broniowsky, PRD'10; Belicka et al, PRC'11

Reggeization in QCD

Reggeization of gluons and quarks in QCD

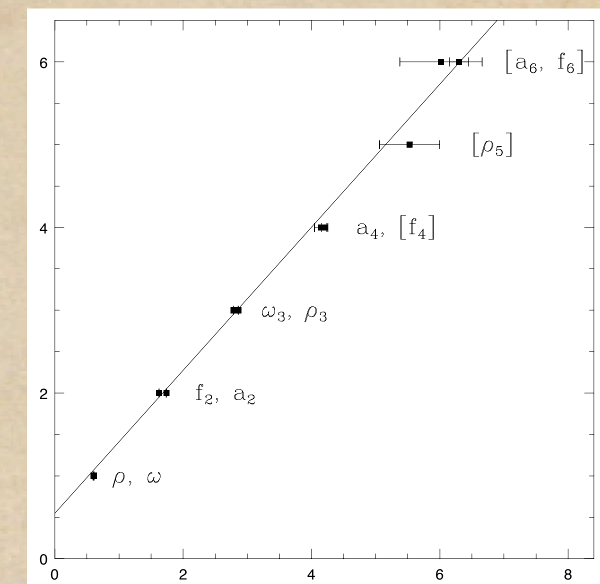
Scattering amplitudes in Regge kinematics $\quad s \to \infty \quad t \sim \text{const.}$



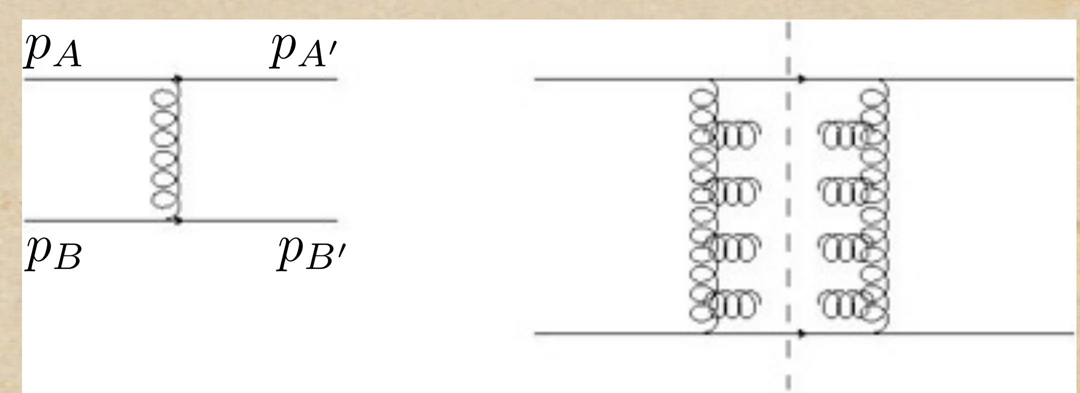
$$f_{M_J}(s,t) \sim \frac{s^J}{t - m^2} \quad \rightarrow \quad f_R(s,t) = (s/s_0)^{\alpha_M(t)}$$

Regge trajectory

$$\alpha_M(t) \approx \alpha_M(0) + \alpha' t$$
$$\alpha_M(M_J^2) = J$$



QCD scattering amplitudes: Multi-Regge kinematics



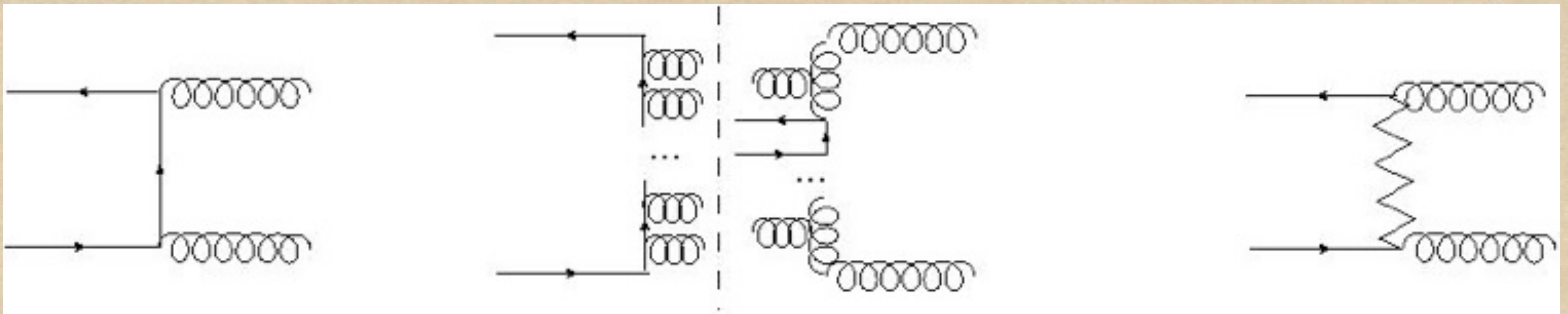
Forward scattering (total cross sections)

Summing up radiative corrections - BFKL Pomeron

$$f_{A'B';AB} \sim \bar{u}(p'_A)\Gamma_{AA'}u(p_A)\bar{u}(p'_B)\Gamma_{BB'}u(p_B)\frac{1}{t} \sim \frac{s}{t}$$
$$\rightarrow \bar{u}(p'_A)\Gamma_{AA'}(t)u(p_A)\bar{u}(p'_B)\Gamma_{BB'}(t)u(p_B)(s/s_0)^{\alpha_P(t)}$$

Fadin, Kuraev, Lipatov, PLB'75;
Lipatov, Yad. Fiz. '76;
Kuraev, Lipatov, Fadin, JETP'76, '77;
Balitsky, Lipatov, Yad. Fiz.'78;

Reggeization of gluons and quarks in QCD



$$f_{GG';Q\bar{Q}}(s,t=-q_\perp^2) \sim \bar{v}(p_{\bar{Q}})\Gamma_{\bar{Q}G'}\frac{\not{q}_\perp}{t-m^2}\Gamma_{QG}u(p_Q) \sim \frac{\sqrt{s}}{t-m^2}$$
$$\to \bar{v}(p_{\bar{Q}})\Gamma_{\bar{Q}G'}\not{q}_\perp\Gamma_{QG}u(p_Q)(s/s_0)^{\alpha_q(t)-1/2}$$

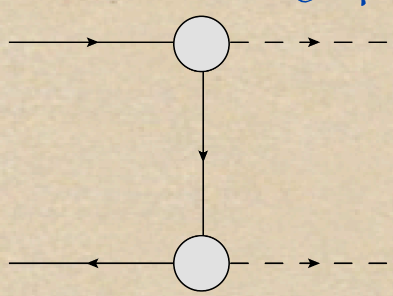
Fadin, Sherman, JETP Lett. '76;
Fadin, Sherman, JETP '77;
Lipatov, Vyazovsky, NPB '01;
Fadin, Fiore, PRD '01;
Kotsky et al, NPB '03;
Bogdan et al, JHEP '02;
Bogdan, Fadin, Yad. Fiz. '05;
Bogdan, Fadin, NPB '06

Pomeron ~ compound state of two reggeized gluons;
Reggeons ~ colorless states of reggeized quark and antiquark

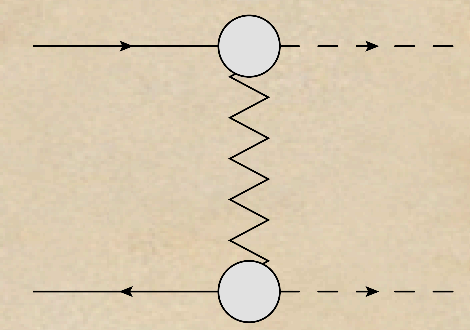
Phenomenology of reggeized quark exchanges:
Meson Regge trajectories

Phenomenology of reggeized quark exchanges

Postulate: similar asymptotic behavior for $q\bar{q}$ -> hadrons, gamma's etc.



$$\frac{\not q}{q^2} \;\to\; \mathcal{P}^{\pm}_{q,R} = \xi^{\pm}_{\alpha_q(q^2)-\frac{1}{2}}\left(\frac{s}{s_0}\right)^{\alpha_q(q^2)-\frac{1}{2}} \not q$$



$$\begin{aligned} f_{q\bar{q}\to\pi\pi}(s,t) &= \boxed{\beta^2_{qq\pi}}\,\bar{v}(p_2)\Gamma_{qq\pi}(p_2,p_{\pi_2})\boxed{\frac{(\not p_1 - \not p_{\pi_1})}{t-m^2}}\Gamma_{qq\pi}(p_1,p_{\pi_1})u(p_1) \\ &\to \boxed{\tilde{\beta}^2_{qq\pi}(t)}\,\bar{v}(p_2)\Gamma_{qq\pi}(p_2,p_{\pi_2})\boxed{\mathcal{P}^{\pm}_{q,R}(s,t)}\Gamma_{qq\pi}(p_1,p_{\pi_1})u(p_1) \end{aligned}$$

The main object of interest: quark trajectory $\qquad \alpha_q(t) = \alpha_q(0) + \alpha'_q t$

The standard Regge pre-factor $\qquad \xi^{\pm}_{\alpha(t)} = \dfrac{1 \pm e^{-i\pi\alpha(t)}}{\sin(\pi\alpha(t))}\dfrac{\pi\alpha'}{\Gamma(\alpha(t)+1)}$

Careful: complex phase would require "colored resonances" in the crossed channel $q\pi \to q\pi$ - unphysical

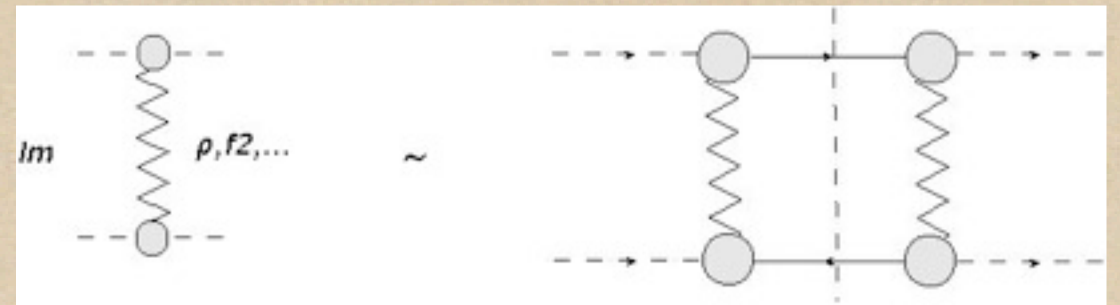
To avoid any redundant parameters: simple ansatz

$$\boxed{\mathcal{P}_R(s,t) \;=\; \pi\alpha'_q\left(\frac{s}{s_0}\right)^{\alpha_q(t)-\frac{1}{2}} \not q}$$

Phenomenology of reggeized quark exchanges

The parameters of the Regge quark exchange: $\boxed{\alpha_q(t) = \alpha_q(0) + \alpha'_q t \qquad \tilde{\beta}_{q\bar{q}\pi}(t) \sim \tilde{\beta}_{q\bar{q}\pi} \times e^{b_\pi t}}$

Employ phenomenology: identify meson Regge exchanges with reggeized quark-antiquark exchanges



$$\begin{aligned}
\mathrm{Im}T_{\pi\pi\to\pi\pi}(s+i\epsilon,t) &= \frac{3N_c\beta_\pi^4\pi{\alpha'_q}^2}{1024 f_\pi^4\tilde{b}_{\pi\pi}^4}\left(\frac{s}{s_0}\right)^{2\alpha_u(0)-1} e^{\frac{2}{3}\tilde{b}_{\pi\pi}t}\\
\mathrm{Im}T_{\pi\pi\to K\bar{K}}(s+i\epsilon,t) &= \frac{3N_c\beta_\pi^2\beta_K^2\pi{\alpha'_q}^2}{1024 f_\pi^2 f_K^2\tilde{b}_{\pi K}^4}\left(\frac{s}{s_0}\right)^{\alpha_u(0)+\alpha_s(0)-1} e^{\frac{2}{3}\tilde{b}_{\pi K}t}
\end{aligned}$$

$$\begin{aligned}
\tilde{\beta}_{qq\pi} &= \frac{\beta_\pi}{2f_\pi}\\
\tilde{\beta}_{qqK} &= \frac{\beta_K}{2f_K}\\
\tilde{b}_{\pi\pi} &= b_{\pi\pi} + \alpha'_q\ln(s/s_0)
\end{aligned}$$

Compare to the well-known trajectories $\rho,\ f_2,\ K^*,\ \phi$

$$\begin{aligned}
2\alpha_u(0)-1 &\approx \alpha_\rho(0)\approx\alpha_{f_2}(0)\approx 0.5\\
\alpha_u(0)+\alpha_s(0)-1 &\approx \alpha_{K^*}(0)\approx 0.3\\
2\alpha_s(0)-1 &\approx \alpha_\phi(0)\approx 0.1\\
\alpha'_q &\approx (1/2){\alpha'}_\rho \approx 0.45\,\mathrm{GeV}^{-2}
\end{aligned}$$

$$\begin{aligned}
\alpha_u(0)=\alpha_d(0) &\approx 0.75\\
\alpha_s(0) &\approx 0.55\\
\alpha'_{u,d,s} &\approx \alpha'_\rho/2 \approx 0.45\,\mathrm{GeV}^{-2}
\end{aligned}$$

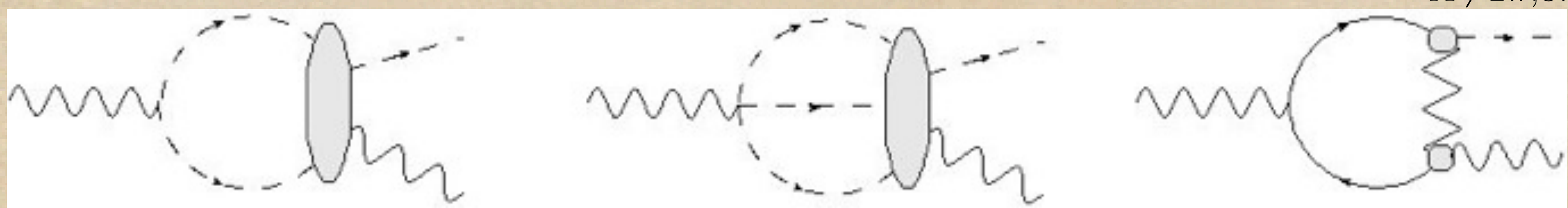
u,d quarks reggeize strongly
s quark reggeizes very mildly

Application to triangle diagrams: pion form factors with unitarity and Regge asymptotics

Pion transition FF with unitarity and Regge

Dispersion representation: $$ReF_{\pi\gamma}(s) = \frac{1}{\pi}\int_{s_{th}} ds' \frac{ImF_{\pi\gamma}(s')}{s'-s}$$

Unitarity in the timelike region: $$ImF_{\pi\gamma} = t^*_{2\pi,\pi\gamma}\rho_{2\pi}F_\pi + t^*_{3\pi,\pi\gamma}\rho_{3\pi}F_{3\pi} + \sum_{X\neq 2\pi,3\pi} t^*_{X,\pi\gamma}\rho_X F_X$$



Hadronic contributions: pure VDM (**ω≈3π**) and VDM + rescattering (**ρ≈2π**)

Truong, PRD '02
Holstein, PRD '96

Quark-hadron duality: $$\sum_{X\neq 2\pi,3\pi} t^*_{X,\pi\gamma}\rho_X F_X \approx \int d\Omega_q t^*_{q\bar{q},\pi\gamma}\rho_q F_q \theta(s-\mu^2)$$

$$\begin{aligned} \mathrm{Im}_{q\bar{q}} F_{\gamma^*\pi^0\gamma}(s+i\epsilon) &= -\frac{N_c(e_u^2-e_d^2)\beta_\pi\pi^2\alpha'_q}{2s_0}\left(\frac{s}{s_0}\right)^{\alpha_q(0)-\frac{3}{2}}\int_{-s}^{0} dt_2 e^{\tilde{b}_{\pi\pi}t_2}t_2 \\ &= \frac{(4\pi f_\pi)^2}{3\beta_\pi\sqrt{\pi}s_0}\sqrt{\mathrm{Im}T_{\pi\pi}(s+i\epsilon,t=0)}\left(\frac{s}{s_0}\right)^{-1} \approx \frac{0.25GeV^2}{s_0\beta_\pi}\left(\frac{s}{s_0}\right)^{\frac{\alpha_\rho(0)}{2}-1} \end{aligned}$$

Energy dependence is fixed by the asymptotics of **ππ** scattering

$$sF_{\gamma^*\pi^0\gamma}(s\to\infty) \sim \left(\frac{s}{1GeV^2}\right)^{\frac{\alpha_\rho(0)}{2}} \approx \left(\frac{s}{1GeV^2}\right)^{0.25}$$

compares well to BaBar

Pion e-m FF with unitarity and Regge

Dispersion representation: $$Re F_\pi(s) = 1 + \frac{s}{\pi}\int_{s_{thr}} \frac{ds'}{s'}\frac{ImF_\pi(s')}{s'-s}$$

Unitarity in the timelike region:

$$ImF_\pi = t^*_{2\pi,2\pi}\rho_{2\pi}F_\pi + t^*_{K\bar K,2\pi}\rho_{2K}F_K + \sum_{X\neq 2\pi, K\bar K} t^*_{X,2\pi}\rho_X F_X$$

Hadronic contributions: N/D approach $$F_\pi(s) = N(s)/D(s)$$

$$\begin{aligned} N(s) &= \sum_{X\neq 2\pi}\frac{1}{\pi}\int_{s_i} ds' \frac{D(s')Re\left[t^*_{X,2\pi}(s')\rho_X(s')F_X(s')\right]}{[1-it^*_{2\pi,2\pi}(s')\rho_{2\pi}(s')](s'-s)} \\ D(s) &= \exp\left(-\frac{s}{\pi}\int_{s_{th}} ds' \frac{\phi(s')}{(s'-s)s'}\right) \end{aligned}$$

$$\tan\phi = \frac{\mathrm{Re}t_{2\pi,2\pi}\rho_{2\pi}}{(1-\mathrm{Im}t_{2\pi.2\pi}\rho_{2\pi})}$$

Quark-hadron duality: $$\sum_{X\neq 2\pi, K\bar K} t^*_{X,\pi\pi}\rho_X F_X \approx \int d\Omega_q t^*_{q\bar q,\pi\pi}\rho_q F_q \theta(s-\mu^2)$$

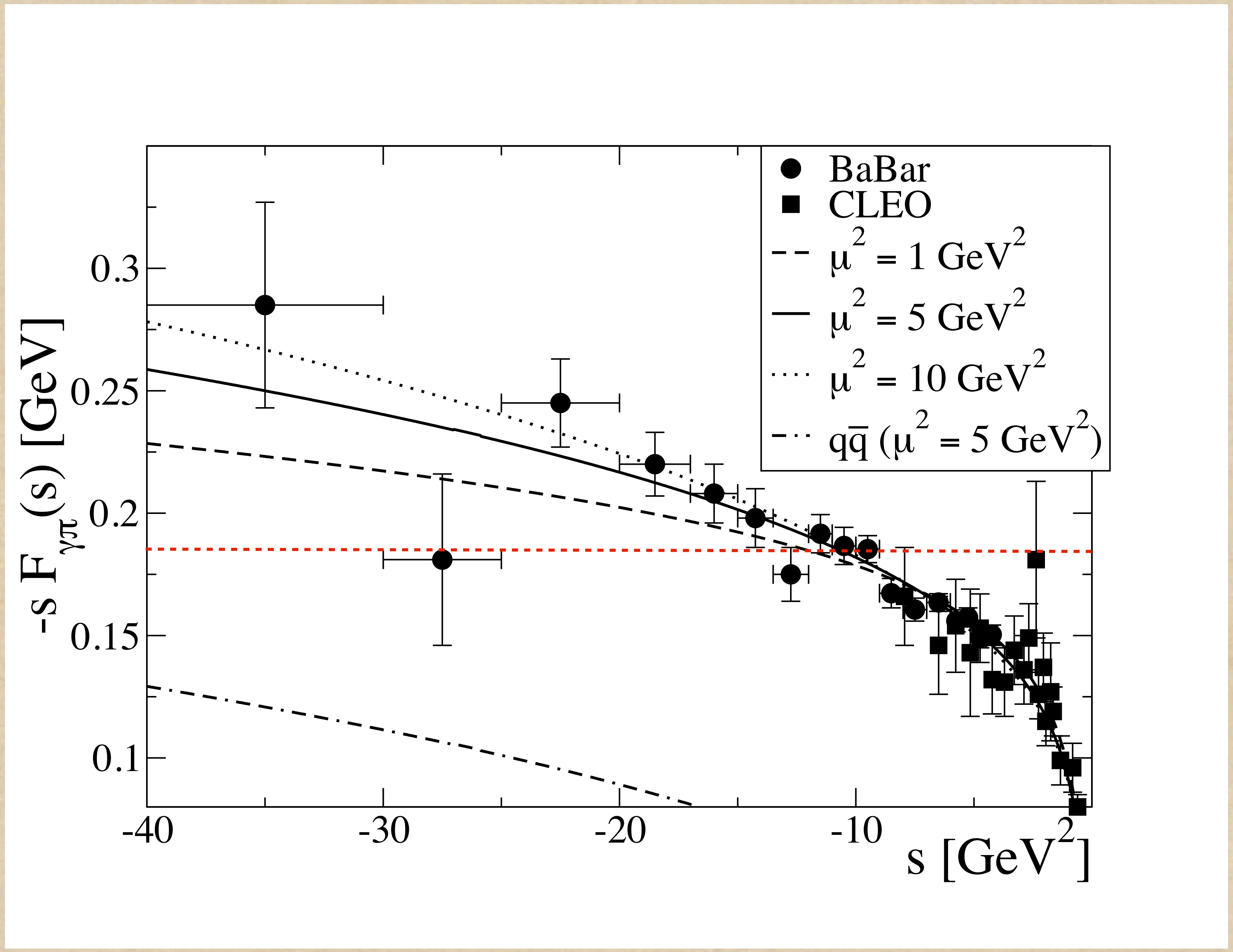
$$\begin{aligned} \mathrm{Im}F_\pi(s+i\epsilon) &= \frac{N_c(e_u-e_d)\beta_\pi^2\pi\alpha'_q}{16\pi f_\pi^2 s_0}\left(\frac{s}{s_0}\right)^{\alpha_q(0)-\frac{3}{2}}\int_{-s}^{0} dt_2 e^{\tilde b_{\pi\pi}t_2}t_2^2 \\ &= \frac{4}{\sqrt{\pi}s_0\tilde b_{\pi\pi}}\sqrt{\mathrm{Im}T_{\pi\pi}(s+i\epsilon, t=0)}\left(\frac{s}{s_0}\right)^{-1} = \frac{0.63}{1+0.125\ln(s/s_0)}\left(\frac{s}{s_0}\right)^{\frac{\alpha_\rho(0)}{2}-1} \end{aligned}$$

We predict the asymptotic behavior $$sF_\pi(s\to\infty) \sim \frac{1}{\ln(s/s_0)}\left(\frac{s}{1GeV^2}\right)^{0.25}$$

Result for the pion form factors

Result for the pion transition FF

Two free parameters: β_π, μ^2

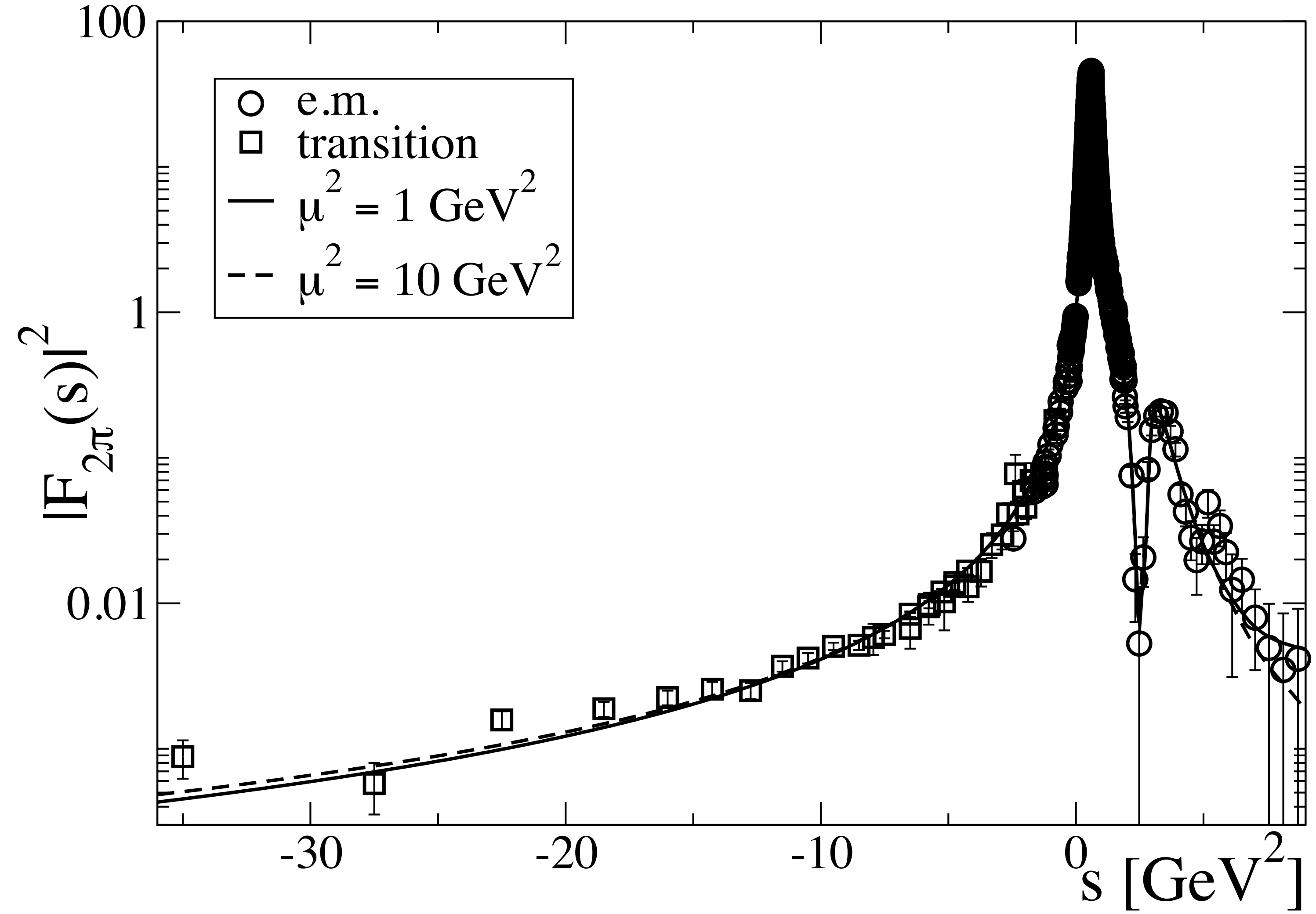


Good fit; even at largest s half of the FF is due to VDM

Combined description of pion FFs

e.-m. and transition FFs on top of each other (both normalized to unity at s=0)

Asymptotic part: parameters from the transition FF fit



Overall: good agreement between the e-m and transition FF data (where they overlap)

Summary

Dispersion analysis of pion form factors:

- Fully unitarized description of hadronic channels

- Employed Regge asymptotics due to colored (quark) Regge exchanges

- Parameters of quark-Regge trajectories: deduced from meson Regge trajectories

- Hadronic contribution (VDM + rescattering): at least as important as the asymptotic part even at largest s

- Asymptotic part: compatible with BaBar results

- Predict pion e.-m. FF to follow similar asymptotics as the pion transition FF

- More work underway