Higher twist parton distributions from light-cone wave functions

arXiv:1103.1269 (Braun, Lautenschlager, Manashov, BP)

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Introduction

Leading twist

- Parton densities, $q(x)$, $\Delta q(x)$, ...
- Functions of one variable.

Twist ≥ 3

- Correlation functions.
- Functions of at least two variables.
- Models wanted!

This work: expansion of Nucleon state in Fock components with 3 and 4 constituents at $\mu^2 = 1\,\mathrm{GeV}^2$.

G. P. Lepage, S. J. Brodsky, PRD22, 2157 (1980)
J. Bolz P. Kroll, ZPA356, 327 (1996)
M. Diehl, T. Feldmann, R. Jakob, P. Kroll, EPJ C 8,409 (1999)
X. d. Ji, J. P. Ma, F. Yuan, EPJ C 33, 75 (2004).
B. Pasquini, S. Cazzaniga, S. Boffi, PRD 78, 034025 (2008)

Nucleon wave functions, 3q

$$|p,+\rangle_{uud} = -\frac{\epsilon^{ijk}}{\sqrt{6}} \int [\mathcal{D}X]_3 \Psi^{(0)}_{123}(X) \times \Big(u^\dagger_{i\uparrow}(1) u^\dagger_{j\downarrow}(2) d^\dagger_{k\uparrow}(3) - u^\dagger_{i\uparrow}(1) d^\dagger_{j\downarrow}(2) u^\dagger_{k\uparrow}(3) \Big) |0\rangle$$

Wave function

$$\Psi^{(0)}_{123}(X) = \frac{1}{4\sqrt{6}} \phi(x_1, x_2, x_3) \frac{(16\pi^2 a_3^2)^2}{x_1 x_2 x_3} \exp\left[-a_3^2 \sum_i k_{\perp i}^2 / x_i \right]$$

Transverse size $a_3 \approx 0.73\,\mathrm{GeV}^{-1}$

- Here: $\phi(x_1, x_2, x_3)$ is the same as tw-3 nucleon distribution amplitude. Can be estimated using lattice methods, SVZ sum rules, ... $\rightarrow$ see arXiv:1103.1269

$$\phi(x_1, x_2, x_3) = 120 f_N x_1 x_2 x_3 \left[1 + a\frac{3}{4}(x_1 - x_3) + b\frac{1}{4}(x_1 + x_3 - 2x_2) + \dots\right]$$

- SVZ: $f_N = 5.0\,\mathrm{GeV}^2$, Lattice: $a, b = \mathcal{O}(1)$

Nucleon wave functions, 3q+1g

$$|p,+\rangle_{uudg_\downarrow} = \epsilon^{ijk} \int [\mathcal{D}X]_4 \, \Psi^{\downarrow}_{1234}(X) \times g^{a,\dagger}_{\downarrow}(4) \, [t^a u_\uparrow(1)]^\dagger_i \, u^\dagger_{j\uparrow}(2) \, d^\dagger_{k\uparrow}(3) \, |0\rangle$$

$$|p,+\rangle_{uudg^\uparrow} = \epsilon^{ijk} \int [\mathcal{D}X]_4 \, g^{a,\dagger}_{\uparrow}(4) \times$$
$$\Big\{ \Psi^{\uparrow(1)}_{1234}(X) \, [t^a u_\downarrow(1)]^\dagger_i \Big(u^\dagger_{j\uparrow}(2) d^\dagger_{k\downarrow}(3) - d^\dagger_{j\uparrow}(2) u^\dagger_{k\downarrow}(3) \Big)$$
$$+ \Psi^{\uparrow(2)}_{1234}(X) u^\dagger_{i\downarrow}(1) \Big([t^a u_\downarrow(2)]^\dagger_j \, d^\dagger_{k\uparrow}(3) - [t^a d_\downarrow(2)]^\dagger_j \, u^\dagger_{k\uparrow}(3) \Big) \Big\} \, |0\rangle$$

$$\Psi^{\downarrow,\uparrow(1,2)}_{1234}(X) = \frac{1}{\sqrt{2x_4}} \, \psi^{\downarrow,\uparrow(1,2)}(x_1, x_2, x_3 x_4) \, \frac{(16\pi^2 a_4^2)^3}{x_1 x_2 x_3 x_4} \exp\left[-a_4^2 \sum_i k^2_{\perp i}/x_i \right]$$

$$\psi^{\downarrow,\uparrow(1,2)}(x_1, x_2, x_3 x_4) = N^{\downarrow,\uparrow(1,2)} x_1 x_2 x_3 x_4^2$$

General strategy

Wanted: $\langle p| \mathcal{O}(z_1, \ldots) |p\rangle$

$$|p\rangle = |uud\rangle + |uudg \uparrow\rangle + |uudg \downarrow\rangle$$
$$\mathcal{O} = \prod (q, \bar{q}, f, \ldots)$$

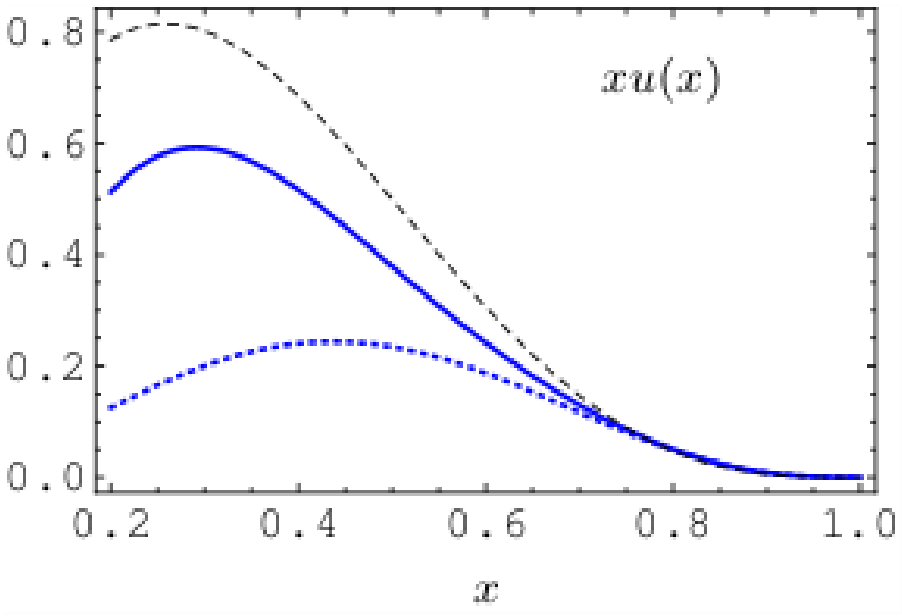
Canonical expansion (quasipartonic operators)

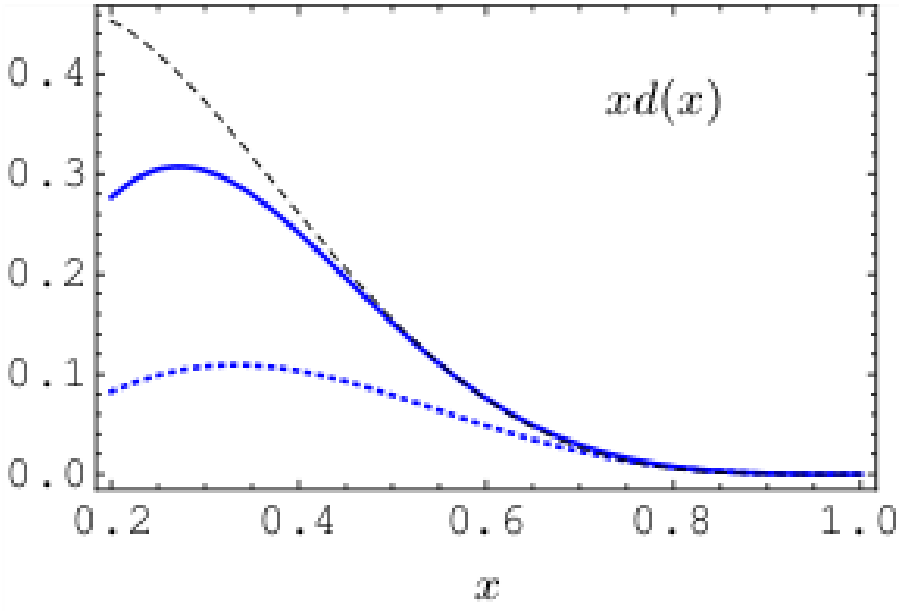
$$q_+^{\downarrow(\uparrow)}(x) = \int \frac{dp_+}{\sqrt{2p_+}} \frac{d^2p_\perp}{(2\pi)^3} \theta(p_+) \left[e^{-ipx} b_{\downarrow(\uparrow)}(p) + e^{+ipx} d^\dagger_{\uparrow(\downarrow)}(p) \right]$$

$$\{b_\lambda(p), b^\dagger_{\lambda'}(p')\} = 2p_+(2\pi)^3 \delta_{\lambda,\lambda'} \delta(p_+ - p'_+) \delta^2(p_\perp - p'_\perp)$$

etc.

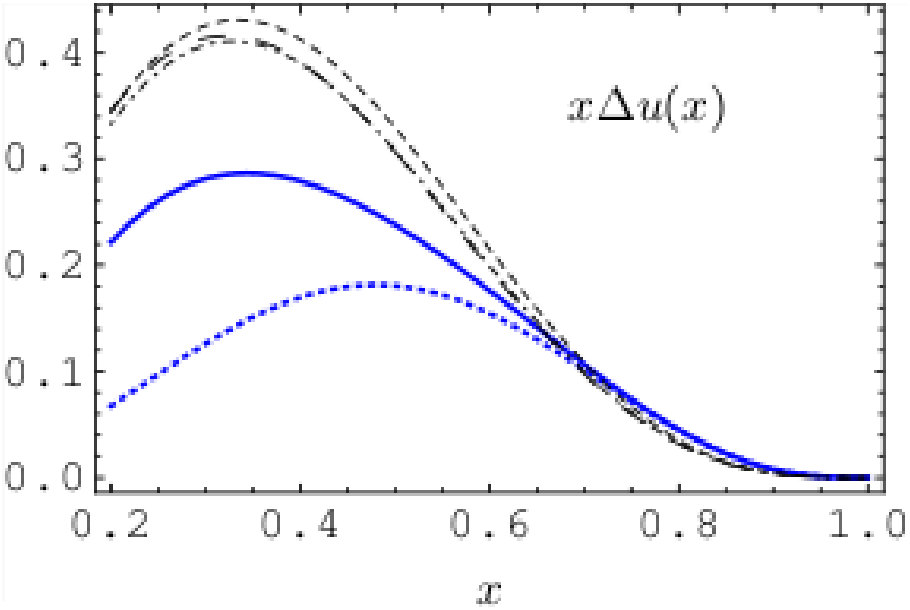
Unpolarized quark distributions vs. GRV

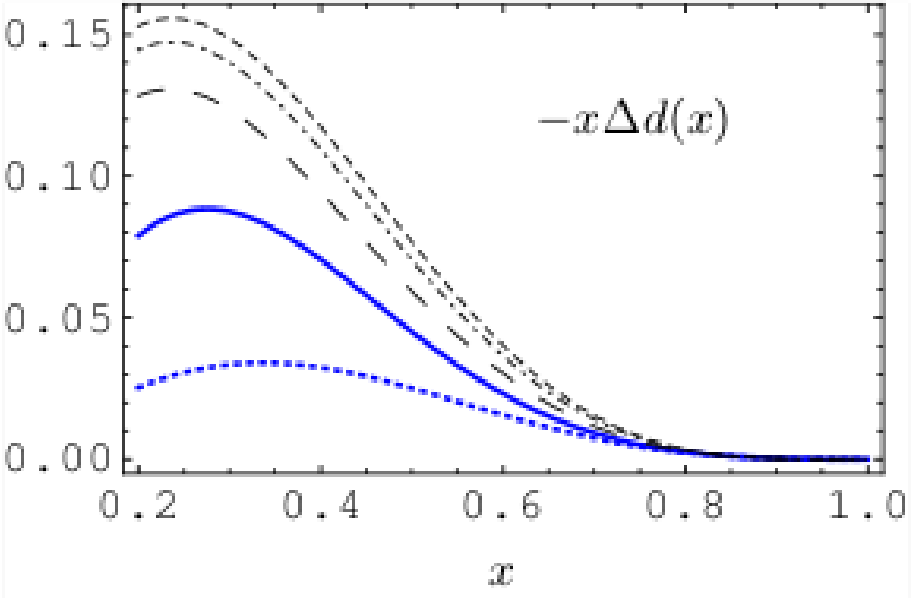




$$q(x) = \int \frac{dz}{2\pi} e^{2iP_+xz} \langle P| \bar{q}_+^{\uparrow}(-zn) q_+^{\uparrow}(zn) + \bar{q}_+^{\downarrow}(-zn) q_+^{\downarrow}(zn) |P\rangle$$

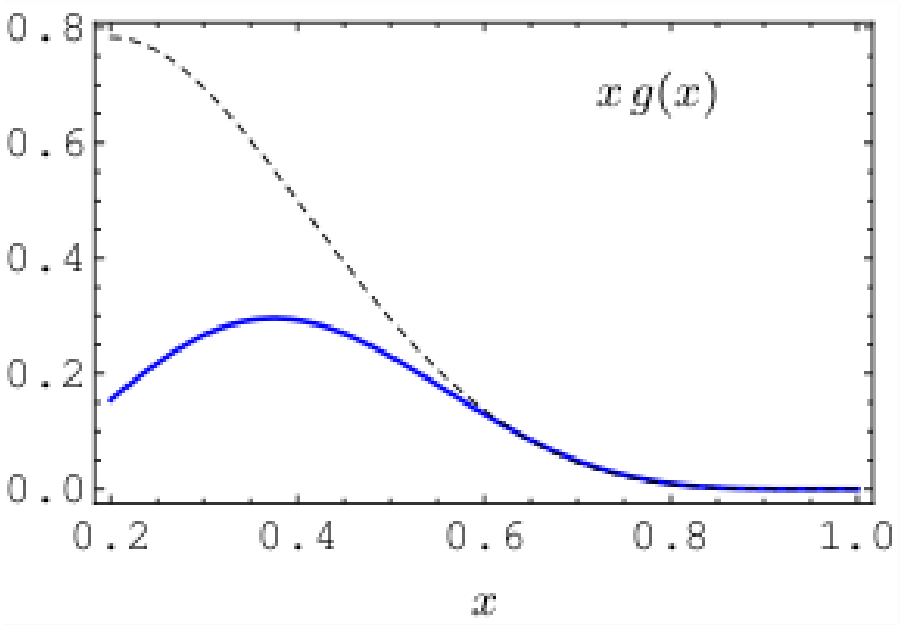
Polarized quark distributions

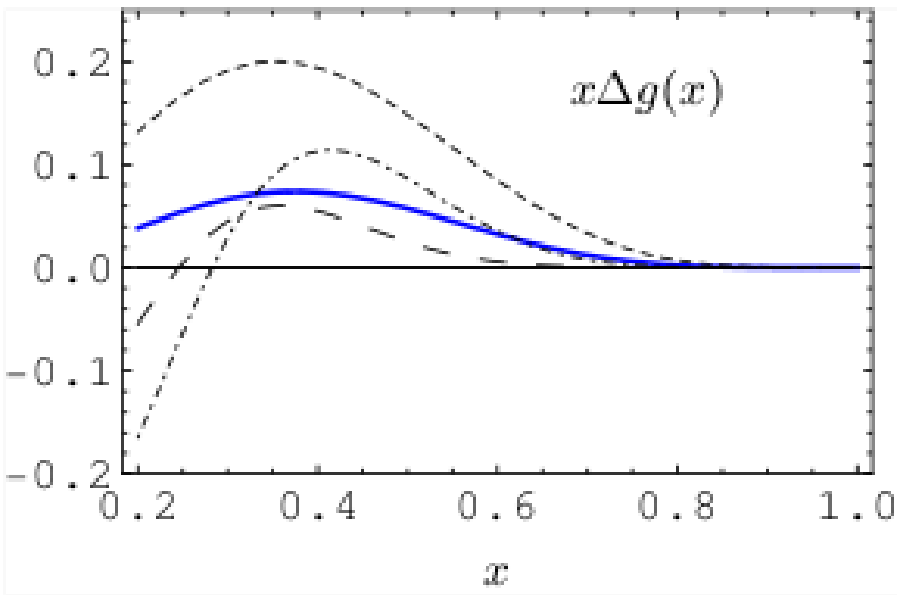




$$\Delta q(x) = \int \frac{dz}{2\pi} e^{2iP_+xz} \langle P,+| \bar{q}_+^{\uparrow}(-zn) q_+^{\uparrow}(zn) - \bar{q}_+^{\downarrow}(-zn) q_+^{\downarrow}(zn) |P,+\rangle$$

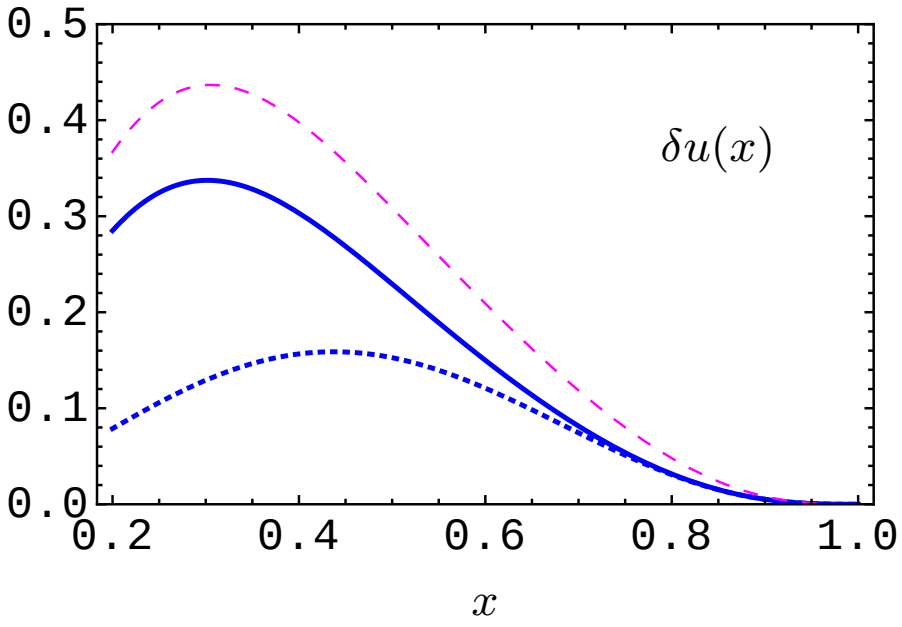
Gluon distributions

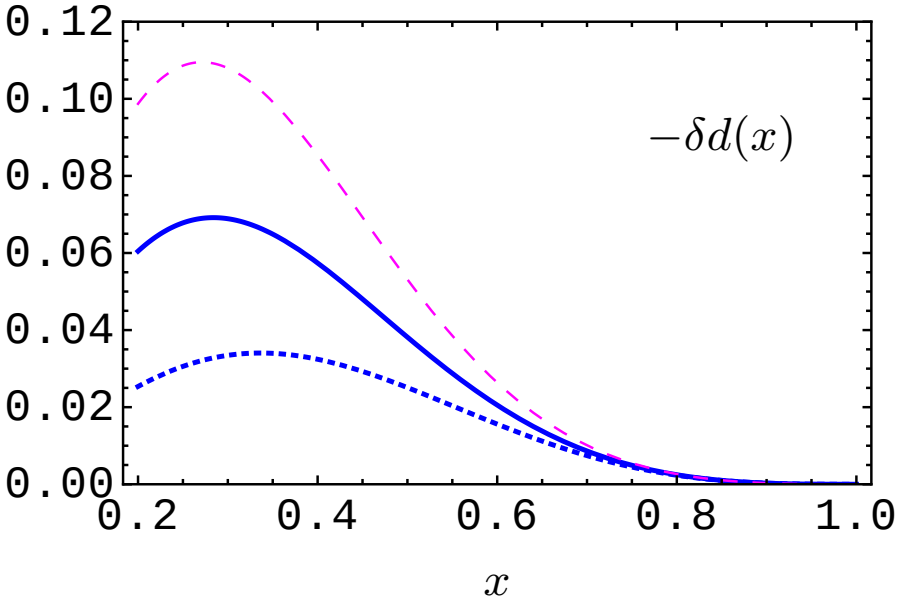




$$\begin{pmatrix} xg(x) \\ x\Delta g(x) \end{pmatrix} = \int \frac{dz}{2\pi P_+} e^{2iP_+xz} \langle f^a_{++}(-zn)\bar{f}^a_{++}(zn) \pm \bar{f}^a_{++}(-zn) f^a_{++}(zn)\rangle$$

Transversity distributions



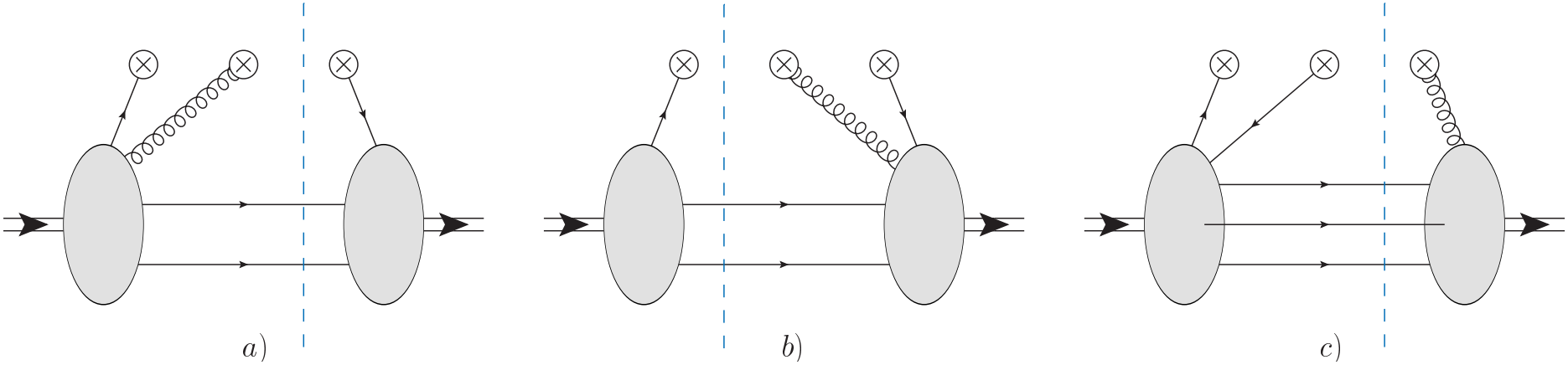


$$\delta q(x) = \int \frac{dz}{2\pi} e^{2iP_+xz} \langle P,+| \bar{q}_+^{\uparrow}(-zn) q_+^{\downarrow}(zn) |P,-\rangle$$

$$q(x) + \Delta q(x) \geq 2|\delta q(x)|$$

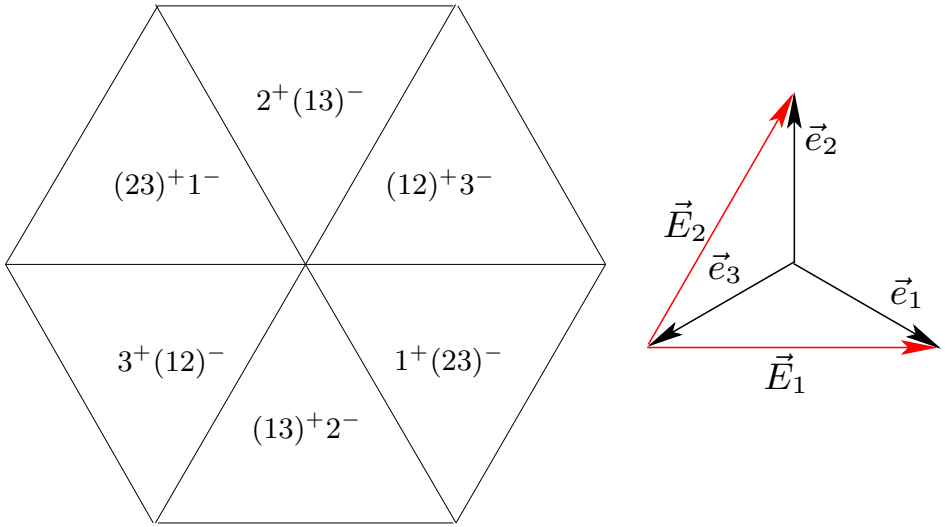
J. Soffer, PRL 74, 1292 (1995)

X. Artru, M. Elchikh, J. M. Richard *et al.*, Phys. Rept. **470**, 1 (2009).



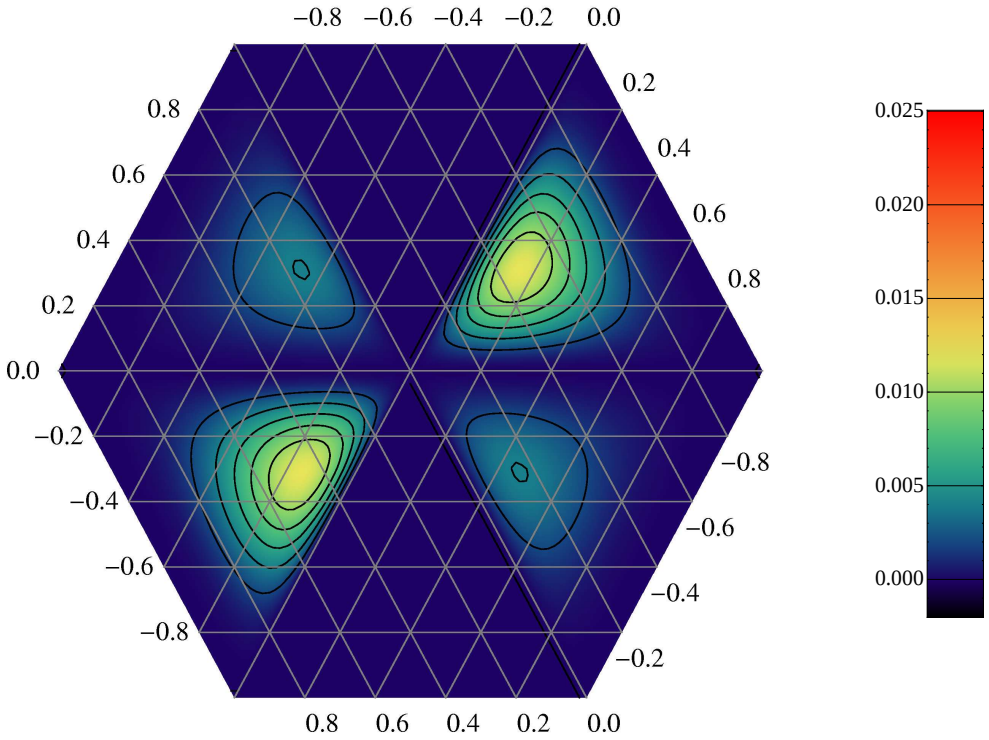
Tw-3 Matrix element of the type

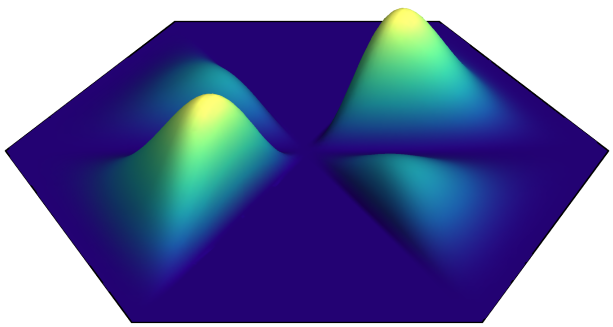
$$\langle p|\,\bar{q}Fq\,|p\rangle = \langle uudg|\,\bar{q}Fq\,|uud\rangle + \langle uud|\,\bar{q}Fq\,|uudg\rangle \overset{\text{F.T.}}{\to} \mathcal{Q}(x_1, x_2, x_3)$$



$$x_1 + x_2 + x_3 = 0\,, \qquad x_1\vec{e}_1 + x_2\vec{e}_1 + x_3\vec{e}_3 = x_1\vec{E}_1 + x_2\vec{E}_2$$

E.g. for down quarks, $\mathcal{Q}_d^+(\mathbf{x})$ at $1\,\mathrm{GeV}^2$

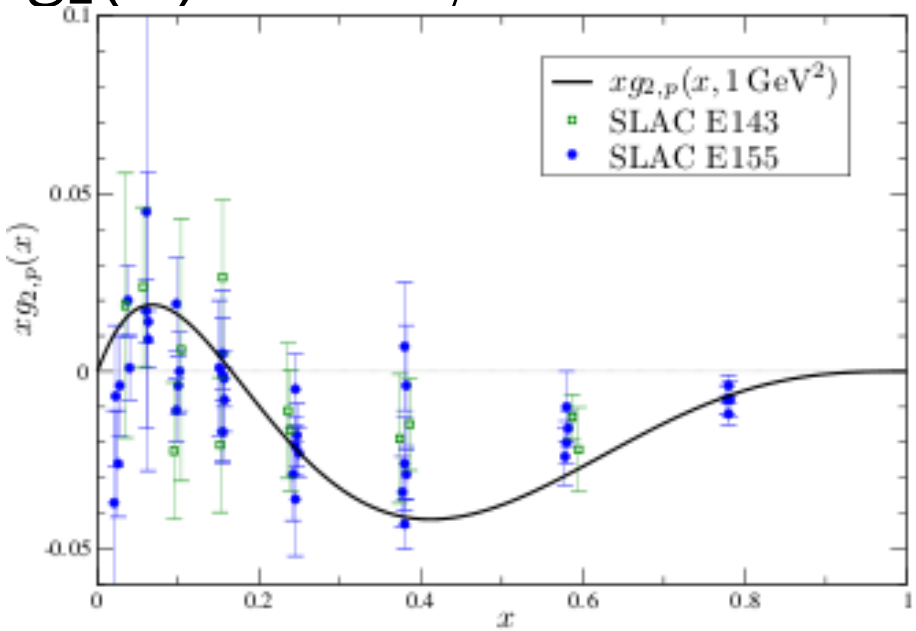


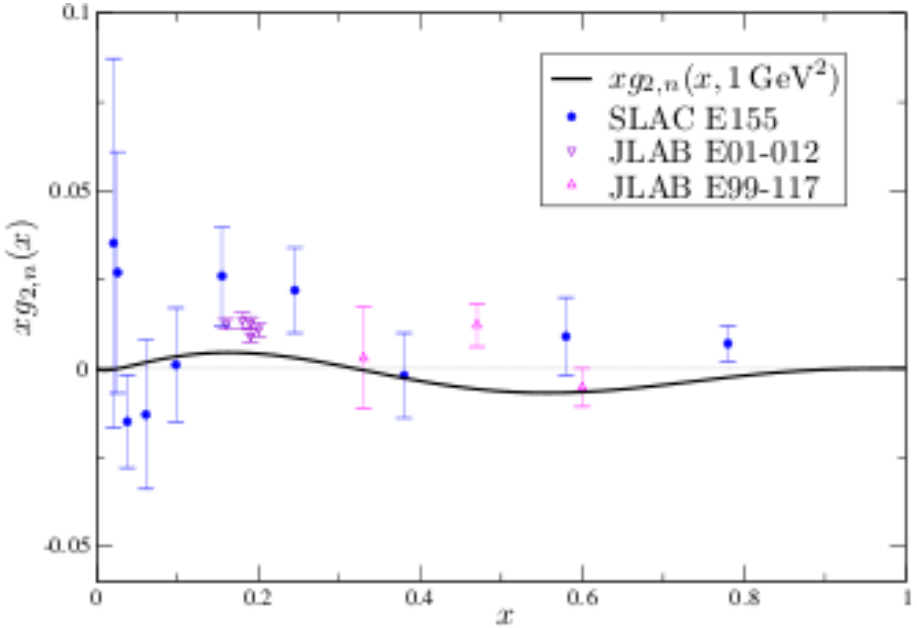


$$\langle p,+|\,\bar{d}_+^{\uparrow}(z_1)f_{++}(z_2)d_+^{\uparrow}(z_3)\,|p,-\rangle + (\uparrow\leftrightarrow\downarrow) + (hc.) =$$

$$= -ip_+^2 \int \mathcal{D}x\, e^{-ip_+(\mathbf{x}\cdot\mathbf{z})}\,\mathcal{Q}_d^+(\mathbf{x})$$

$xg_2(x)$ Proton/Neutron



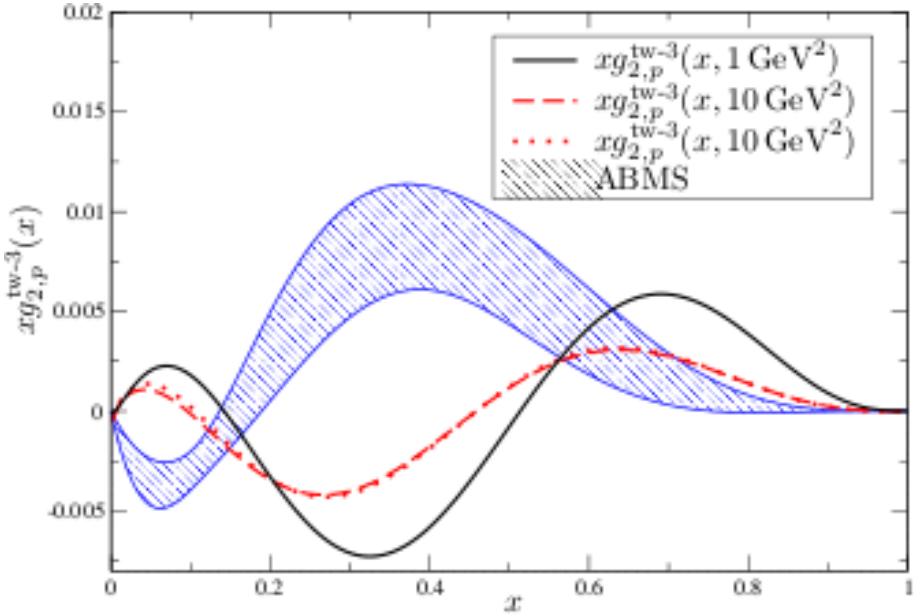


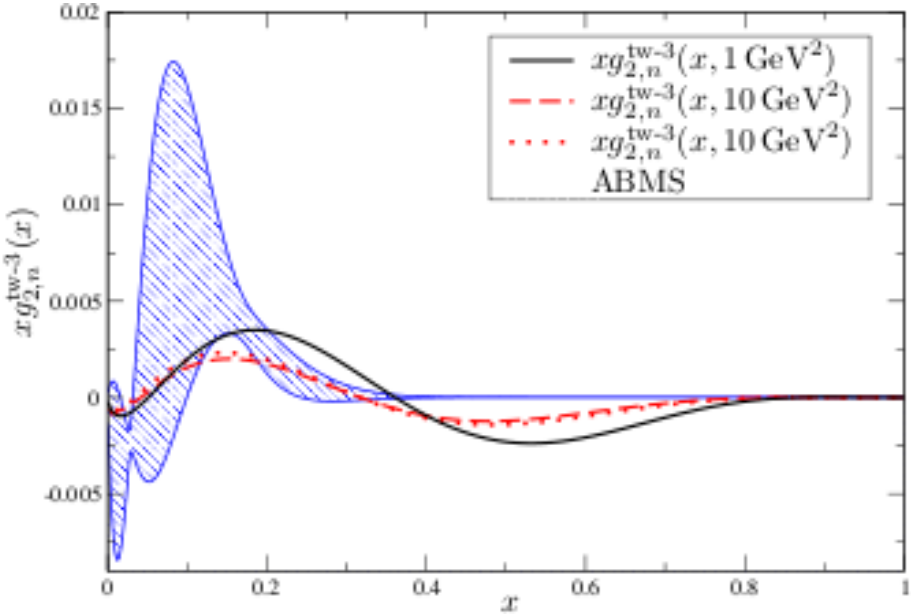
$$g_2(x_B, Q^2) = g_2^{WW}(x_B, Q^2) + g_2^{tw-3}(x_B, Q^2)$$

$$g_2^{WW}(x_B, Q^2) = -g_1(x, Q^2) + \int_{x_B}^{1} \frac{dy}{y} g_1(y, Q^2)$$

$$g_1(x, Q^2) = \frac{1}{2} \sum_q e_q^2 [\Delta q(x_B, Q^2) + \Delta q(-x_B, Q^2)]$$

$xg_2(x)$ Proton/Neutron - WW breaking





→ A. Accardi, A. Bacchetta, W. Melnitchouk, M. Schlegel, JHEP**0911**, 093 (2009).

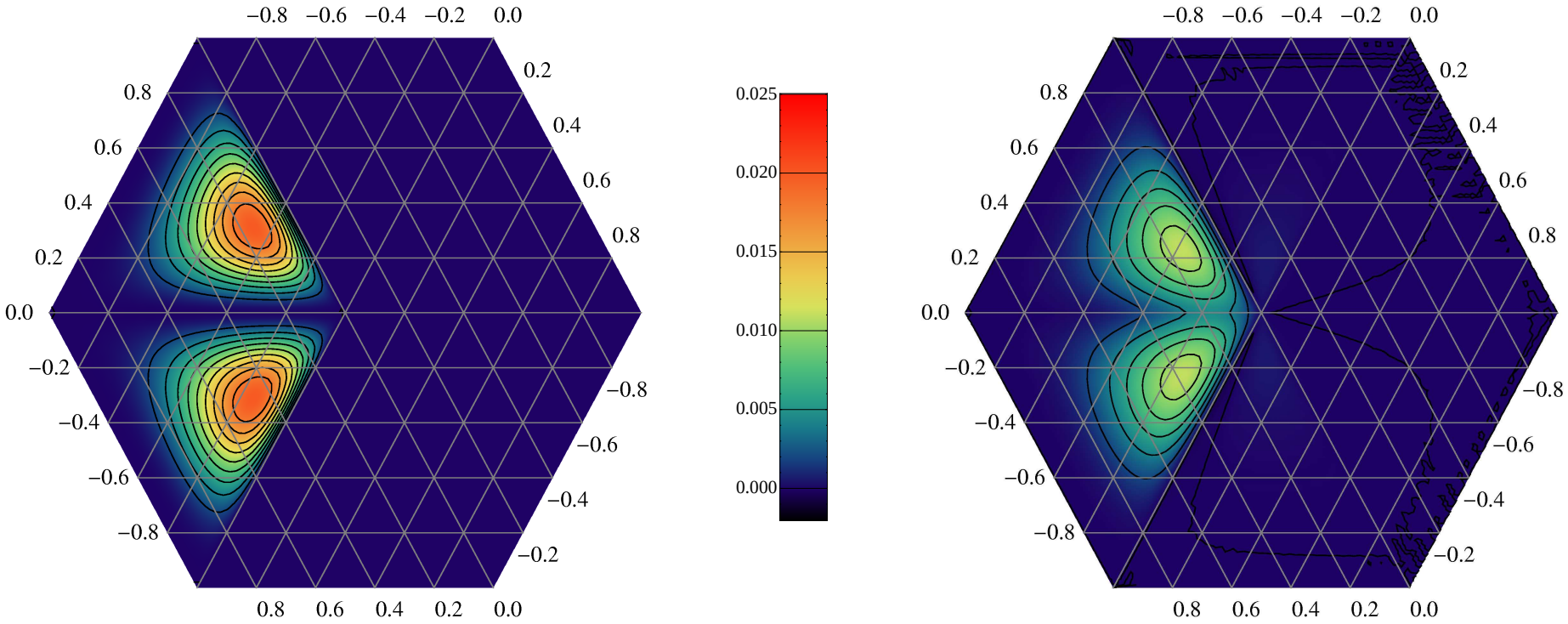
$$g_2^{\text{tw}-3}(x_B, Q^2) = \frac{g}{2m_N} \sum_q e_q^2 \int \mathcal{D}x\, \mathcal{Q}_q^+(\mathbf{x}) \left[\frac{\theta(x_3 - x_B)}{x_2 x_3^2} - \frac{\delta(x_3 - x_B)}{x_2 x_3} - \frac{\theta(x_3 - x_B)}{x_2^2 x_3} + \frac{\theta(x_1 - x_B)}{x_2^2 x_1} \right]$$

SSA

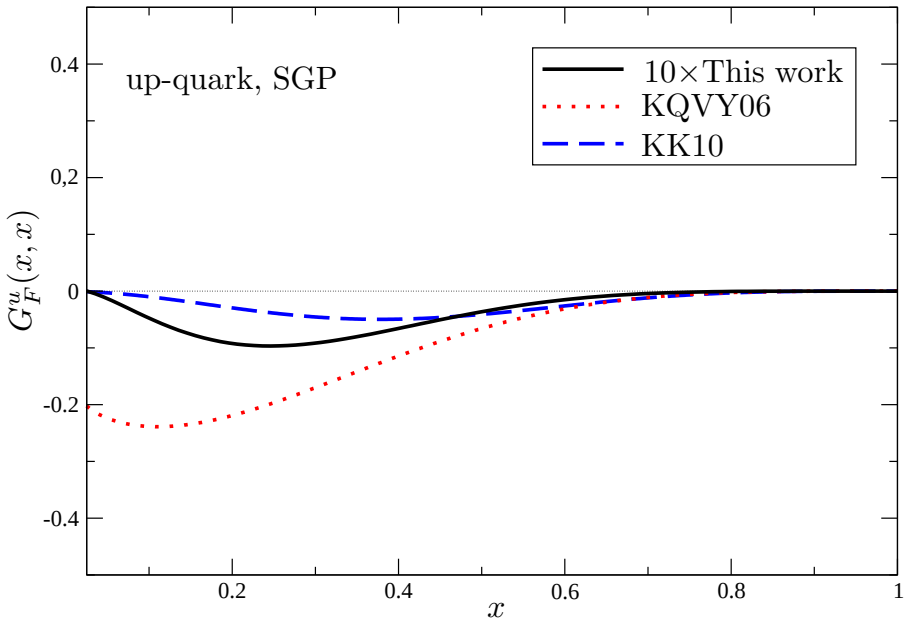
Relevant for SSA: $T_{\bar{q}Fq}(\mathbf{x}) = \frac{-g}{4}[(1+P_{13})\mathcal{Q}_q^+(\mathbf{x}) + (1-P_{13})\mathcal{Q}_q^-(\mathbf{x})]$

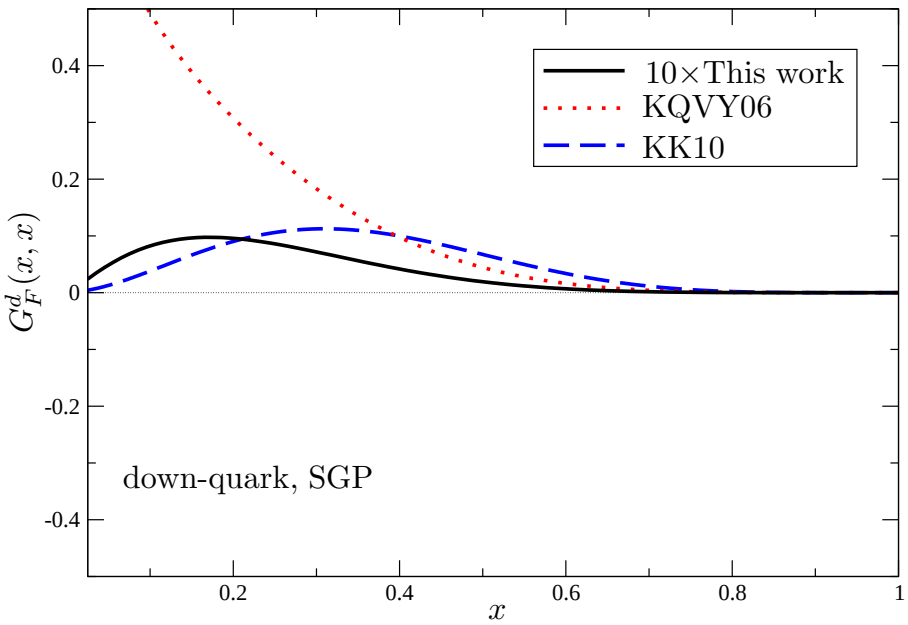
- Vanishes on SGP and SFP lines at $1\,\mathrm{GeV}^2$.
- Nonzero contributions generated by evolution.

Effect of QCD evolution, d-quark ($1\,\mathrm{GeV}^2 \to 10\,\mathrm{GeV}^2$):



SSA, soft-gluon pole





Compared to:

C. Kouvaris, J. -W. Qiu, W. Vogelsang, F. Yuan, PRD74 114013 (2006)

K. Kanazawa, Y. Koike, PRD82 034009 (2010)

Summary

- Reasonable description of parton densities at $x \geq 0.5$.
- Qualitative description of twist-3 observables.
- Room for improvements:
  - $q\bar{q}$ wavefunction.
  - Higher Fock-states.
  - Gerneralization of wavefunctions with free parameters, e.g.

$$\psi(\mathbf{x}) \sim x_1^{p_1} x_2^{p_2} x_3^{p_3} x_4^{p_4}$$

  - GRV approach?