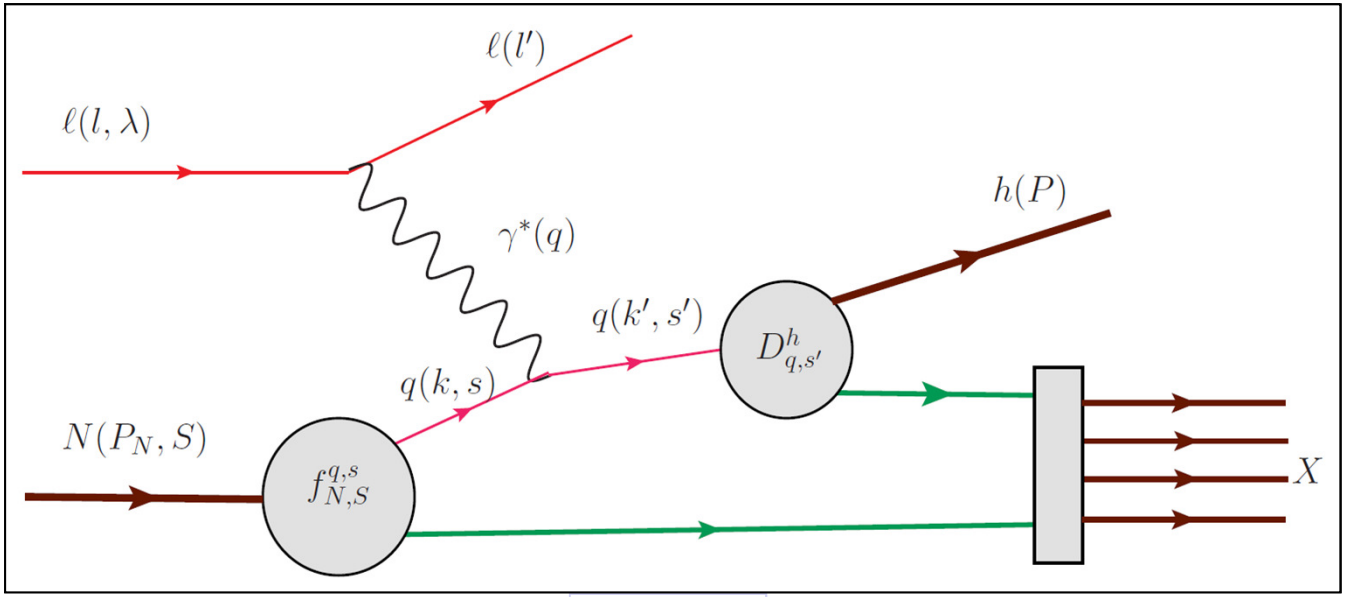
SIDIS in target fragmentation region

Aram Kotzinian

Uni&INFN, Torino & YerPhI, Armenia

- Spin and Transverse Momentum Dependent asymmetries in SIDIS (spinless hadron production)
  - Introduction
    - CFR
  - TFR (based on M.Anselmino, V.Barone and AK, **arXiv:1102.4214;** PLB 699 (2011) 108)
  - TFR+CFR
- Discussion

SIDIS: CFR



$x_F > 0$

AK **arXiv:9412283,** NP B441, 234 (1995)

$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h(P)+X}}{dx\,dQ^2\,d\phi_S\,dz\,d^2P_T} = f_{q,s/N,S} \otimes \frac{d\sigma^{\ell(l,\lambda)+q(k,s)\to\ell(l')+q(k',s')}}{dQ^2} \otimes D^{h_1}_{q,s'}$$

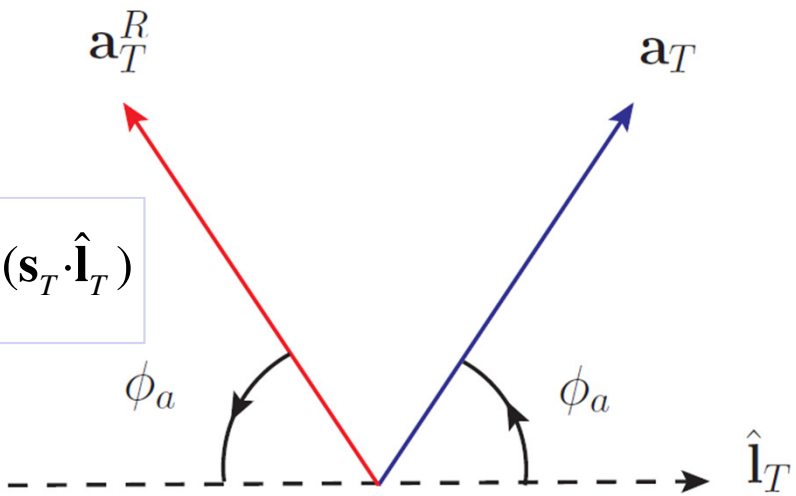
Lepton-quark scattering

$$\frac{d\sigma^{\ell(l,\lambda)+q(k,s)\to\ell(l')+q(k',s')}}{dQ^2} = e_q^2 \frac{2\pi\alpha^2}{\breve{s}^2}\frac{1}{Q^4}\Big((\breve{s}^2+\breve{u}^2)(1+s_L s_L')+(\breve{s}^2-\breve{u}^2)\lambda(s_L+s_L')$$
$$-2\breve{s}\breve{u}(\mathbf{s}_T\cdot\mathbf{s}'_T)-4\breve{u}(\mathbf{s}_T\cdot\mathbf{l}_T)(\mathbf{s}'_T\cdot\mathbf{l}'_T)-4\breve{s}(\mathbf{s}_T\cdot\mathbf{l}'_T)(\mathbf{s}'_T\cdot\mathbf{l}_T)\Big)$$

$$\breve{s}=\frac{Q^2}{y}, \qquad \breve{u}=\frac{Q^2}{y}(1-y)$$

$$\frac{d\sigma_0}{dQ^2}=e_q^2\frac{2\pi\alpha^2}{Q^4}\left(1+(1-y)^2\right), \qquad \frac{d\Delta\sigma}{dQ^2}=\lambda s_L D_{ll}(y)\frac{d\sigma_0}{dQ^2}, \qquad D_{ll}(y)=\frac{y(2-y)}{1+(1-y)^2}$$

$$\mathbf{s}'_T = D_{nn}(y)\mathbf{s}_T^R, \quad D_{nn}(y)=\frac{2(1-y)}{1+(1-y)^2}, \quad \mathbf{s}_T^R=\mathbf{s}_T-2\hat{\mathbf{l}}_T(\mathbf{s}_T\cdot\hat{\mathbf{l}}_T)$$



STMD DFs & FFs

$$\mathcal{P}_N^q(x,\mathbf{k}_T) = f_1^q(x,k_T^2) + f_{1T}^{\perp q}(x,k^2)\frac{\mathbf{k}_T \times \mathbf{S}_T}{M},$$

$$f_1^q(x,k_T^2)s_L(x,\mathbf{k}_T) = g_{1L}^q(x,k_T^2)S_L + g_{1T}^{\perp q}(x,k_T^2)\frac{\mathbf{k}_T \cdot \mathbf{S}_T}{M},$$

$$f_1^q(x,k_T^2)s_T^i(x,\mathbf{k}_T) = h_{1T}^q(x,k_T^2)\mathbf{S}_T + \left(h_{1L}^{\perp q}(x,k_T^2)S_L + h_{1T}^{\perp q}(x,k_T^2)\frac{\mathbf{k}_T \cdot \mathbf{S}_T}{M} \right)\frac{k_T{}^i}{M}$$

$$+ h_1^{\perp q}(x,k_T^2)\frac{\epsilon_T^{ij} k_T{}^j}{M}$$

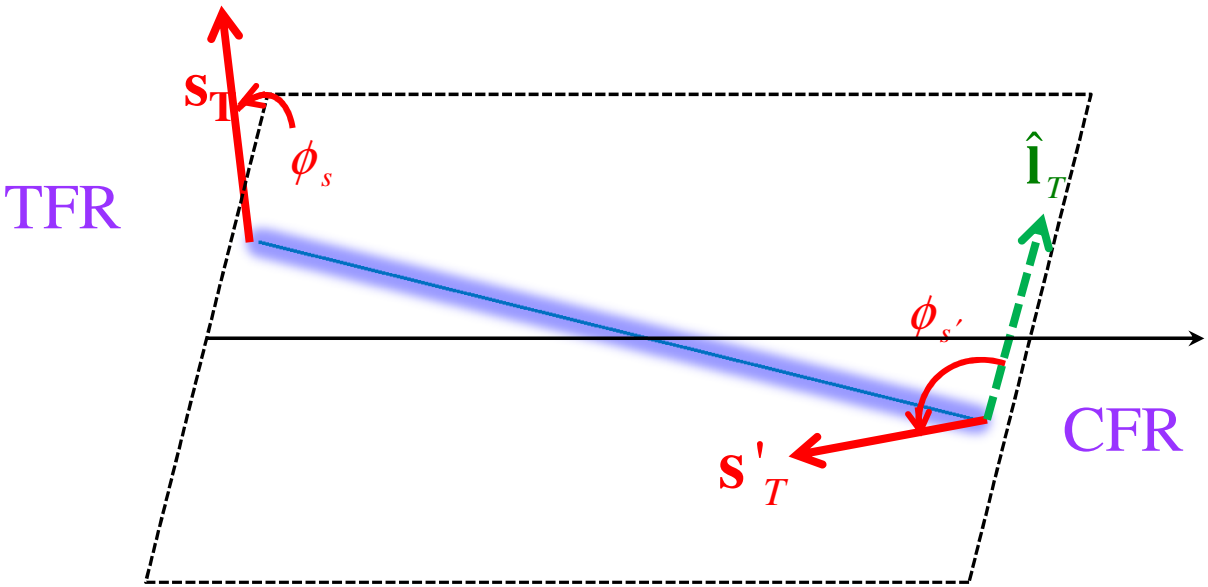
$$\mathbf{a}_T \times \mathbf{b}_T = \epsilon_T^{ij} a_T{}^i b_T{}^j = a_T b_T \sin(\phi_b - \phi_a)$$

$$D_{q,s'}^{h_1}(z,\mathbf{p}_T) = D_1(z,p_T^2) + \frac{\mathbf{p}_T \times \mathbf{s}'_T}{m_h} H_1(z,p_T^2)$$

$$\mathbf{p}_T = \mathbf{P}_T - z\mathbf{k}_T$$

$$\mathbf{s}'_T = D_{nn}(y)\mathbf{s}_T^R, \quad \phi_{s'} = \pi - \phi_s$$

Hadronization in SIDIS



$$\phi_{s'} = \pi - \phi_s$$

For TMD DFs

$$\phi_s = \phi_{S_T} \quad \text{for } h_{1T}$$

$$\phi_s = \phi_{k_T} \quad \text{for } h_{1T}^{\perp}$$

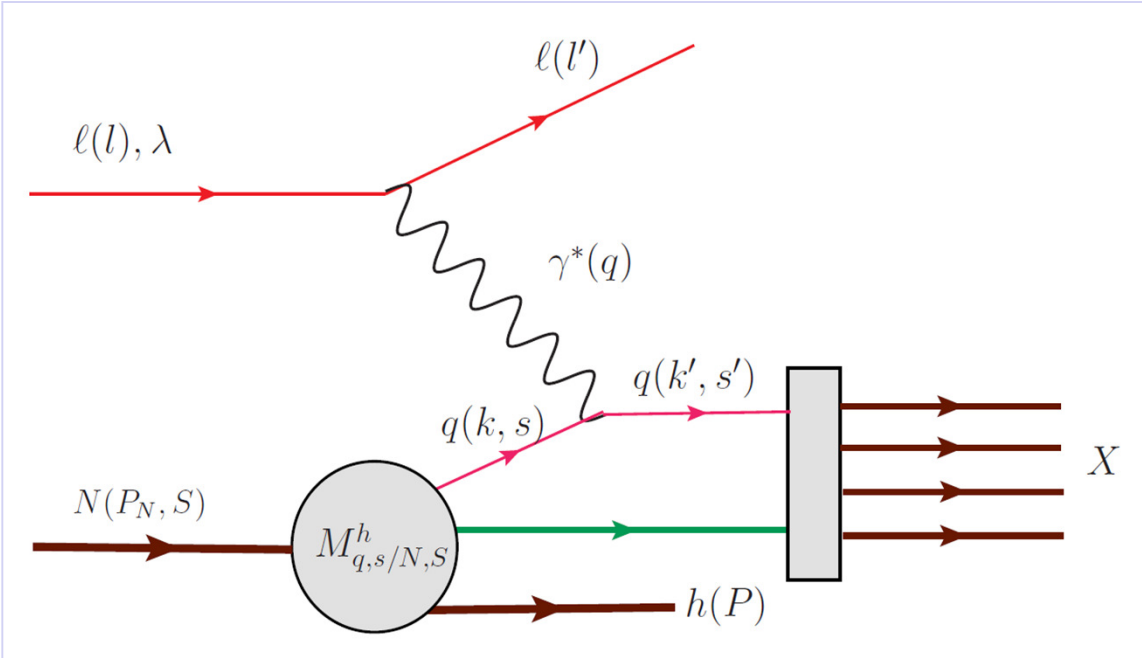
LO cross section in SIDIS CFR

$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h(P)+X}(x_F>0)}{dx\,dQ^2\,d\phi_S\,dz\,d^2P_T}=\frac{\alpha^2 x}{yQ^2}\left(1+(1-y)^2\right)\times$$

$$\times\left[\begin{array}{l} F_{UU,T}+D_{nn}(y)F_{UU}^{\cos 2\phi_h}\cos(2\phi_h)+ \\ S_L D_{nn}(y)F_{UL}^{\sin 2\phi_h}\sin(2\phi_h)+\lambda S_L D_{ll}(y)F_{LL}+ \\ S_T\left(F_{UT,T}^{\sin(\phi_h-\phi_S)}\sin(\phi_h-\phi_S)+D_{nn}(y)\left(\begin{array}{l} F_{UT}^{\sin(\phi_h+\phi_S)}\sin(\phi_h+\phi_S)+ \\ F_{UT}^{\sin(3\phi_h-\phi_S)}\sin(3\phi_h-\phi_S)\end{array}\right)\right)+ \\ \lambda S_T D_{ll}(y)F_{LT}^{\cos(\phi_h-\phi_S)}\cos(\phi_h-\phi_S)\end{array}\right]$$

8 terms out of 18 Structure Functions, 6 azimuthal modulations

SIDIS: TFR



$x_F < 0$

Anselmino, Barone and AK **arXiv:1102.4214;** PL B699 (2011) 108

$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h_1(P_1)+h_2(P_2)+X}}{dx\,dQ^2\,d\phi_S\,dz\,d^2P_{T1}\,d\zeta\,d^2P_{T2}} = M^{h_2}_{q,s/N,S} \otimes \frac{d\sigma^{\ell(l,\lambda)+q(k,s)\to\ell(l')+q(k',s')}}{dQ^2}$$

where $\zeta = \frac{P^-}{P_N^-} \approx x_F(1-x)$

STMD Fracture Functions

$$M^{h}_{q/N,S} = \hat{M} + \frac{\mathbf{P}_T \times \mathbf{S}_T}{m_h}\hat{M}^{h}_{T} + \frac{\mathbf{k}_T \times \mathbf{S}_T}{m_N}\hat{M}^{\perp}_{T} + \frac{S_L(\mathbf{k}_T \times \mathbf{P}_T)}{m_N m_h}\hat{M}^{\perp h}_{L}$$

$$\Delta M^{h}_{q,s_L/N,S} = \hat{M}s_L = S_L \Delta\hat{M}_L + \frac{\mathbf{P}_T \cdot \mathbf{S}_T}{m_h}\Delta\hat{M}^{h}_{T} + \frac{\mathbf{k}_T \cdot \mathbf{S}_T}{m_N}\Delta\hat{M}^{\perp}_{T} + \frac{\mathbf{k}_T \times \mathbf{P}_T}{m_N m_h}\Delta\hat{M}^{\perp h}$$

$$\Delta_T M^{h_2}_{q,\mathbf{s}_T/N,S} = \hat{M}s^i_T = S^i_T \Delta_T\hat{M}_T + \frac{S_L P^i_T}{m_h}\Delta_T\hat{M}^{h}_{L} + \frac{S_L k_T{}^i}{m_N}\Delta_T\hat{M}^{\perp}_{L}$$

$$+\frac{(\mathbf{P}_T \cdot \mathbf{S}_T)P^i_T}{m_h^2}\Delta_T\hat{M}^{hh}_{T} + \frac{(\mathbf{k}_T \cdot \mathbf{S}_T)k^i}{m_N^2}\Delta_T\hat{M}^{\perp\perp}_{T}$$

$$+\frac{(\mathbf{k}_T \cdot \mathbf{S}_T)P^i_T - (\mathbf{P}_T \cdot \mathbf{S}_T)k_T{}^i}{m_N m_h}\Delta_T\hat{M}^{\perp h}_{T}$$

$$+\frac{\epsilon^{ij}_T P_T{}^j}{m_h}\Delta_T\hat{M}^{h} + \frac{\epsilon^{ij}_T k^j_T}{m_N}\Delta_T\hat{M}^{\perp}$$

8 new structures

All $\hat{M}^{\cdots}_{\cdots}$ depend on $x, \zeta, k_T^2, P_T^2, (\mathbf{k}_T \cdot \mathbf{P}_T)$

Integrals of STMD fracture functions

Trentadue & Veneziano (1994)

$$M(x,\zeta) = \int d^2P_T \int d^2k_T \hat{M}(x,k_T^2,\zeta,P_T^2,\mathbf{k}_T\cdot\mathbf{P}_T)$$

$$\int d^2k_T M^h_{q/N,S}(x,k_T^2,\zeta,P_T^2,\mathbf{k}_T\cdot\mathbf{P}_T) = M(x,\zeta,P_T^2) + \frac{\mathbf{P}_T\times\mathbf{S}_T}{m_h} M_T^h(x,\zeta,P_T^2)$$

$$M(x,\zeta,P_T^2) = \int d^2k_T \hat{M}(x,k_T^2,\zeta,P_T^2,\mathbf{k}_T\cdot\mathbf{P}_T)$$

$$M_T^h(x,\zeta,P_T^2) = \int d^2k_T \left(\hat{M}_T^h + \frac{m_h}{m_N}\frac{\mathbf{k}_T\cdot\mathbf{P}_T}{P_T^2}\hat{M}_T^\perp \right)$$

Extended fracture function Grazini, Trentadue & Veneziano (1997)

$$\int d^2k_T \Delta M^h_{q,s_L/N,S}(x,k_T^2,\zeta,P_T^2,\mathbf{k}_T\cdot\mathbf{P}_T) = S_L \Delta M_L(x,\zeta,P_T^2) + \frac{\mathbf{P}_T\cdot\mathbf{S}_T}{m_h}\Delta M_T^h(x,\zeta,P_T^2)$$

$$\Delta M_L(x,\zeta,P_T^2) = \int d^2k_T \Delta\hat{M}_L(x,k_T^2,\zeta,P_T^2,\mathbf{k}_T\cdot\mathbf{P}_T)$$

$$\Delta M_T^h(x,\zeta,P_T^2) = \int d^2k_T \left(\Delta\hat{M}_T^h + \frac{m_h}{m_N}\frac{\mathbf{k}_T\cdot\mathbf{P}_T}{P_T^2}\Delta\hat{M}_T^\perp \right)$$

Sum Rules

$$\sum_h \int \zeta d\zeta \int d^2 P_T \hat{M} = (1-x) f_1(x, k_T^2)$$

$$\sum_h \int \zeta d\zeta \int d^2 P_T \left(\hat{M}_T^{\perp} + \frac{m_N}{m_h} \frac{\mathbf{k}_T \cdot \mathbf{P}}{k_T^2} \hat{M}_T^h \right) = -(1-x) f_{1T}^{\perp}(x, k_T^2)$$

$$\sum_h \int \zeta d\zeta \int d^2 P_T \Delta\hat{M}_L = (1-x) g_{1L}(x, k_T^2)$$

$$\sum_h \int \zeta d\zeta \int d^2 P_T \left(\Delta\hat{M}_T^{\perp} + \frac{m_N}{m_h} \frac{\mathbf{k}_T \cdot \mathbf{P}_T}{k_T^2} \Delta\hat{M}_T^h \right) = (1-x) g_{1T}(x, k_T^2)$$

$$\sum_h \int \zeta d\zeta \int d^2 P_T \left(\Delta_T \hat{M}_L^{\perp} + \frac{m_N}{m_h} \frac{\mathbf{k}_T \cdot \mathbf{P}_T}{k_T^2} \Delta_T \hat{M}_L^h \right) = (1-x) h_{1L}^{\perp}(x, k_T^2)$$

$$\sum_h \int \zeta d\zeta \int d^2 P_T \left(\Delta_T \hat{M}^{\perp} + \frac{m_N}{m_h} \frac{\mathbf{k}_T \cdot \mathbf{P}_T}{k_T^2} \Delta_T \hat{M}^h \right) = -(1-x) h_1^{\perp}(x, k_T^2)$$

$$\sum_h \int \zeta d\zeta \int d^2 P_T \left(\Delta_T \hat{M}_T^{\perp\perp} + \frac{m_N^2}{m_h^2} \frac{2(\mathbf{k}_T \cdot \mathbf{P}_T)^2 - k_T^2 P_T^2}{k_T^4} \Delta_T \hat{M}_T^{hh} \right) = (1-x) h_{1T}^{\perp}(x, k_T^2)$$

$$\sum_h \int \zeta d\zeta \int d^2 P_T \left(\Delta\hat{M}_T + \frac{k_T^2}{2m_N^2} \Delta_T \hat{M}_T^{\perp\perp} + \frac{P_T^2}{2m_h^2} \Delta\hat{M}_T^{hh} \right) = (1-x) h_1(x, k_T^2)$$

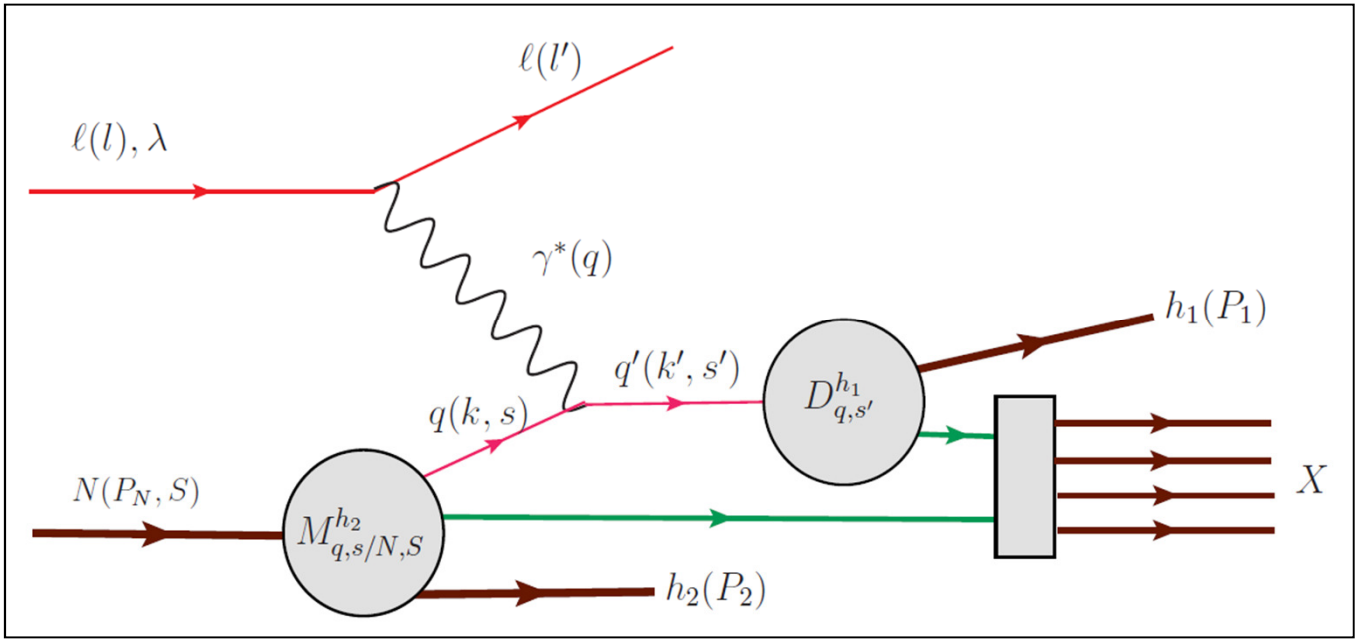
LO cross-section in TFR

$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h(P)+X}(x_F<0)}{dx\,dQ^2\,d\phi_S\,d\zeta\,d^2P_T}=\frac{\alpha^2 x}{yQ^4}\left(1+(1-y)^2\right)\sum_q e_q^2\times$$

$$\times\left[\begin{array}{l} M\left(x,\zeta,P_T^2\right)-S_T\dfrac{P_T}{m_h}M_T^h\left(x,\zeta,P_T^2\right)\sin(\phi_h-\phi_S)+ \\ \lambda D_{ll}(y)\left(S_L\Delta M_L\left(x,\zeta,P_T^2\right)+S_T\dfrac{P_T}{m_h}\Delta M_T^h\left(x,\zeta,P_T^2\right)\cos(\phi_h-\phi_S)\right)\end{array}\right]$$

Only 4 terms out of 18 Structure Functions, 2 azimuthal modulations

DSIDIS: TFR & CFR



$$x_{F2} < 0, \quad x_{F1} > 0$$

$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h_1(P_1)+h_2(P_2)+X}}{dx\,dQ^2\,d\phi_S\,dz\,d^2P_{T1}\,d\zeta\,d^2P_{T2}} = M^{h_2}_{q,s/N,S} \otimes \frac{d\sigma^{\ell(l,\lambda)+q(k,s)\to\ell(l')+q(k',s')}}{dQ^2} \otimes D^{h_1}_{q,s'}$$

$$D^{h_1}_{q,s'}(z,\mathbf{p}_T) = D_1(z,p_T^2) + \frac{\mathbf{p}_T \times \mathbf{s}'_T}{m_h} H_1(z,p_T^2)$$

$$\mathbf{s}'_T = D_{nn}(y)\mathbf{s}^R_T$$

DSIDIS cross-section

$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h_1(P_1)+h_2(P_2)+X}}{dx\,dQ^2\,d\phi_S\,dz\,d^2P_{T1}\,d\zeta\,d^2P_{T2}} =$$

$$= \frac{\alpha^2 x}{Q^4 y}\left(1+(1-y)^2\right)\left(\begin{array}{l} M^{h_2}_{q/N,S} \otimes D^{h_1}_{1q} + \lambda D_{ll}(y)\Delta M^{h_2}_{q,s_L/N,S} \otimes D^{h_1}_{1q} \\ +\Delta_T M^{h_2}_{q,\mathbf{s}_T/N,S} \otimes \dfrac{\mathbf{p}_T \times \mathbf{s}'_T}{m_{h_1}} H^{h_1}_{1q} \end{array}\right)$$

$$= \frac{\alpha^2 x}{Q^4 y}\left(1+(1-y)^2\right)\left(\begin{array}{l} \sigma_{UU} + S_L\sigma_{UL} + S_T\sigma_{UT} + \\ \lambda D_{ll}\left(\sigma_{LU} + S_L\sigma_{LL} + S_T\sigma_{UT}\right) \end{array}\right)$$

σ_{UU}

$$\sigma_{UU} = F_0^{\hat{M}\cdot D_1} - D_{nn}\left(\begin{array}{l} \dfrac{P_{T1}^2}{m_1 m_N} F_{kp1}^{\Delta_T \hat{M}^{\perp}\cdot H_1}\cos(2\phi_1) \\ + \dfrac{P_{T1}P_{T2}}{m_1 m_2} F_{p1}^{\Delta_T \hat{M}^{h}\cdot H_1}\cos(\phi_1+\phi_2) \\ + \left(\dfrac{P_{T2}^2}{m_1 m_N} F_{kp2}^{\Delta_T \hat{M}^{\perp}\cdot H_1} + \dfrac{P_{T2}^2}{m_1 m_2} F_{p2}^{\Delta_T \hat{M}^{h}\cdot H_1}\right)\cos(2\phi_2) \end{array}\right)$$

σ_{UL}

$$\sigma_{UL} = -\frac{P_{T1}P_{T2}}{m_2 m_N} F_{k1}^{\hat{M}_L^{\perp h}\cdot D_1} \sin(\phi_1 - \phi_2)$$

$$+D_{nn}\left(\begin{array}{l} \dfrac{P_{T1}^2}{m_1 m_N} F_{kp1}^{\Delta_T \hat{M}_L^{\perp}\cdot H_1} \sin(2\phi_1) \\ +\dfrac{P_{T1}P_{T2}}{m_1 m_2} F_{p1}^{\Delta_T \hat{M}_L^{h}\cdot H_1} \sin(\phi_1 + \phi_2) \\ +\left(\dfrac{P_{T2}^2}{m_1 m_N} F_{kp2}^{\Delta_T \hat{M}_L^{\perp}\cdot H_1} + \dfrac{P_{T2}^2}{m_1 m_2} F_{p2}^{\Delta_T \hat{M}_L^{h}\cdot H_1}\right)\sin(2\phi_2) \end{array}\right)$$

σ_{UT}

$$
\begin{aligned}
\sigma_{UT} = & -\frac{P_{T1}}{m_N} F_{k1}^{\hat{M}_T^{\perp}\cdot D_1} \sin(\phi_1 - \phi_S) \\
& -\left(\frac{P_{T2}}{m_2} F_0^{\hat{M}_T^{h}\cdot D_1} + \frac{P_{T2}}{m_N} F_{k2}^{\hat{M}_T^{\perp}\cdot D_1} \right) \sin(\phi_2 - \phi_S) \\
& + D_{nn}(y) \left[\begin{aligned}
& \left(\begin{aligned} & \frac{P_{T1}}{m_1} F_{p1}^{\Delta_T \hat{M}_T \cdot H_1} + \frac{P_{T1}P_{T2}^2}{2m_1 m_2^2} F_{p1}^{\Delta_T \hat{M}_T^{hh}\cdot H_1} - \frac{P_{T1}P_{T2}^2}{2m_1 m_2 m_N} F_{kp3}^{\Delta_T \hat{M}_T^{\perp h}\cdot H_1} \\ & + \frac{P_{T1}^3}{2m_1 m_N^2} F_{kkp1}^{\Delta_T \hat{M}_T^{\perp\perp}\cdot H_1} + \frac{P_{T1}P_{T2}^2}{2m_1 m_N^2} F_{kkp4}^{\Delta_T \hat{M}_T^{\perp\perp}\cdot H_1} + \frac{P_{T1}}{m_1 m_N^2} F_{kkp5}^{\Delta_T \hat{M}_T^{\perp\perp}\cdot H_1} \end{aligned} \right) \sin(\phi_1 + \phi_S) \\
& + \left(\begin{aligned} & \frac{P_{T2}}{m_1} F_{p2}^{\Delta_T \hat{M}_T \cdot H_1} + \frac{P_{T2}^3}{2m_1 m_2^2} F_{p2}^{\Delta_T \hat{M}_T^{hh}\cdot H_1} + \frac{P_{T1}^2 P_{T2}}{2m_1 m_2 m_N} F_{kp1}^{\Delta_T \hat{M}_T^{\perp h}\cdot H_1} + \frac{P_{T2}}{m_1 m_2 m_N} F_{kp4}^{\Delta_T \hat{M}_T^{\perp h}\cdot H_1} \\ & + \frac{P_{T1}^2 P_{T2}}{2m_1 m_N^2} F_{kkp2}^{\Delta_T \hat{M}_T^{\perp\perp}\cdot H_1} + \frac{P_{T2}^3}{2m_1 m_N^2} F_{kkp3}^{\Delta_T \hat{M}_T^{\perp\perp}\cdot H_1} \frac{P_{T2}}{m_1 m_N^2} F_{kkp6}^{\Delta_T \hat{M}_T^{\perp\perp}\cdot H_1} \end{aligned} \right) \sin(\phi_2 + \phi_S) \\
& + \frac{P_{T1}^3}{2m_1 m_N^2} F_{kkp1}^{\Delta_T \hat{M}_T^{\perp\perp}\cdot H_1} \sin(3\phi_1 - \phi_S) \\
& + \left(\frac{P_{T2}^3}{2m_1 m_2^2} F_{p2}^{\Delta_T \hat{M}_T^{hh}\cdot H_1} + \frac{P_{T2}^3}{2m_1 m_N^2} F_{kkp3}^{\Delta_T \hat{M}_T^{\perp\perp}\cdot H_1} \right) \sin(3\phi_2 - \phi_S) \\
& + \left(\frac{P_{T1}P_{T2}^2}{2m_1 m_2^2} F_{p1}^{\Delta_T \hat{M}_T^{hh}\cdot H_1} + \frac{P_{T1}P_{T2}^2}{2m_1 m_N^2} F_{kkp4}^{\Delta_T \hat{M}_T^{\perp\perp}\cdot H_1} \right) \sin(\phi_1 + 2\phi_2 - \phi_S) \\
& - \frac{P_{T1}^2 P_{T2}}{2m_1 m_2 m_N} F_{kp1}^{\Delta_T \hat{M}_T^{\perp h}\cdot H_1} \sin(2\phi_1 - \phi_2 + \phi_S) \\
& - \frac{P_{T1}P_{T2}^2}{2m_1 m_2 m_N} F_{kp3}^{\Delta_T \hat{M}_T^{\perp h}\cdot H_1} \sin(\phi_1 - 2\phi_2 - \phi_S) \\
& + \frac{P_{T1}^2 P_{T2}}{2m_1 m_N^2} F_{kkp2}^{\Delta_T \hat{M}_T^{\perp\perp}\cdot H_1} \sin(2\phi_1 + \phi_2 - \phi_S)
\end{aligned} \right]
\end{aligned}
$$

σ_{LU}, σ_{LL}, σ_{LT}

$$\sigma_{LU} = -\frac{P_{T1}P_{T2}}{m_2 m_N} F_{k1}^{\Delta\hat{M}^{\perp h}\cdot D_1} \sin(\phi_1 - \phi_2)$$

$$\sigma_{LL} = F_0^{\Delta\hat{M}_L\cdot D_1}$$

$$\sigma_{LT} = \frac{P_{T1}}{m_N} F_{k1}^{\Delta\hat{M}_T^{\perp}\cdot D_1} \cos(\phi_1 - \phi_S) + \left(\frac{P_{T2}}{m_2} F_0^{\Delta\hat{M}_T^{h}\cdot D_1} + \frac{P_{T2}}{m_N} F_{k2}^{\Delta\hat{M}_T^{\perp}\cdot D_1} \right) \cos(\phi_2 - \phi_S)$$

CONCLUSIONS

- New members appeared in the polarized TMDs family -- 16 LO STMD fracture functions
- For hadron produced in the TFR, only 4 k_T-integrated fracture functions of unpolarized and longitudinally polarized quarks are probed.
  - SSA contains only a Sivers-type modulation $\sin(\varphi_h-\varphi_S)$ but no Collins-type $\sin(\varphi_h+\varphi_S)$ or $\sin(3\varphi_h-\varphi_S)$
  - The eventual observation of Collins-type asymmetry will indicate that LO factorized approach fails and long range correlations between the struck quark polarization and P_T of produced in TFR hadron might be important.
- DSIDIS cross section at LO contains 2 azimuthal independent and 20 azimuthally modulated terms. (Much more in SIDY)
- The ideal place to test the fracture functions factorization and measure these new functions are JLab12 and EIC facilities with full coverage of phase space
- To do
  - Factorization proof (SIDIS, DSIDIS, SIDY)
    - Structure of Wilson lines. SIDIS ←→ DY: sign changes of some fracture functions?
    - Higher twist
    - Polarized hadron production
  - Phenomenology: parameterizations , simple models ...

Convolutions & tensorial decomposition

$$C[\hat{M}\cdot Dw] = \sum_a e_a^2 \int d^2k_T\, d^2p_T\, \delta^{(2)}(z\mathbf{k}_T + \mathbf{p}_T - \mathbf{P}_{T1})\hat{M}_a(x,\zeta,k_T^2,P_{T2}^2,\mathbf{k}_T\cdot\mathbf{P}_{T2})D_a(z,p_T^2)w$$

$$C[\hat{M}\cdot D] = F_0^{\hat{M}\cdot D}$$

$$C[\hat{M}\cdot Dk^i] = P_{T1}^i F_{k1}^{\hat{M}\cdot D} + P_{T2}^i F_{k2}^{\hat{M}\cdot D}$$

$$C[\hat{M}\cdot Dp^i] = P_{T1}^i F_{p1}^{\hat{M}\cdot D} + P_{T2}^i F_{p2}^{\hat{M}\cdot D}$$

$$C[\hat{M}\cdot Dk^ik^j] = P_{T1}^iP_{T1}^j F_{kk1}^{\hat{M}\cdot D} + P_{T2}^iP_{T2}^j F_{kk2}^{\hat{M}\cdot D} + \delta^{ij} F_{kk3}^{\hat{M}\cdot D}$$

$$C[\hat{M}\cdot Dk^ip^j] = P_{T1}^iP_{T1}^j F_{kp1}^{\hat{M}\cdot D} + P_{T2}^iP_{T2}^j F_{kp2}^{\hat{M}\cdot D} + \left(P_{T1}^iP_{T2}^j - P_{T1}^jP_{T2}^i\right)F_{kp3}^{\hat{M}\cdot D} + \delta^{ij} F_{kp4}^{\hat{M}\cdot D}$$

$$C[\hat{M}\cdot Dk^ik^jp^k] = P_{T1}^iP_{T1}^jP_{T1}^k F_{kkp1}^{\hat{M}\cdot D} + P_{T1}^iP_{T1}^jP_{T2}^k F_{kkp2}^{\hat{M}\cdot D} + P_{T2}^iP_{T2}^jP_{T2}^k F_{kkp3}^{\hat{M}\cdot D}$$

$$+P_{T2}^iP_{T2}^jP_{T1}^k F_{kkp4}^{\hat{M}\cdot D} + P_{T1}^k\delta^{ij} F_{kkp5}^{\hat{M}\cdot D} + P_{T2}^k\delta^{ij} F_{kkp6}^{\hat{M}\cdot D}$$

where $F_{\ldots}^{\hat{M}\cdot D}$ depend on $x, z, \zeta, P_{T1}^2, P_{T2}^2, \left(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2}\right)$

Structure functions

$$F_{k1}^{\hat{M}\cdot D} = C\left[\hat{M}\cdot D\frac{(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})(\mathbf{P}_{T2}\cdot\mathbf{k})-(\mathbf{P}_{T1}\cdot\mathbf{k})\mathbf{P}_{T2}^2}{\left(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2}\right)^2-\mathbf{P}_{T1}^2\mathbf{P}_{T2}^2}\right]$$

$$F_{k2}^{\hat{M}\cdot D} = C\left[\hat{M}\cdot D\frac{(\mathbf{P}_{T1}\cdot\mathbf{k})(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})-(\mathbf{P}_{T2}\cdot\mathbf{k})\mathbf{P}_{T1}^2}{\left(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2}\right)^2-\mathbf{P}_{T1}^2\mathbf{P}_{T2}^2}\right]$$

$$F_{kk1}^{\hat{M}\cdot D} = C\left[\hat{M}\cdot D\frac{\left(-2\left(\mathbf{P}_{T1}\cdot\mathbf{k}\right)^2+\mathbf{k}^2\mathbf{P}_{T1}^2\right)\mathbf{P}_{T2}^4+\left(2\left(\mathbf{P}_{T2}\cdot\mathbf{k}\right)^2-\mathbf{k}^2\mathbf{P}_{T2}^2\right)\left(2\left(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2}\right)^2-\mathbf{P}_{T1}^2\mathbf{P}_{T2}^2\right)}{4\left(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2}\right)^2\left(\left(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2}\right)^2-\mathbf{P}_{T1}^2\mathbf{P}_{T2}^2\right)}\right]$$

$$F_{kk2}^{\hat{M}\cdot D} = C\left[\hat{M}\cdot D\frac{\left(2\left(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2}\right)^2-\mathbf{P}_{T1}^2\mathbf{P}_{T2}^2\right)\left(\mathbf{P}_{T1}\cdot\mathbf{k}\right)^2+\mathbf{P}_{T1}^2\left(\mathbf{P}_{T1}^2\mathbf{P}_{T2}^2-\left(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2}\right)^2\right)\mathbf{k}^2-\left(\mathbf{P}_{T2}\cdot\mathbf{k}\right)^2\mathbf{P}_{T1}^4}{2\left(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2}\right)^2\left(\left(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2}\right)^2-\mathbf{P}_{T1}^2\mathbf{P}_{T2}^2\right)}\right]$$

$$F_{kk3}^{\hat{M}\cdot D} = C\left[\hat{M}\cdot D\frac{\left(\left(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2}\right)^2+\mathbf{P}_{T1}^2\mathbf{P}_{T2}^2\right)\mathbf{k}^2-\left(\mathbf{P}_{T2}\cdot\mathbf{k}\right)^2\mathbf{P}_{T1}^2-\left(\mathbf{P}_{T1}\cdot\mathbf{k}\right)^2\mathbf{P}_{T2}^2}{2\left(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2}\right)^2}\right]$$