Single transverse-spin asymmetry in open charm production in SIDIS

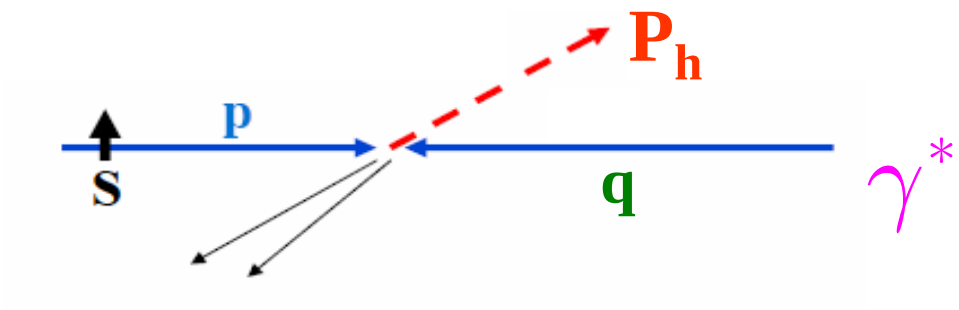
Kazuhiro Tanaka (Juntendo U)

with H. Beppu, Y. Koike, S. Yoshida (Niigata U)

PRD82 ('10) 054005
arXiv:1104.0798

numerical estimates

SSA in **SIDIS** $ep^{\uparrow} \rightarrow ehX$



$$d\sigma \sim \vec{S}_{\perp} \cdot \left(\vec{p} \times \vec{P}_h \right)$$

$$A = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}}$$

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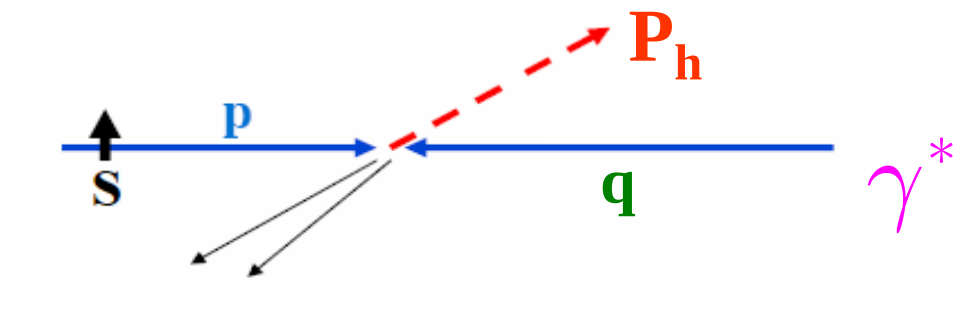

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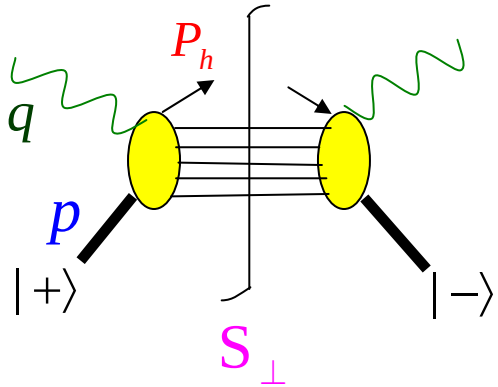
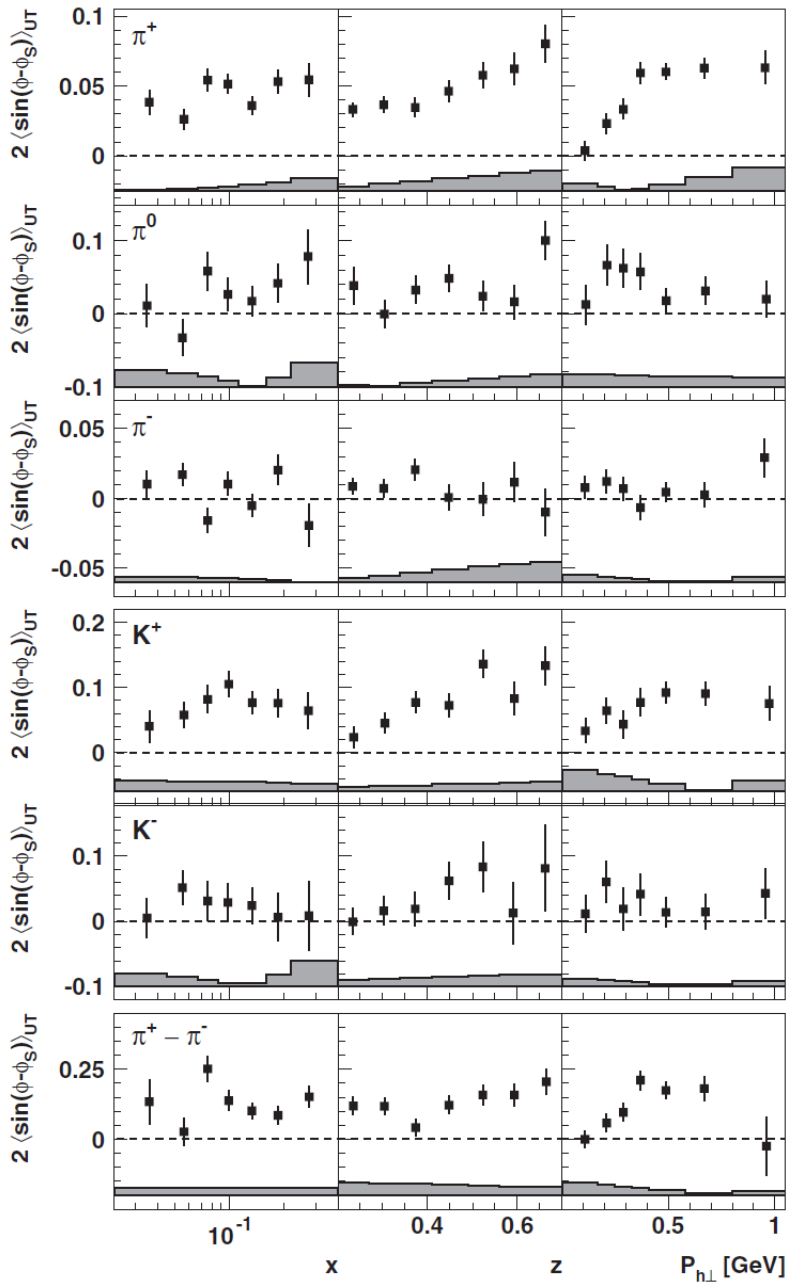


1. $P_{h\perp} \neq 0$: $k_{\perp}$ of quark or gluon
2. proton helicity flip
3. interaction phase: beyond Born

Sivers asymmetry



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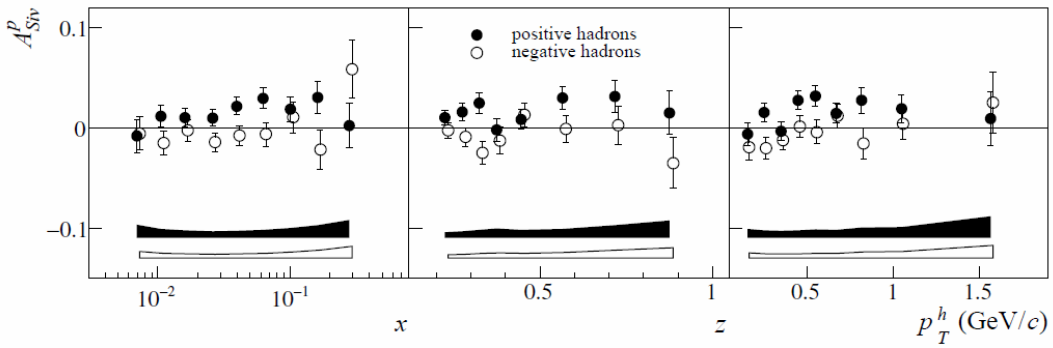

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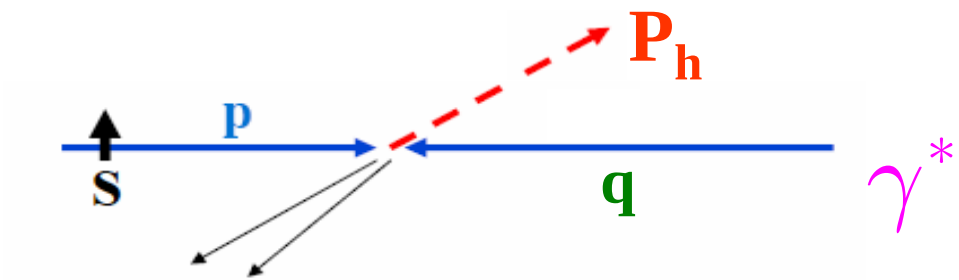
SSA in **SIDIS** $ep^{\uparrow} \to ehX$



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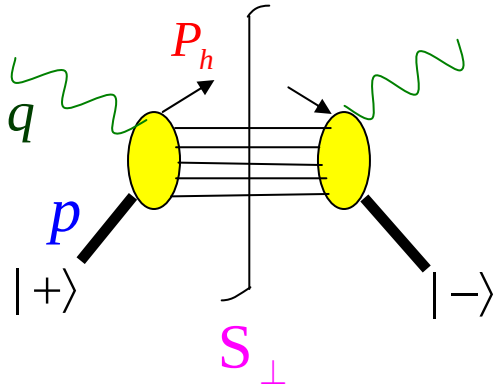
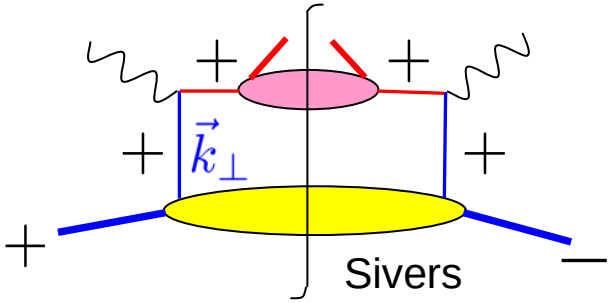


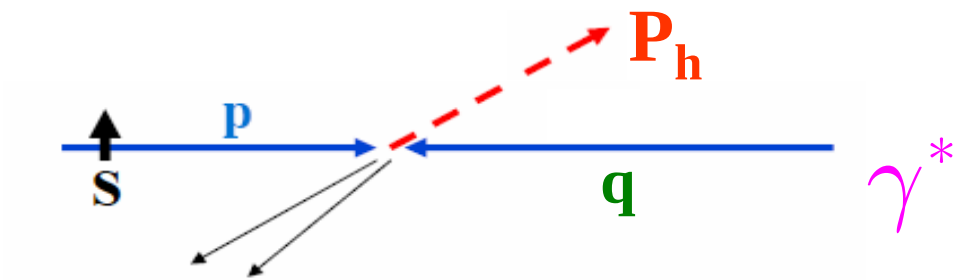
1. $P_{h\perp} \neq 0$: $k_{\perp}$ of quark or gluon **NP**
2. proton helicity flip **NP**
3. interaction phase: beyond Born **NP**



TMD-factorization

$$P_{h\perp} \sim \Lambda_{\mathrm{QCD}}$$

SSA in **SIDIS** $ep^{\uparrow} \to ehX$



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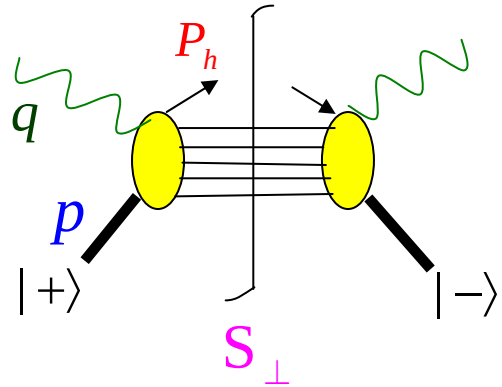

1. $P_{h\perp} \neq 0$: $k_{\perp}$ of quark or gluon **P**
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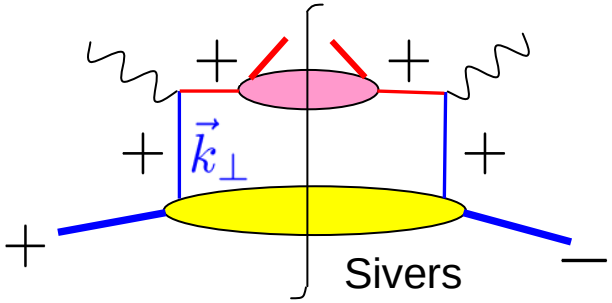

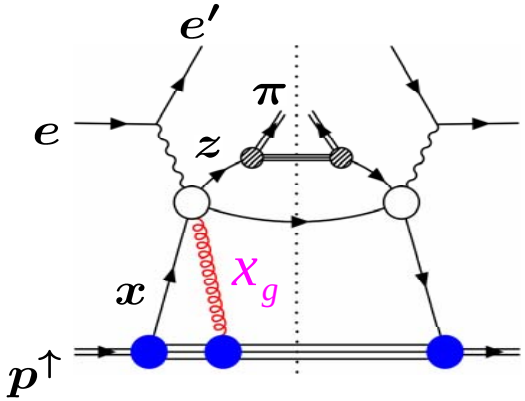

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twist-3 collinear-factorization

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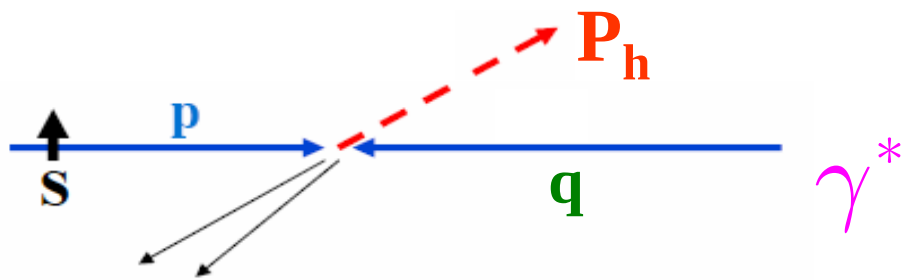

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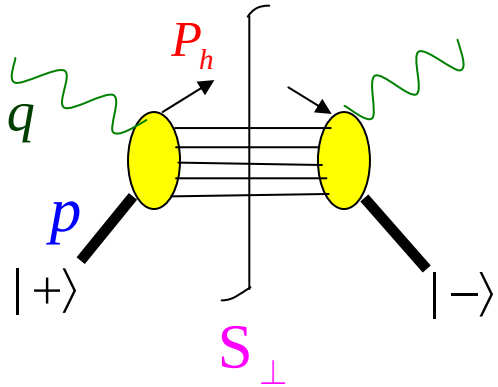

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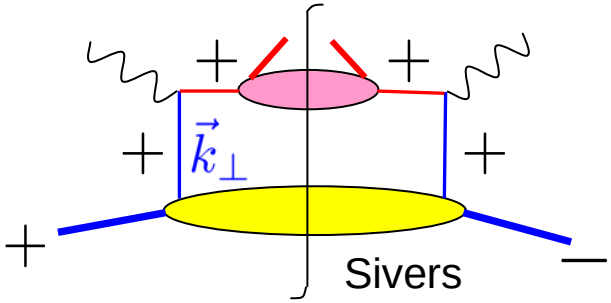

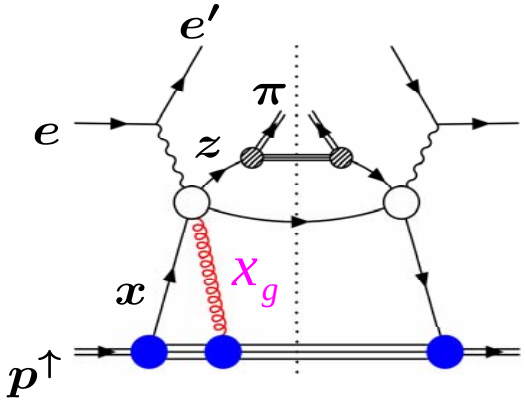

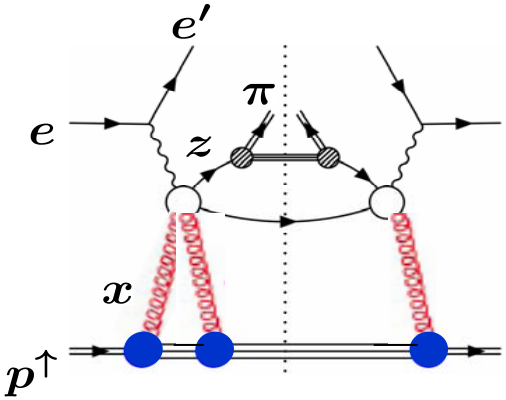

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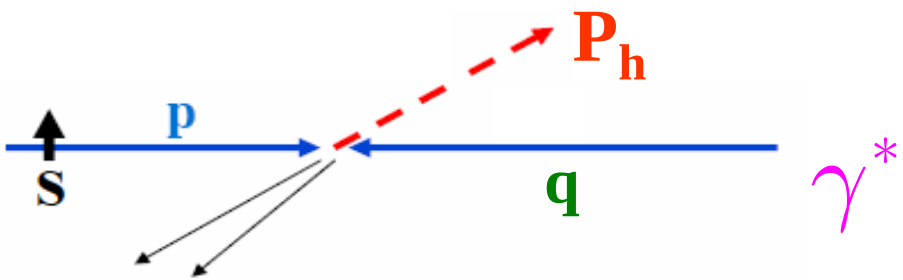

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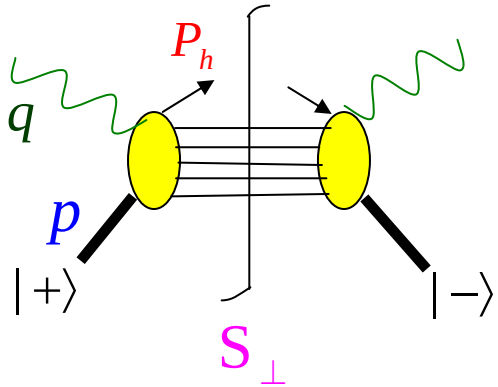

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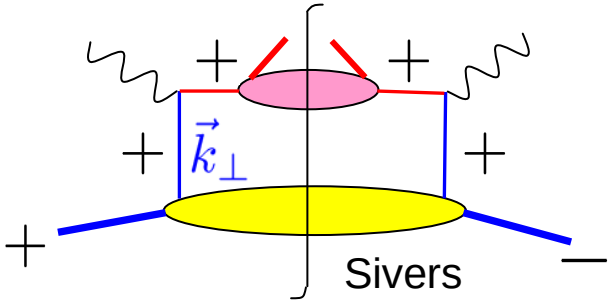
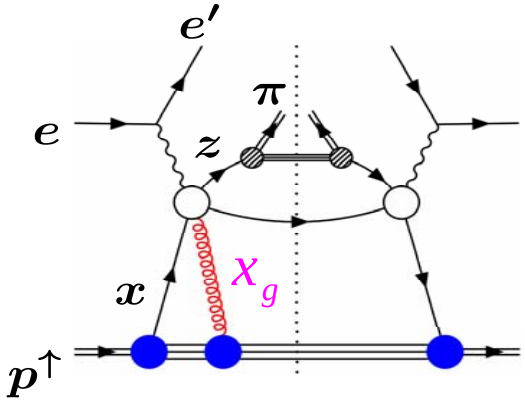
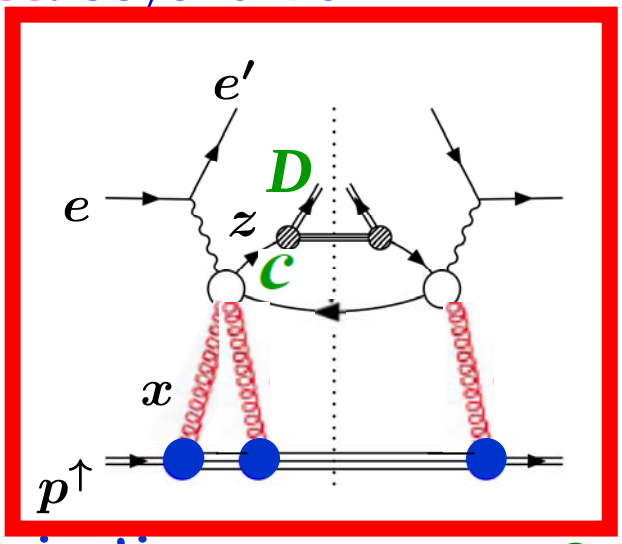






TMD-factorization

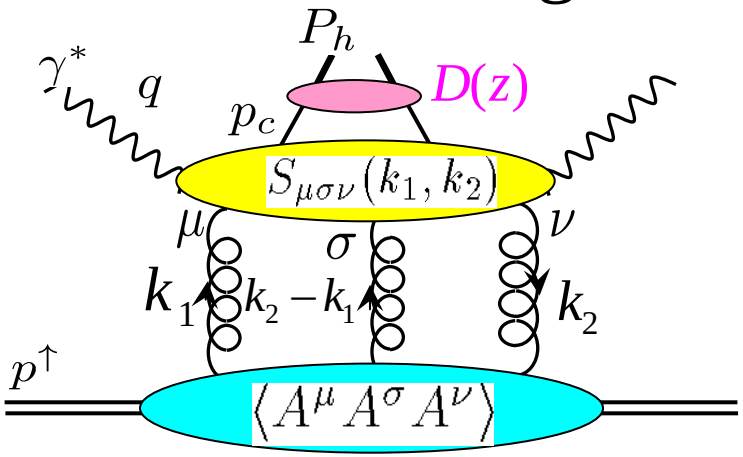
$$P_{h\perp} \sim \Lambda_{QCD}$$

twist-3 collinear-factorization

$$P_{h\perp} \gg \Lambda_{QCD}$$

$$m_D \neq 0$$
$$m_c \neq 0$$

SSA from multi-gluon correlation in SIDIS at $P_{h\perp} \gg \Lambda_{\rm QCD}$



1. $P_{h\perp}$ by hard interaction
2. proton helicity flip

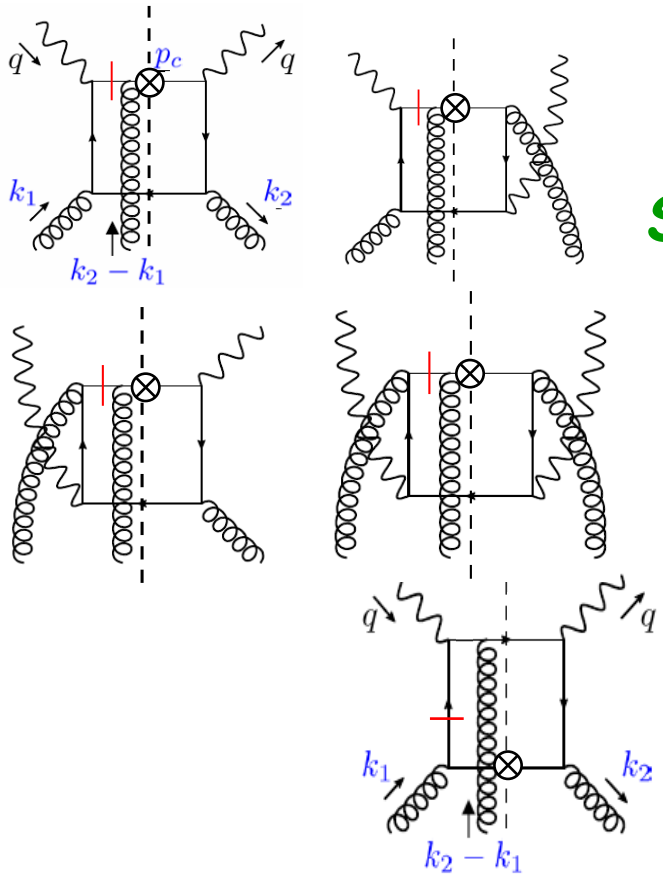
$$\langle p\,S\,|\,A^\mu(0)A^\sigma(\zeta n)A^\nu(\lambda n)\,|\,p\,S\rangle$$

$$= g_\perp^{\nu\mu} S_{\perp\alpha}\varepsilon^{\sigma pn\alpha} H + \cdots$$

$$n^\mu = \left(0, n^-, \mathbf{0}_\perp\right) \qquad p\cdot n = p^+ n^- = 1$$

$$\varepsilon^{\sigma pn\alpha} \equiv \varepsilon^{\sigma\mu\nu\alpha} p_\mu n_\nu = \varepsilon^{\sigma -+\alpha}$$

$$g_\perp^{\mu\nu} \equiv g^{\mu\nu} - p^\mu n^\nu - n^\mu p^\nu$$



SGP

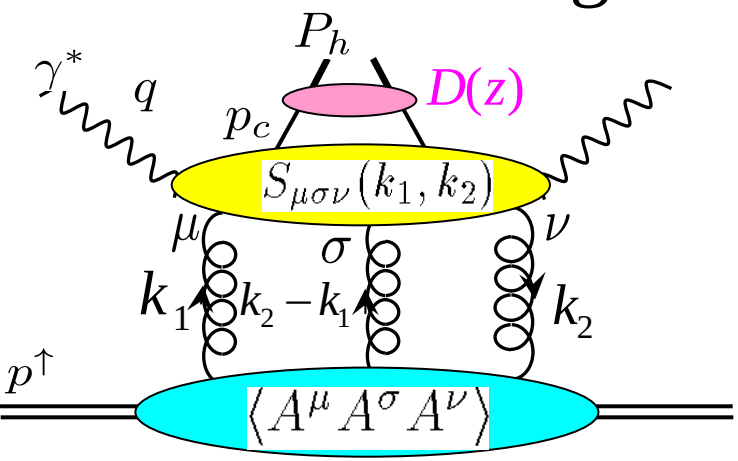
$k_2 - k_1 = 0$

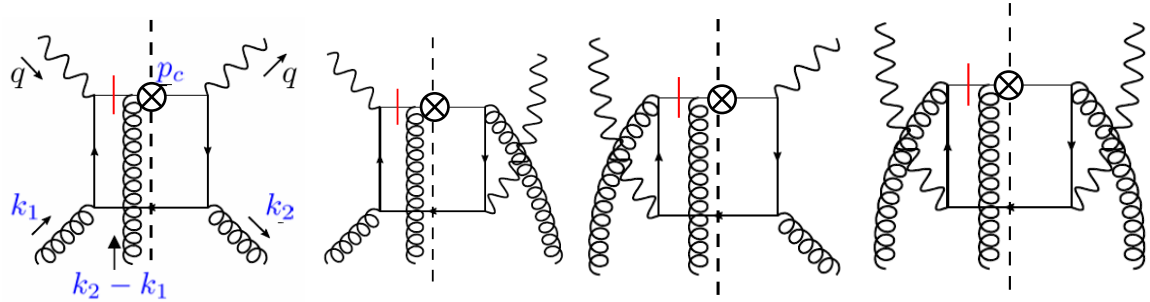
$k_1 = 0$

3. phase from parton propagator

$$\frac{1}{k^2 + i\varepsilon} = \mathrm{P}\frac{1}{k^2} - i\pi\delta(k^2)$$

SSA from multi-gluon correlation in SIDIS at $P_{h\perp} \gg \Lambda_{\rm QCD}$





SGP $k_2 - k_1 = 0$

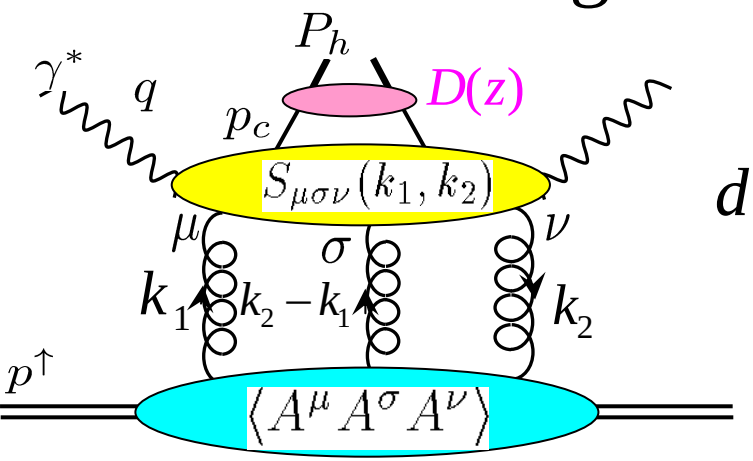
Collinear expansion:

$$S_{\mu\sigma\nu}(k_1,k_2) = S_{\mu\sigma\nu}(x_1 p, x_2 p) + \left.\frac{\partial S_{\mu\sigma\nu}(k_1,k_2)}{\partial k_1^\alpha}\right|_{k_i = x_i p} (k_1 - x_1 p)^\alpha + \left.\frac{\partial S_{\mu\sigma\nu}(k_1,k_2)}{\partial k_2^\alpha}\right|_{k_i = x_i p} (k_2 - x_2 p)^\alpha + \cdots$$

$$\cdots + \frac{1}{2}\left.\frac{\partial^3 S_{\mu\sigma\nu}(k_1,k_2)}{\partial k_1^\alpha \partial k_2^\beta \partial k_2^\gamma}\right|_{k_i = x_i p} (k_1 - x_1 p)^\alpha (k_2 - x_2 p)^\beta (k_2 - x_2 p)^\gamma + \frac{1}{6}\left.\frac{\partial^3 S_{\mu\sigma\nu}(k_1,k_2)}{\partial k_2^\alpha \partial k_2^\beta \partial k_2^\gamma}\right|_{k_i = x_i p} (k_2 - x_2 p)^\alpha (k_2 - x_2 p)^\beta (k_2 - x_2 p)^\gamma$$

$$A^\sigma = (A\cdot n)\, p^\sigma + A_\perp^\sigma + (A\cdot p) n^\sigma$$

SSA from multi-gluon correlation in SIDIS at $P_{h\perp} \gg \Lambda_{\rm QCD}$



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$$d\sigma_{\text{twist-3}} \sim \int dx_1 dx_2 dz \left.\frac{\partial S_{\mu\beta\nu}(k_1,k_2)\,p^\beta}{\partial k_{2\perp}^\sigma}\right|_{k_i = x_i p} \otimes \begin{Bmatrix} N_{\nu\mu\sigma}(x_1,x_2) \\ O_{\nu\mu\sigma}(x_1,x_2) \end{Bmatrix} \otimes D(z)$$

$$i\int \frac{d\lambda}{2\pi}\int \frac{d\zeta}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \langle p\, S_\perp | F_a^{\mu n}(0) g F_c^{\sigma n}(\zeta n) F_b^{\nu n}(\lambda n) | p\, S_\perp \rangle$$
$$= \frac{1}{24} i f^{abc} N^{\nu\mu\sigma}(x_1,x_2) + \frac{3}{40} d^{abc} O^{\nu\mu\sigma}(x_1,x_2)$$

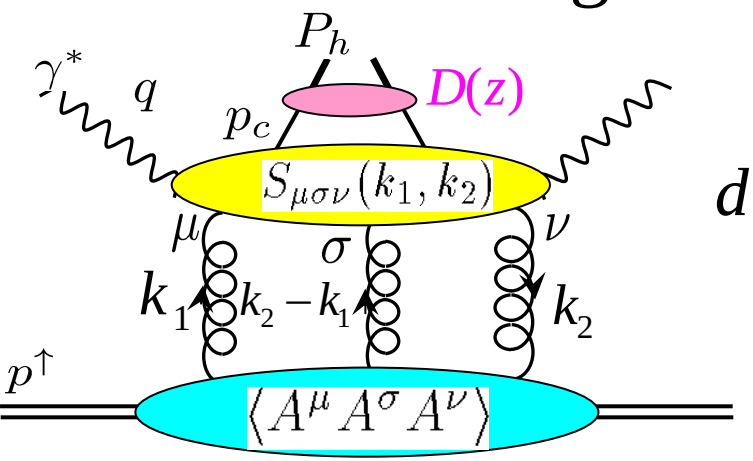
$S_{\mu\sigma\nu}(k_1,k_2)$ **in Feynman gauge calculation:**

$$\left.\frac{\partial S_{\mu\beta\nu}(k_1,k_2)\,p^\beta}{\partial k_1^\sigma}\right|_{k_i = x_i p} = -\left.\frac{\partial S_{\mu\beta\nu}(k_1,k_2)\,p^\beta}{\partial k_2^\sigma}\right|_{k_i = x_i p}$$

, **Ward identity**

$$\begin{cases} k_1^\mu S_{\mu\sigma\nu}(k_1,k_2) = 0 \\ k_2^\nu S_{\mu\sigma\nu}(k_1,k_2) = 0 \\ (k_2-k_1)^\sigma S_{\mu\sigma\nu}(k_1,k_2) = 0 \end{cases}$$

SSA from multi-gluon correlation in SIDIS at $P_{h\perp} \gg \Lambda_{QCD}$



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$$N^{\nu\mu\sigma}(x_1,x_2) = 2iM_N\left[g_\perp^{\nu\mu} S_{\perp\alpha}\varepsilon^{\sigma pn\alpha} N(x_1,x_2) - g_\perp^{\mu\sigma} S_{\perp\alpha}\varepsilon^{\nu pn\alpha} N(x_2,x_2-x_1) - g_\perp^{\sigma\nu} S_{\perp\alpha}\varepsilon^{\mu pn\alpha} N(x_1,x_1-x_2)\right]$$
$$O^{\nu\mu\sigma}(x_1,x_2) = 2iM_N\left[g_\perp^{\nu\mu} S_{\perp\alpha}\varepsilon^{\sigma pn\alpha} O(x_1,x_2) + g_\perp^{\mu\sigma} S_{\perp\alpha}\varepsilon^{\nu pn\alpha} O(x_2,x_2-x_1) + g_\perp^{\sigma\nu} S_{\perp\alpha}\varepsilon^{\mu pn\alpha} O(x_1,x_1-x_2)\right]$$

Permutation symmetry

$S_{\mu\sigma\nu}(k_1,k_2)$ **in Feynman gauge calculation:**

$$\left.\frac{\partial S_{\mu\beta\nu}(k_1,k_2)\,p^\beta}{\partial k_1^\sigma}\right|_{k_i=x_i p} = -\left.\frac{\partial S_{\mu\beta\nu}(k_1,k_2)\,p^\beta}{\partial k_2^\sigma}\right|_{k_i=x_i p}, \quad \textbf{Ward identity} \begin{cases} k_1^\mu S_{\mu\sigma\nu}(k_1,k_2) = 0 \\ k_2^\nu S_{\mu\sigma\nu}(k_1,k_2) = 0 \\ (k_2-k_1)^\sigma S_{\mu\sigma\nu}(k_1,k_2) = 0 \end{cases}$$

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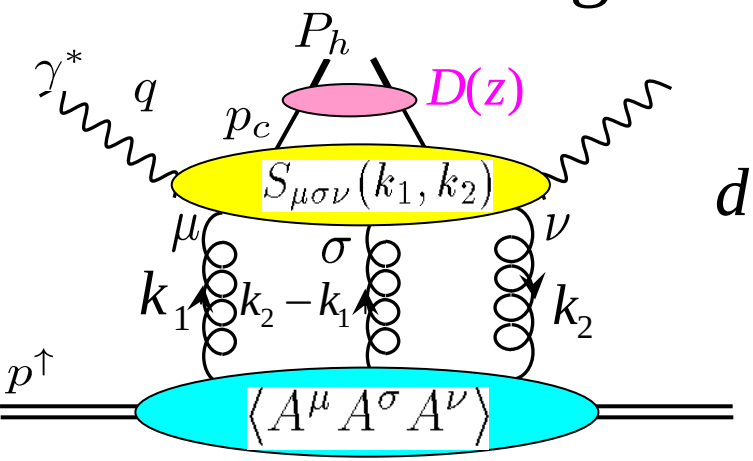

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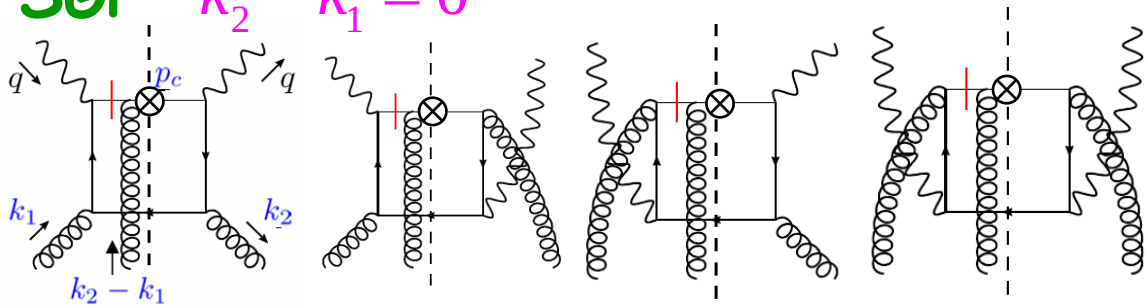
$$O^{\nu\mu\sigma}(x,x) = 2iM_N\left[g_\perp^{\nu\mu}S_{\perp\alpha}\varepsilon^{\sigma pn\alpha}O(x,x) + g_\perp^{\mu\sigma}S_{\perp\alpha}\varepsilon^{\nu pn\alpha}O(x,0) + g_\perp^{\sigma\nu}S_{\perp\alpha}\varepsilon^{\mu pn\alpha}O(x,0)\right]$$

Permutation symmetry

SGP $k_2 - k_1 = 0$

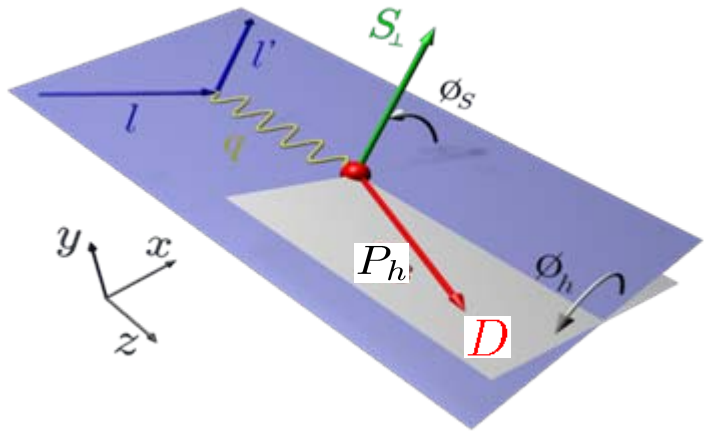


$N(x,x),\ O(x,x)$

$N(x,0),\ O(x,0)$

new!

⋆ Kinematics for $e(\ell) + p(p, S_\perp) \to e(\ell') + D(P_h) + X$



"Trento convention"

$$S_{ep} = (\ell + p)^2$$
$$q = \ell - \ell'$$
$$x_{bj} = \frac{Q^2}{2p\cdot q}$$
$$z_f = \frac{p\cdot P_h}{p\cdot q}$$

$P_{h\perp}$: $\perp$-mom. of final D

ϕ_h: azimuth. angle of hadron plane

ϕ_S: azimuth. angle of $\vec{S}_\perp$

- **twist-2 unpol. cross section**

$$\frac{d^5\sigma^{\text{unpol}}_{\text{twist-2}}}{dQ^2 dx_{bj} dz_f dP^2_{h\perp} d\phi_h} = \sigma^U_1 + \sigma^U_2 \cos(\phi_h) + \sigma^U_3 \cos(2\phi_h)$$

Final form of twist-3 SSA for $ep^{\uparrow} \to eDX$

$$\frac{d^5\sigma_{\text{twist-3}}}{dx_{bj}dQ^2dz_f dP_{h\perp}^2 d\phi_h} = \frac{\alpha_{em}^2\alpha_s e_c^2 M_N}{8\pi x_{bj}^2 z_f^2 S_{ep}^2 Q^2}\left(\frac{-\pi}{2}\right)\sum_{k=1,\cdots,4,8,9}\mathcal{A}_k\mathcal{S}_k\int\frac{dx}{x}\int\frac{dz}{z}\delta\left(\frac{P_{h\perp}^2}{z_f^2Q^2}-\left(1-\frac{1}{\hat{x}}\right)\left(1-\frac{1}{\hat{z}}\right)+\frac{m_c^2}{\hat{z}^2Q^2}\right)$$

$$\times\sum_{a=c,\bar{c}}D_a(z)\left[\delta_a\left\{\left(\frac{dO(x,x)}{dx}-\frac{2O(x,x)}{x}\right)\Delta\hat{\sigma}_k^1+\left(\frac{dO(x,0)}{dx}-\frac{2O(x,0)}{x}\right)\Delta\hat{\sigma}_k^2+\frac{O(x,x)}{x}\Delta\hat{\sigma}_k^3+\frac{O(x,0)}{x}\Delta\hat{\sigma}_k^4\right\}\right.$$

$$\left.+\left\{\left(\frac{dN(x,x)}{dx}-\frac{2N(x,x)}{x}\right)\Delta\hat{\sigma}_k^1-\left(\frac{dN(x,0)}{dx}-\frac{2N(x,0)}{x}\right)\Delta\hat{\sigma}_k^2+\frac{N(x,x)}{x}\Delta\hat{\sigma}_k^3-\frac{N(x,0)}{x}\Delta\hat{\sigma}_k^4\right\}\right]$$

$$\mathcal{A}_1 = 1+\cosh^2\psi,\quad \mathcal{A}_2=-2,\quad \mathcal{A}_3=-\cos\phi_h\sinh 2\psi,\quad \mathcal{A}_4=\cos 2\phi_h\sinh^2\psi,$$

$$\mathcal{A}_8=-\sin\phi_h\sinh 2\psi,\quad \mathcal{A}_9=\sin 2\phi_h\sinh^2\psi.$$

$$\cosh\psi = \frac{2x_{bj}S_{ep}}{Q^2}-1$$

$$\mathcal{S}_k=\sin(\phi_h-\phi_S)\quad (k=1,2,3,4),\qquad \mathcal{S}_k=\cos(\phi_h-\phi_S)\quad (k=8,9)$$

$$\hat{x}=\frac{x_{bj}}{x},\ \hat{z}=\frac{z_f}{z}$$

$$\delta_c=1,\ \delta_{\bar{c}}=-1$$

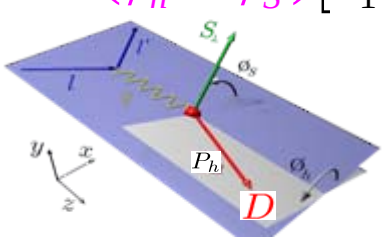
- Receives contribution from 4 functions $O(x,x)$, $O(x,0)$, $N(x,x)$ and $N(x,0)$, and all of them contribute both as derivative and nonderivative terms.

$$\Delta\hat{\sigma}_k^1 \neq \Delta\hat{\sigma}_k^2$$
$$\Delta\hat{\sigma}_k^3 \neq \Delta\hat{\sigma}_k^4$$

-5 structure functions with different azimuthal dependence

similar as $ep^{\uparrow}\to e\pi X$ from quark-gluon correlation

$$=\sin(\phi_h-\phi_S)\left[\mathcal{F}_1+\mathcal{F}_2\cos\phi_h+\mathcal{F}_3\cos 2\phi_h\right]+\cos(\phi_h-\phi_S)\left[\mathcal{F}_4\sin\phi_h+\mathcal{F}_5\sin 2\phi_h\right]$$



Final form of twist-3 SSA for $ep^{\uparrow} \to eDX$

$$\frac{d^5\sigma_{\text{twist-3}}}{dx_{bj}dQ^2dz_f dP_{h\perp}^2 d\phi_h} = \frac{\alpha_{em}^2\alpha_s e_c^2 M_N}{8\pi x_{bj}^2 z_f^2 S_{ep}^2 Q^2}\left(\frac{-\pi}{2}\right)\sum_{k=1,\cdots,4,8,9}\mathcal{A}_k\mathcal{S}_k\int\frac{dx}{x}\int\frac{dz}{z}\delta\left(\frac{P_{h\perp}^2}{z_f^2Q^2}-\left(1-\frac{1}{\hat{x}}\right)\left(1-\frac{1}{\hat{z}}\right)+\frac{m_c^2}{\hat{z}^2Q^2}\right)$$

$$\times\sum_{a=c,\bar{c}}D_a(z)\left[\delta_a\left\{\left(\frac{dO(x,x)}{dx}-\frac{2O(x,x)}{x}\right)\Delta\hat{\sigma}_k^1+\left(\frac{dO(x,0)}{dx}-\frac{2O(x,0)}{x}\right)\Delta\hat{\sigma}_k^2+\frac{O(x,x)}{x}\Delta\hat{\sigma}_k^3+\frac{O(x,0)}{x}\Delta\hat{\sigma}_k^4\right\}\right.$$

$$\left.+\left\{\left(\frac{dN(x,x)}{dx}-\frac{2N(x,x)}{x}\right)\Delta\hat{\sigma}_k^1-\left(\frac{dN(x,0)}{dx}-\frac{2N(x,0)}{x}\right)\Delta\hat{\sigma}_k^2+\frac{N(x,x)}{x}\Delta\hat{\sigma}_k^3-\frac{N(x,0)}{x}\Delta\hat{\sigma}_k^4\right\}\right]$$

$$\mathcal{A}_1 = 1+\cosh^2\psi,\quad \mathcal{A}_2=-2,\quad \mathcal{A}_3=-\cos\phi_h\sinh 2\psi,\quad \mathcal{A}_4=\cos 2\phi_h\sinh^2\psi,$$

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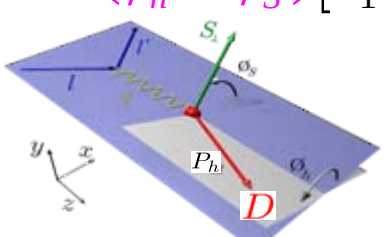


absent from the results by Kang, Qiu, PRD78 ('08) 034005

Beppu, Koike, Tanaka, Yoshida, PRD82 ('10) 054005

$$\Delta\hat{\sigma}_1^1 = \frac{8q_T\hat{x}}{Q^6(1-\hat{z})^3\hat{z}^2}\{Q^4\hat{z}(1-\hat{z})(1-2\hat{z}+2\hat{z}^2-2\hat{x}+2\hat{x}^2+12\hat{x}\,\hat{z}(1-\hat{x})(1-\hat{z})) + 2m_c^2Q^2\hat{x}(2\hat{z}(1-\hat{z})+\hat{x}(1-8\hat{z}+8\hat{z}^2))-4m_c^4\hat{x}^2\},$$

$$\Delta\hat{\sigma}_2^1 = \frac{64q_T\hat{x}^2}{Q^4(1-\hat{z})^2\hat{z}}\{Q^2\hat{z}(1-\hat{x})(1-\hat{z})-m_c^2\hat{x}\},$$

$$\Delta\hat{\sigma}_3^1 = \frac{16\hat{x}}{Q^5(1-\hat{z})^3\hat{z}^3}(1-2\hat{z})\{Q^2\hat{z}(1-\hat{x})(1-\hat{z})-m_c^2\hat{x}\}\{Q^2\hat{z}(1-2\hat{x})(1-\hat{z})-2m_c^2\hat{x}\},$$

$$\Delta\hat{\sigma}_4^1 = \frac{32q_T\hat{x}^2}{Q^6(1-\hat{z})^3\hat{z}^2}\{Q^2\hat{z}(1-\hat{x})(1-\hat{z})-m_c^2\hat{x}\}\{Q^2\hat{z}(1-\hat{z})+m_c^2\}, \qquad \Delta\hat{\sigma}_8^1 = \Delta\hat{\sigma}_9^1 = 0,$$

$$\hat{x} = \frac{x_{bj}}{x}, \quad \hat{z} = \frac{z_f}{z}$$

$$q_T \equiv \frac{P_{h\perp}}{z_f}$$

$$\Delta\hat{\sigma}_1^2 = \frac{8q_T\hat{x}}{Q^6(1-\hat{z})^3\hat{z}^2}\{Q^4\hat{z}(1-\hat{z})(1-2\hat{z}+2\hat{z}^2-4\hat{x}+4\hat{x}^2+24\hat{x}\,\hat{z}(1-\hat{x})(1-\hat{z})) + 4m_c^2Q^2\hat{x}(2\hat{z}(1-\hat{z})+\hat{x}(1-8\hat{z}+8\hat{z}^2))-8m_c^4\hat{x}^2\},$$

$$\Delta\hat{\sigma}_2^2 = 2\Delta\hat{\sigma}_2^1,$$

$$\Delta\hat{\sigma}_3^2 = 2\Delta\hat{\sigma}_3^1,$$

$$\Delta\hat{\sigma}_4^2 = -\frac{16q_T\hat{x}}{Q^6(1-\hat{z})^3\hat{z}^2}(Q^2\hat{z}(1-2\hat{x})(1-\hat{z})-2m_c^2\hat{x})^2,$$

$$\Delta\hat{\sigma}_8^2 = \frac{16\hat{x}}{Q^3(1-\hat{z})^2\hat{z}^2}(1-2\hat{z})(Q^2\hat{z}(1-\hat{x})(1-\hat{z})-m_c^2\hat{x}),$$

$$\Delta\hat{\sigma}_9^2 = -\frac{16q_T\hat{x}}{Q^4(1-\hat{z})^2\hat{z}}(Q^2\hat{z}(1-2\hat{x})(1-\hat{z})-2m_c^2\hat{x}),$$

$$\Delta\hat{\sigma}_1^3 = \frac{16q_T\hat{x}^2}{Q^6(1-\hat{z})^3\hat{z}^2}(Q^2\hat{z}(1-2\hat{x})(1-\hat{z})-2m_c^2\hat{x})(Q^2(1-6\hat{z}+6\hat{z}^2)-2m_c^2),$$

$$\Delta\hat{\sigma}_2^3 = -\frac{64q_T\hat{x}^2}{Q^4(1-\hat{z})^2\hat{z}}(Q^2\hat{z}(1-2\hat{x})(1-\hat{z})-2m_c^2\hat{x}),$$

$$\Delta\hat{\sigma}_3^3 = -\frac{8\hat{x}}{Q^5(1-\hat{z})^3\hat{z}^3}(1-2\hat{z})\{Q^4\hat{z}^2(1-\hat{z})^2(1-8\hat{x}+8\hat{x}^2)-8m_c^2Q^2\hat{x}\,\hat{z}(1-2\hat{x})(1-\hat{z})+8m_c^4\hat{x}^2\},$$

$$\Delta\hat{\sigma}_4^3 = -\frac{32q_T\hat{x}^2}{Q^6(1-\hat{z})^3\hat{z}^2}(Q^2\hat{z}(1-\hat{z})+m_c^2)(Q^2\hat{z}(1-2\hat{x})(1-\hat{z})-2m_c^2\hat{x}),$$

$$\Delta\hat{\sigma}_8^3 = -\frac{8\hat{x}}{Q^3(1-\hat{z})^2\hat{z}^2}(1-2\hat{z})(Q^2\hat{z}(1-2\hat{x})(1-\hat{z})-2m_c^2\hat{x}),$$

$$\Delta\hat{\sigma}_9^3 = -\frac{32q_T\hat{x}^2}{Q^4(1-\hat{z})^2\hat{z}}(Q^2\hat{z}(1-\hat{z})+m_c^2),$$

$$\Delta\hat{\sigma}_1^4 = \frac{16q_T\hat{x}^2}{Q^6(1-\hat{z})^3\hat{z}^2}(Q^2\hat{z}(1-4\hat{x})(1-\hat{z})-4m_c^2\hat{x})(Q^2(1-6\hat{z}+6\hat{z}^2)-2m_c^2),$$

$$\Delta\hat{\sigma}_2^4 = -\frac{64q_T\hat{x}^2}{Q^4(1-\hat{z})^2\hat{z}}(Q^2\hat{z}(1-4\hat{x})(1-\hat{z})-4m_c^2\hat{x}),$$

$$\Delta\hat{\sigma}_3^4 = \frac{8\hat{x}}{Q^5(1-\hat{z})^3\hat{z}^3}(1-2\hat{z})\{Q^4\hat{z}^2(1-\hat{z})^2(1+12\hat{x}-16\hat{x}^2)+4m_c^2Q^2\hat{x}\,\hat{z}(3-8\hat{x})(1-\hat{z})-16m_c^4\hat{x}^2\},$$

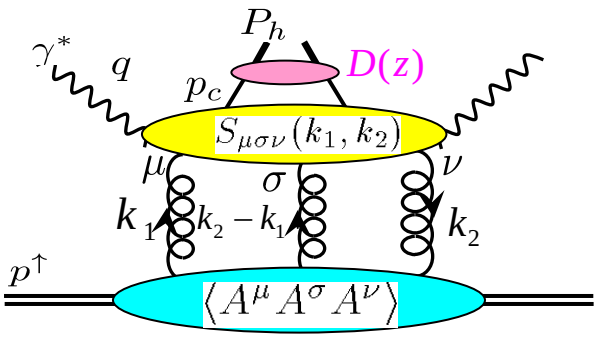
$$\Delta\hat{\sigma}_4^4 = -\frac{32q_T\hat{x}}{Q^6(1-\hat{z})^3\hat{z}^2}(Q^4\hat{z}^2(1-\hat{z})^2(1+\hat{x}-4\hat{x}^2)+m_c^2Q^2\hat{x}\,\hat{z}(1-8\hat{x})(1-\hat{z})-4m_c^4\hat{x}^2),$$

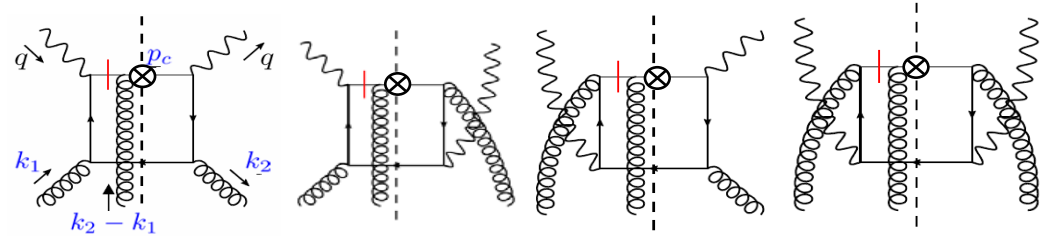
$$\Delta\hat{\sigma}_8^4 = \frac{8\hat{x}}{Q^3(1-\hat{z})^2\hat{z}^2}(1-2\hat{z})(Q^2\hat{z}(1+2\hat{x})(1-\hat{z})+2m_c^2\hat{x}),$$

$$\Delta\hat{\sigma}_9^4 = -\frac{32q_T\hat{x}}{Q^4(1-\hat{z})^2\hat{z}}(Q^2\hat{z}(1-\hat{z})(1+\hat{x})+m_c^2\hat{x}),$$

$$\left(m_c \to 0 \text{ for } ep^{\uparrow} \to e\pi X \text{ case}\right)$$

$$d\sigma_{\text{twist-3}} \sim \int dx_1 dx_2 dz \left.\frac{\partial S_{\mu\beta\nu}(k_1,k_2)\, p^{\beta}}{\partial k_{2\perp}^{\sigma}}\right|_{k_i = x_i p} \otimes \begin{Bmatrix} N_{\nu\mu\sigma}(x_1,x_2) \\ O_{\nu\mu\sigma}(x_1,x_2) \end{Bmatrix} \otimes D(z)$$



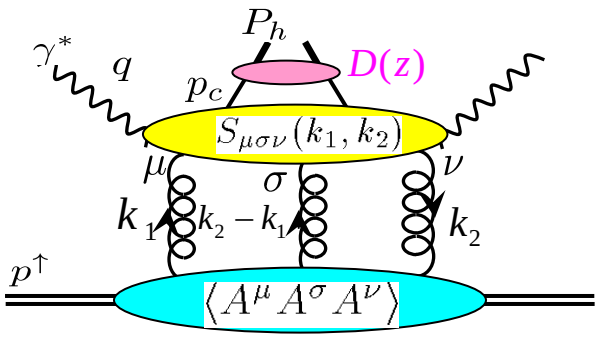


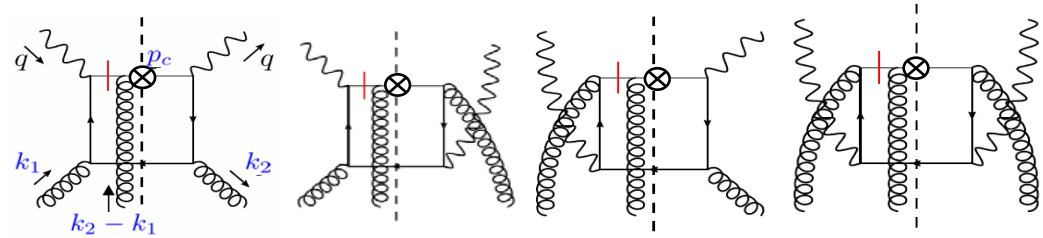
SGP $k_2 - k_1 = 0$

$$N^{\nu\mu\sigma}(x_1,x_2) = 2iM_N \left[g_{\perp}^{\nu\mu} S_{\perp\alpha} \varepsilon^{\sigma pn\alpha} N(x_1,x_2) - g_{\perp}^{\mu\sigma} S_{\perp\alpha} \varepsilon^{\nu pn\alpha} N(x_2,x_2-x_1) - g_{\perp}^{\sigma\nu} S_{\perp\alpha} \varepsilon^{\mu pn\alpha} N(x_1,x_1-x_2) \right]$$

$$O^{\nu\mu\sigma}(x_1,x_2) = 2iM_N \left[g_{\perp}^{\nu\mu} S_{\perp\alpha} \varepsilon^{\sigma pn\alpha} O(x_1,x_2) + g_{\perp}^{\mu\sigma} S_{\perp\alpha} \varepsilon^{\nu pn\alpha} O(x_2,x_2-x_1) + g_{\perp}^{\sigma\nu} S_{\perp\alpha} \varepsilon^{\mu pn\alpha} O(x_1,x_1-x_2) \right]$$

$$d\sigma_{\text{twist-3}} \sim \int dx_1 dx_2 dz \left.\frac{\partial S_{\mu\beta\nu}(k_1,k_2)\,p^\beta}{\partial k_{2\perp}^\sigma}\right|_{k_i = x_i p} \otimes \begin{Bmatrix} N_{\nu\mu\sigma}(x_1,x_2) \\ O_{\nu\mu\sigma}(x_1,x_2) \end{Bmatrix} \otimes D(z)$$





SGP $k_2 - k_1 = 0$

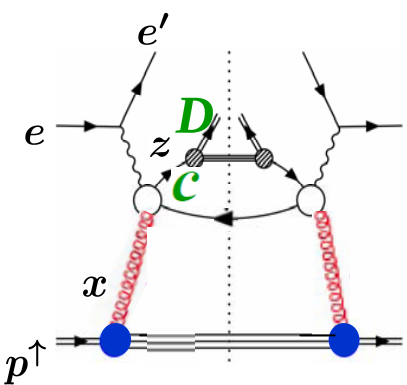
$$N^{\nu\mu\sigma}(x,x) = 2iM_N\left[g_\perp^{\nu\mu} S_{\perp\alpha}\varepsilon^{\sigma pn\alpha} N(x,x) - g_\perp^{\mu\sigma} S_{\perp\alpha}\varepsilon^{\nu pn\alpha} N(x,0) - g_\perp^{\sigma\nu} S_{\perp\alpha}\varepsilon^{\mu pn\alpha} N(x,0)\right]$$

$$O^{\nu\mu\sigma}(x,x) = 2iM_N\left[g_\perp^{\nu\mu} S_{\perp\alpha}\varepsilon^{\sigma pn\alpha} O(x,x) + g_\perp^{\mu\sigma} S_{\perp\alpha}\varepsilon^{\nu pn\alpha} O(x,0) + g_\perp^{\sigma\nu} S_{\perp\alpha}\varepsilon^{\mu pn\alpha} O(x,0)\right]$$

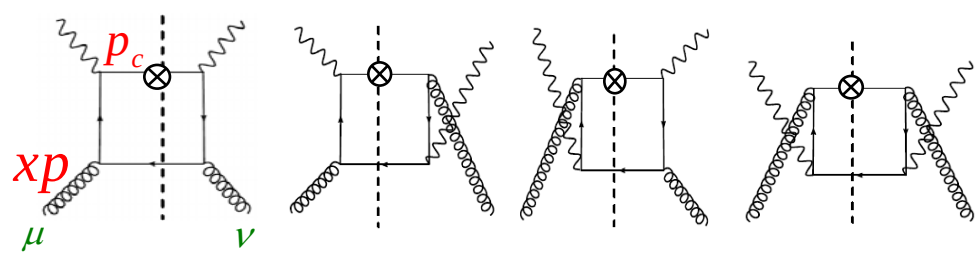
Master formula

Koike, Tanaka, Yoshida, arXiv:1104.0798

$$d\sigma_{\text{twist-3}} \sim -i\pi \int dx dz \frac{\partial \mathcal{H}_{\mu\nu}(xp, p_c)}{\partial p_c^\sigma} \otimes \begin{Bmatrix} N_{\nu\mu\sigma}(x,x) \\ O_{\nu\mu\sigma}(x,x) \end{Bmatrix} \otimes D(z)$$



$\mathcal{H}_{\mu\nu}(xp, p_c) =$



2→2 Born subprocess

Final form of twist-3 SSA for $ep^{\uparrow} \to eDX$

$$\frac{d^5\sigma_{\text{twist-3}}}{dx_{bj}dQ^2dz_f dP_{h\perp}^2 d\phi_h} = \frac{\alpha_{em}^2\alpha_s e_c^2 M_N}{8\pi x_{bj}^2 z_f^2 S_{ep}^2 Q^2}\left(\frac{-\pi}{2}\right)\sum_{k=1,\cdots,4,8,9} \mathcal{A}_k \mathcal{S}_k \int\frac{dx}{x}\int\frac{dz}{z}\delta\left(\frac{P_{h\perp}^2}{z_f^2Q^2}-\left(1-\frac{1}{\hat{x}}\right)\left(1-\frac{1}{\hat{z}}\right)+\frac{m_c^2}{\hat{z}^2Q^2}\right)$$

$$\times\sum_{a=c,\bar{c}} D_a(z)\left[\delta_a\left\{\left(\frac{dO(x,x)}{dx}-\frac{2O(x,x)}{x}\right)\Delta\hat{\sigma}_k^1+\left(\frac{dO(x,0)}{dx}-\frac{2O(x,0)}{x}\right)\Delta\hat{\sigma}_k^2+\frac{O(x,x)}{x}\Delta\hat{\sigma}_k^3+\frac{O(x,0)}{x}\Delta\hat{\sigma}_k^4\right\}\right.$$

$$\left.+\left\{\left(\frac{dN(x,x)}{dx}-\frac{2N(x,x)}{x}\right)\Delta\hat{\sigma}_k^1-\left(\frac{dN(x,0)}{dx}-\frac{2N(x,0)}{x}\right)\Delta\hat{\sigma}_k^2+\frac{N(x,x)}{x}\Delta\hat{\sigma}_k^3-\frac{N(x,0)}{x}\Delta\hat{\sigma}_k^4\right\}\right]$$

$$\Delta\hat{\sigma}_k^1 = \frac{2P_{h\perp}\hat{x}}{Q^2 z_f(1-\hat{z})}\hat{\sigma}_k^U, \qquad \Delta\hat{\sigma}_k^3 = \frac{2P_{h\perp}}{z_f Q^2}\left(\frac{z_f^2Q^2}{\hat{z}}\frac{\partial\hat{\sigma}_k^U}{\partial P_{h\perp}^2}-\frac{\hat{x}^2}{1-\hat{z}}\frac{\partial\hat{\sigma}_k^U}{\partial\hat{x}}\right)-\frac{2^{k-8}z_f\hat{\sigma}_{k-5}^U}{\hat{z}P_{h\perp}}$$

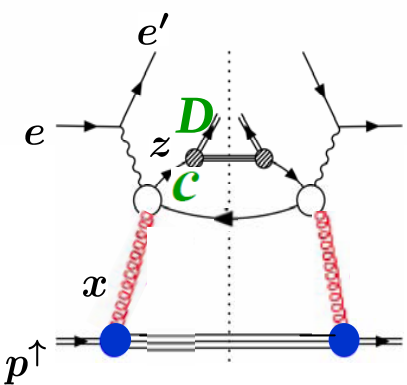
$$\Delta\hat{\sigma}_k^{2,4} \Leftarrow \mathcal{H}_{11}(xp,p_c),\ \mathcal{H}_{22}(xp,p_c),\ \mathcal{H}_{12}(xp,p_c)+\mathcal{H}_{21}(xp,p_c)$$

twist-2 unpol. cross section

$$\frac{d^5\sigma_{\text{twist-2}}^{\text{unpol}}}{dx_{bj}dQ^2dz_f dP_{h\perp}^2 d\phi_h} = \frac{\alpha_{em}^2\alpha_s e_c^2}{8\pi x_{bj}^2 z_f^2 S_{ep}^2 Q^2}\frac{1}{4}\sum_{k=1}^{4}\mathcal{A}_k\int\frac{dx}{x}\int\frac{dz}{z}\delta\left(\frac{P_{h\perp}^2}{z_f^2Q^2}-\left(1-\frac{1}{\hat{x}}\right)\left(1-\frac{1}{\hat{z}}\right)+\frac{m_c^2}{\hat{z}^2Q^2}\right)$$

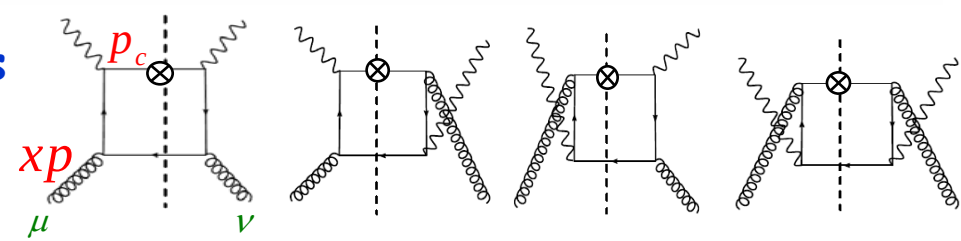
$$\times\sum_{a=c,\bar{c}} D_a(z)g(x)\,\hat{\sigma}_k^U$$

$$\hat{\sigma}_k^U \sim g_\perp^{\nu\mu}\mathcal{H}_{\mu\nu}(xp,p_c)$$



2→2 Born subprocess

$$\mathcal{H}_{\mu\nu}(xp,p_c) =$$



Estimates of twist-3 SSA for $ep^{\uparrow} \to eD^0X$

$$\frac{d^5\sigma_{\text{twist-3}}}{dx_{bj}dQ^2dz_f dP_{h\perp}^2 d\phi_h} = \frac{\alpha_{em}^2\alpha_s e_c^2 M_N}{8\pi x_{bj}^2 z_f^2 S_{ep}^2 Q^2}\left(\frac{-\pi}{2}\right)\sum_{k=1,\cdots,4,8,9}\mathcal{A}_k\mathcal{S}_k\int\frac{dx}{x}\int\frac{dz}{z}\delta\left(\frac{P_{h\perp}^2}{z_f^2Q^2}-\left(1-\frac{1}{\hat{x}}\right)\left(1-\frac{1}{\hat{z}}\right)+\frac{m_c^2}{\hat{z}^2Q^2}\right)$$

$$\times\sum_{a=c,\bar{c}}D_a(z)\left[\delta_a\left\{\left(\frac{dO(x,x)}{dx}-\frac{2O(x,x)}{x}\right)\Delta\hat{\sigma}_k^1+\left(\frac{dO(x,0)}{dx}-\frac{2O(x,0)}{x}\right)\Delta\hat{\sigma}_k^2+\frac{O(x,x)}{x}\Delta\hat{\sigma}_k^3+\frac{O(x,0)}{x}\Delta\hat{\sigma}_k^4\right\}\right.$$

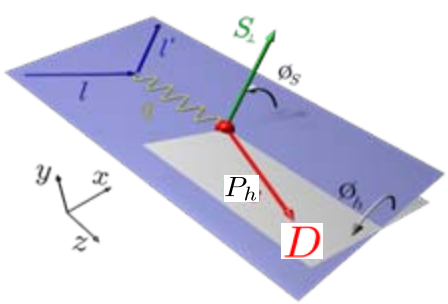
$$\left.+\left\{\left(\frac{dN(x,x)}{dx}-\frac{2N(x,x)}{x}\right)\Delta\hat{\sigma}_k^1-\left(\frac{dN(x,0)}{dx}-\frac{2N(x,0)}{x}\right)\Delta\hat{\sigma}_k^2+\frac{N(x,x)}{x}\Delta\hat{\sigma}_k^3-\frac{N(x,0)}{x}\Delta\hat{\sigma}_k^4\right\}\right]$$

$$=\sin(\phi_h-\phi_S)\left[\mathcal{F}_1+\mathcal{F}_2\cos\phi_h+\mathcal{F}_3\cos 2\phi_h\right]+\cos(\phi_h-\phi_S)\left[\mathcal{F}_4\sin\phi_h+\mathcal{F}_5\sin 2\phi_h\right]$$

$$A_N:\frac{\mathcal{F}_1}{\sigma_1^U},\ \frac{\mathcal{F}_{2,3,4,5}}{2\sigma_1^U}$$

$$\left(\frac{d^5\sigma_{\text{twist-2}}^{\text{unpol}}}{dQ^2dx_{bj}dz_f dP_{h\perp}^2 d\phi_h}=\sigma_1^U+\sigma_2^U\cos\phi_h+\sigma_3^U\cos 2\phi_h\right)$$

$$\langle 1\rangle,\langle\cos\phi\rangle,\langle\cos 2\phi\rangle,\langle\sin\phi\rangle,\langle\sin 2\phi\rangle$$



Estimates of twist-3 SSA for $ep^{\uparrow} \to eD^0X$

$$\frac{d^5\sigma_{\text{twist-3}}}{dx_{bj}dQ^2dz_f dP_{h\perp}^2 d\phi_h} = \frac{\alpha_{em}^2\alpha_s e_c^2 M_N}{8\pi x_{bj}^2 z_f^2 S_{ep}^2 Q^2}\left(\frac{-\pi}{2}\right)\sum_{k=1,\cdots,4,8,9} \mathcal{A}_k \mathcal{S}_k \int\frac{dx}{x}\int\frac{dz}{z}\delta\left(\frac{P_{h\perp}^2}{z_f^2Q^2}-\left(1-\frac{1}{\hat{x}}\right)\left(1-\frac{1}{\hat{z}}\right)+\frac{m_c^2}{\hat{z}^2Q^2}\right)$$

$$\times\sum_{a=c,\bar{c}} D_a(z)\left[\delta_a\left\{\left(\frac{dO(x,x)}{dx}-\frac{2O(x,x)}{x}\right)\Delta\hat{\sigma}_k^1+\left(\frac{dO(x,0)}{dx}-\frac{2O(x,0)}{x}\right)\Delta\hat{\sigma}_k^2+\frac{O(x,x)}{x}\Delta\hat{\sigma}_k^3+\frac{O(x,0)}{x}\Delta\hat{\sigma}_k^4\right\}\right.$$

$$\left.+\left\{\left(\frac{dN(x,x)}{dx}-\frac{2N(x,x)}{x}\right)\Delta\hat{\sigma}_k^1-\left(\frac{dN(x,0)}{dx}-\frac{2N(x,0)}{x}\right)\Delta\hat{\sigma}_k^2+\frac{N(x,x)}{x}\Delta\hat{\sigma}_k^3-\frac{N(x,0)}{x}\Delta\hat{\sigma}_k^4\right\}\right]$$

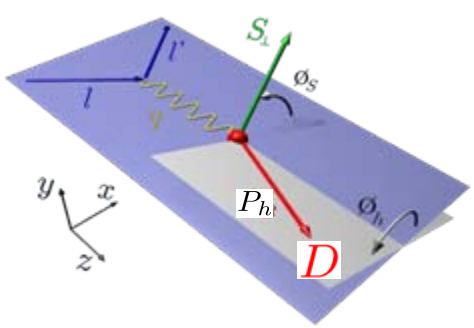
$$=\sin(\phi_h-\phi_S)\left[\mathcal{F}_1+\mathcal{F}_2\cos\phi_h+\mathcal{F}_3\cos 2\phi_h\right]+\cos(\phi_h-\phi_S)\left[\mathcal{F}_4\sin\phi_h+\mathcal{F}_5\sin 2\phi_h\right]$$

$$A_N : \frac{\mathcal{F}_1}{\sigma_1^U},\ \frac{\mathcal{F}_{2,3,4,5}}{2\sigma_1^U}$$

$$\left(\frac{d^5\sigma_{\text{twist-2}}^{\text{unpol}}}{dQ^2dx_{bj}dz_f dP_{h\perp}^2 d\phi_h} = \sigma_1^U+\sigma_2^U\cos\phi_h+\sigma_3^U\cos 2\phi_h\right)$$

$$\langle 1\rangle, \langle\cos\phi\rangle, \langle\cos 2\phi\rangle, \langle\sin\phi\rangle, \langle\sin 2\phi\rangle$$

$$\mu=\sqrt{Q^2+m_c^2+P_{h\perp}^2}$$



$$O(x,x)=O(x,0)=\frac{\sqrt{4\pi\alpha_s(\mu)}}{8\pi M_N}\left[(0.07\ \text{GeV})\times xg(x,\mu)\right]$$

CTEQ6L

$D_c(z)$: **Kneesch-Kniehl-Kramer-Schienbein08**

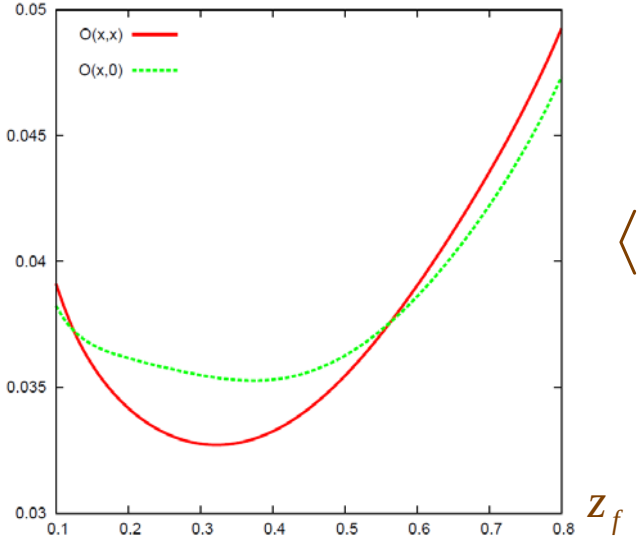
(cf: Kang, Qiu, PRD78 ('08) 034005)

$$\frac{d^5\sigma_{\text{twist-3}}}{dx_{bj}dQ^2dz_f dP_{h\perp}^2 d\phi_h} = \frac{\alpha_{em}^2\alpha_s e_c^2 M_N}{8\pi x_{bj}^2 z_f^2 S_{ep}^2 Q^2}\left(\frac{-\pi}{2}\right)\sum_{k=1,\cdots,4,8,9}\mathcal{A}_k\mathcal{S}_k\int\frac{dx}{x}\int\frac{dz}{z}\delta\left(\frac{P_{h\perp}^2}{z_f^2Q^2}-\left(1-\frac{1}{\hat{x}}\right)\left(1-\frac{1}{\hat{z}}\right)+\frac{m_c^2}{\hat{z}^2Q^2}\right)$$

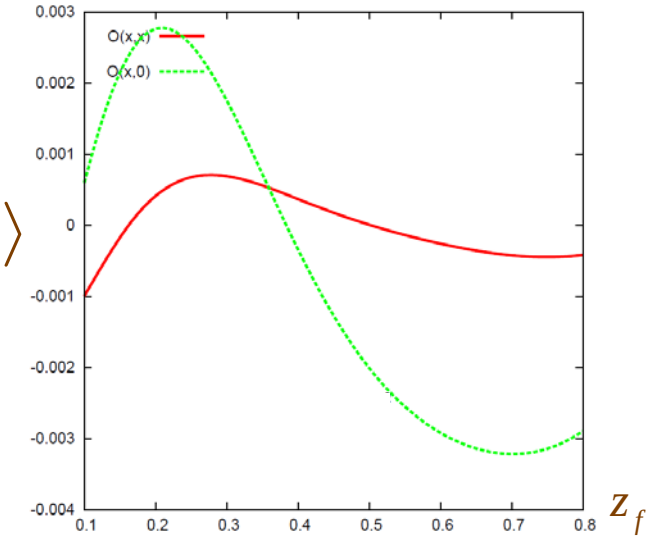
$$\times\sum_{a=c,\bar{c}}D_a(z)\left[\delta_a\left\{\left(\frac{dO(x,x)}{dx}-\frac{2O(x,x)}{x}\right)\Delta\hat{\sigma}_k^1+\left(\frac{dO(x,0)}{dx}-\frac{2O(x,0)}{x}\right)\Delta\hat{\sigma}_k^2+\frac{O(x,x)}{x}\Delta\hat{\sigma}_k^3+\frac{O(x,0)}{x}\Delta\hat{\sigma}_k^4\right\}\right.$$

$$\left.+\left\{\left(\frac{dN(x,x)}{dx}-\frac{2N(x,x)}{x}\right)\Delta\hat{\sigma}_k^1-\left(\frac{dN(x,0)}{dx}-\frac{2N(x,0)}{x}\right)\Delta\hat{\sigma}_k^2+\frac{N(x,x)}{x}\Delta\hat{\sigma}_k^3-\frac{N(x,0)}{x}\Delta\hat{\sigma}_k^4\right\}\right]$$

$\langle 1\rangle$



$\langle\cos\phi\rangle$

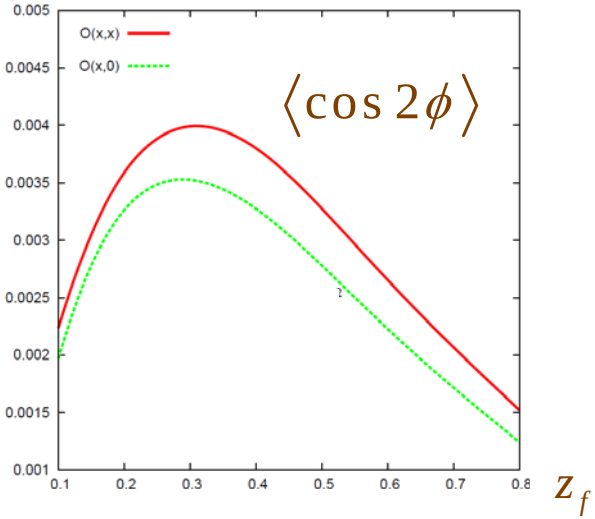


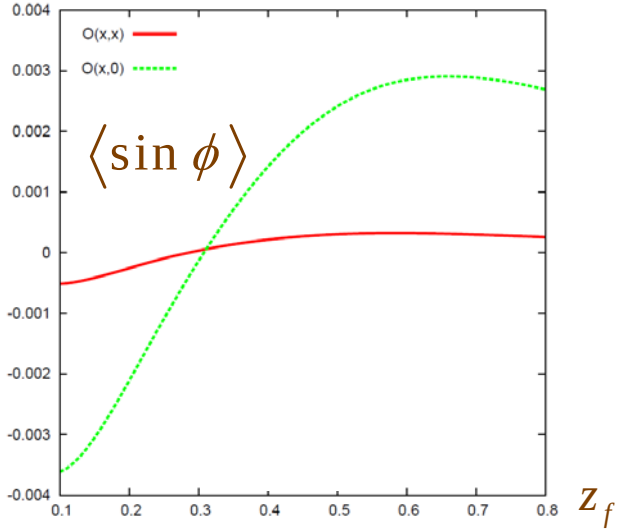
$S_{ep} = 2500\ \text{GeV}^2$

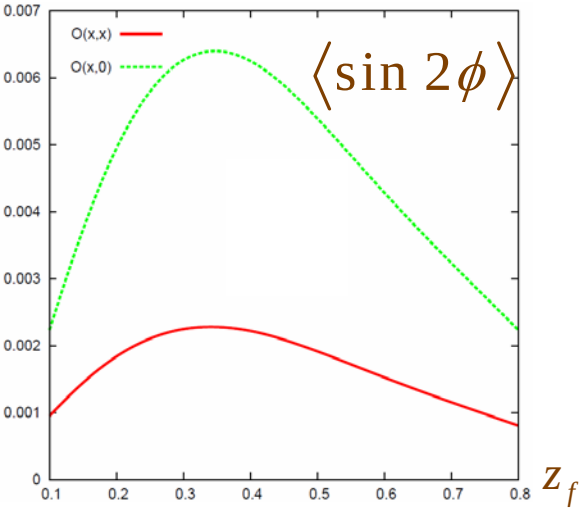
$Q = 4\ \text{GeV}$

$x_{bj} = 0.01$

$P_{h\perp} = 2\ \text{GeV}$





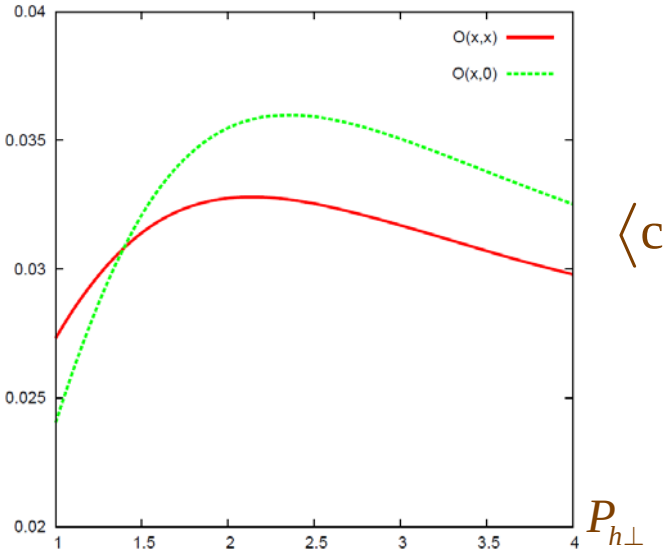


$$\frac{d^5\sigma_{\text{twist-3}}}{dx_{bj}dQ^2dz_f dP_{h\perp}^2 d\phi_h} = \frac{\alpha_{em}^2\alpha_s e_c^2 M_N}{8\pi x_{bj}^2 z_f^2 S_{ep}^2 Q^2}\left(\frac{-\pi}{2}\right)\sum_{k=1,\cdots,4,8,9}\mathcal{A}_k\mathcal{S}_k\int\frac{dx}{x}\int\frac{dz}{z}\delta\left(\frac{P_{h\perp}^2}{z_f^2Q^2}-\left(1-\frac{1}{\hat{x}}\right)\left(1-\frac{1}{\hat{z}}\right)+\frac{m_c^2}{\hat{z}^2Q^2}\right)$$

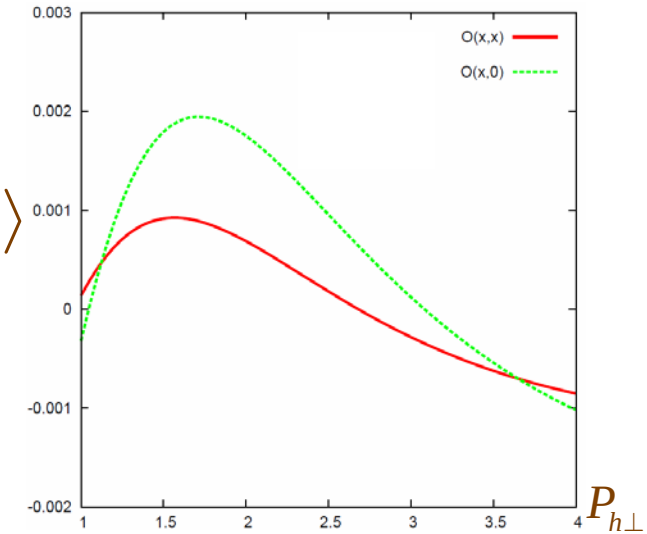
$$\times\sum_{a=c,\bar{c}}D_a(z)\left[\delta_a\left\{\left(\frac{dO(x,x)}{dx}-\frac{2O(x,x)}{x}\right)\Delta\hat{\sigma}_k^1+\left(\frac{dO(x,0)}{dx}-\frac{2O(x,0)}{x}\right)\Delta\hat{\sigma}_k^2+\frac{O(x,x)}{x}\Delta\hat{\sigma}_k^3+\frac{O(x,0)}{x}\Delta\hat{\sigma}_k^4\right\}\right.$$

$$\left.+\left\{\left(\frac{dN(x,x)}{dx}-\frac{2N(x,x)}{x}\right)\Delta\hat{\sigma}_k^1-\left(\frac{dN(x,0)}{dx}-\frac{2N(x,0)}{x}\right)\Delta\hat{\sigma}_k^2+\frac{N(x,x)}{x}\Delta\hat{\sigma}_k^3-\frac{N(x,0)}{x}\Delta\hat{\sigma}_k^4\right\}\right]$$

$\langle 1\rangle$



$\langle\cos\phi\rangle$

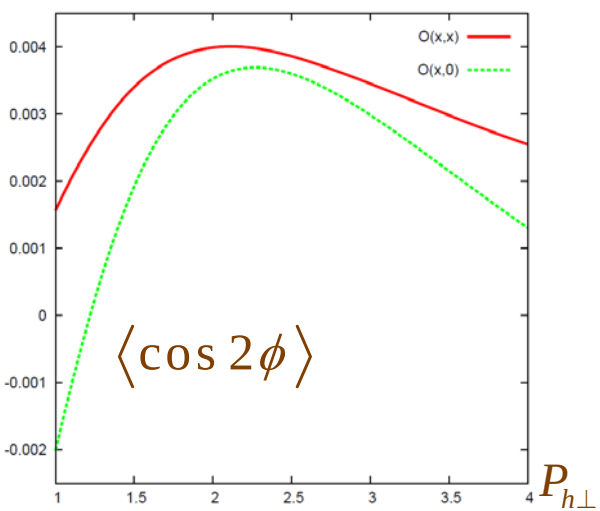


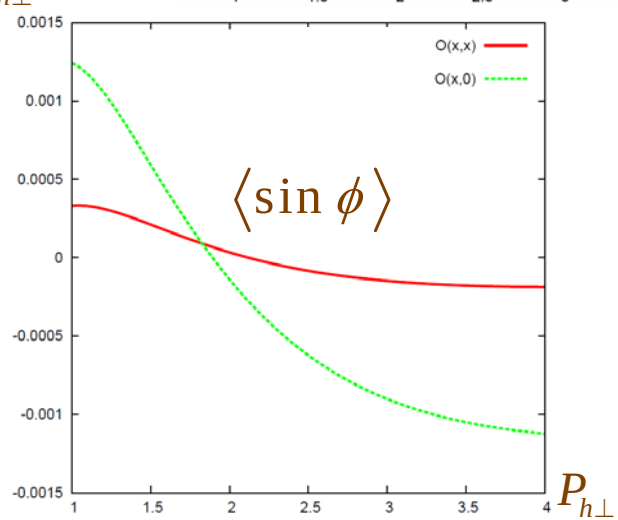
$S_{ep} = 2500\ \text{GeV}^2$

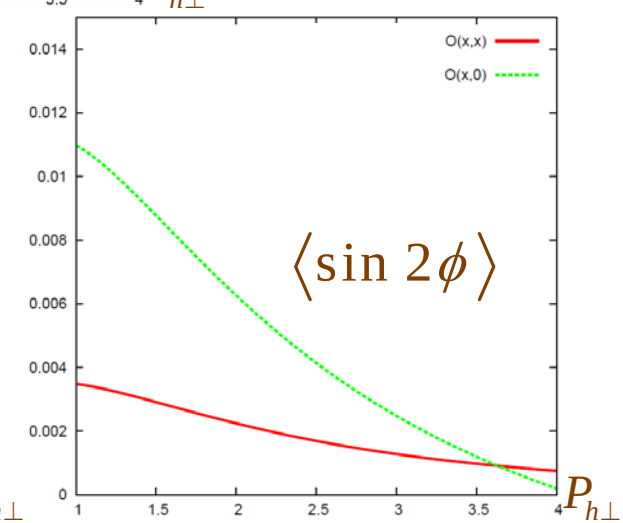
$Q = 4\ \text{GeV}$

$x_{bj} = 0.01$

$z_f = 0.3$





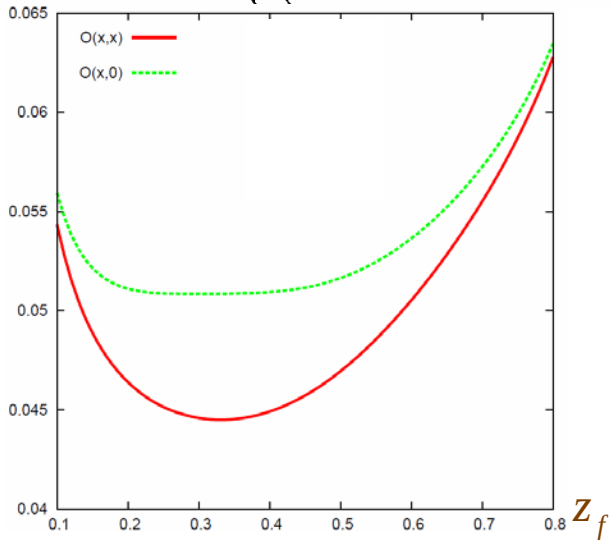


$$\frac{d^5\sigma_{\text{twist-3}}}{dx_{bj}dQ^2dz_f dP_{h\perp}^2 d\phi_h} = \frac{\alpha_{em}^2\alpha_s e_c^2 M_N}{8\pi x_{bj}^2 z_f^2 S_{ep}^2 Q^2}\left(\frac{-\pi}{2}\right)\sum_{k=1,\cdots,4,8,9}\mathcal{A}_k\mathcal{S}_k\int\frac{dx}{x}\int\frac{dz}{z}\delta\left(\frac{P_{h\perp}^2}{z_f^2Q^2}-\left(1-\frac{1}{\hat{x}}\right)\left(1-\frac{1}{\hat{z}}\right)+\frac{m_c^2}{\hat{z}^2Q^2}\right)$$

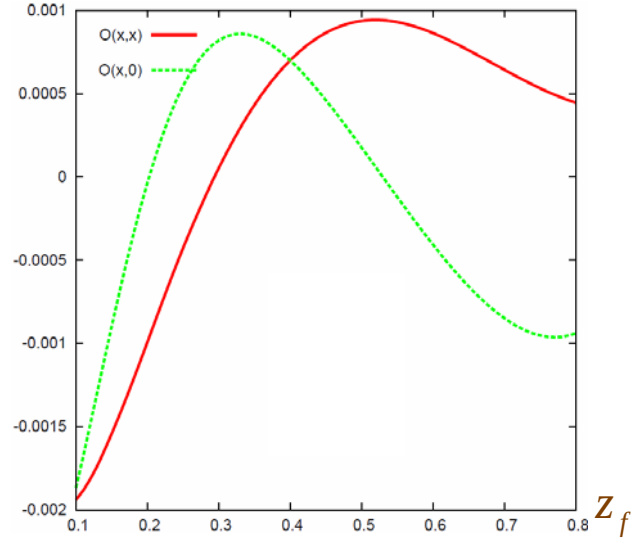
$$\times\sum_{a=c,\bar{c}}D_a(z)\left[\delta_a\left\{\left(\frac{dO(x,x)}{dx}-\frac{2O(x,x)}{x}\right)\Delta\hat{\sigma}_k^1+\left(\frac{dO(x,0)}{dx}-\frac{2O(x,0)}{x}\right)\Delta\hat{\sigma}_k^2+\frac{O(x,x)}{x}\Delta\hat{\sigma}_k^3+\frac{O(x,0)}{x}\Delta\hat{\sigma}_k^4\right\}\right.$$

$$\left.+\left\{\left(\frac{dN(x,x)}{dx}-\frac{2N(x,x)}{x}\right)\Delta\hat{\sigma}_k^1-\left(\frac{dN(x,0)}{dx}-\frac{2N(x,0)}{x}\right)\Delta\hat{\sigma}_k^2+\frac{N(x,x)}{x}\Delta\hat{\sigma}_k^3-\frac{N(x,0)}{x}\Delta\hat{\sigma}_k^4\right\}\right]$$

$\langle 1\rangle$



$\langle\cos\phi\rangle$

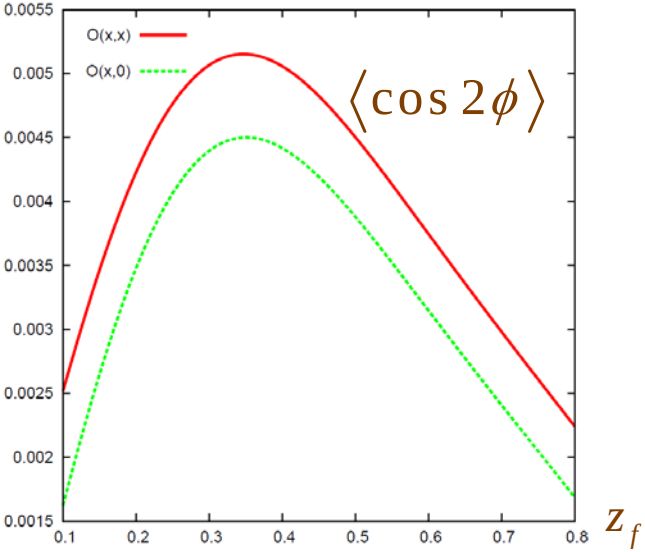


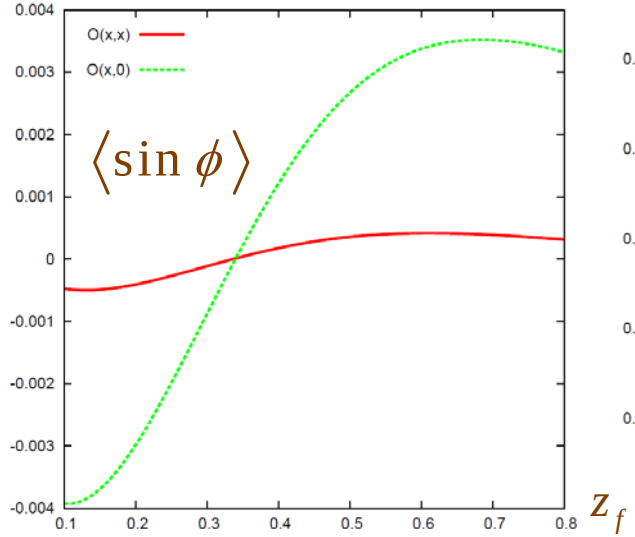
$S_{ep} = 1000\ \text{GeV}^2$

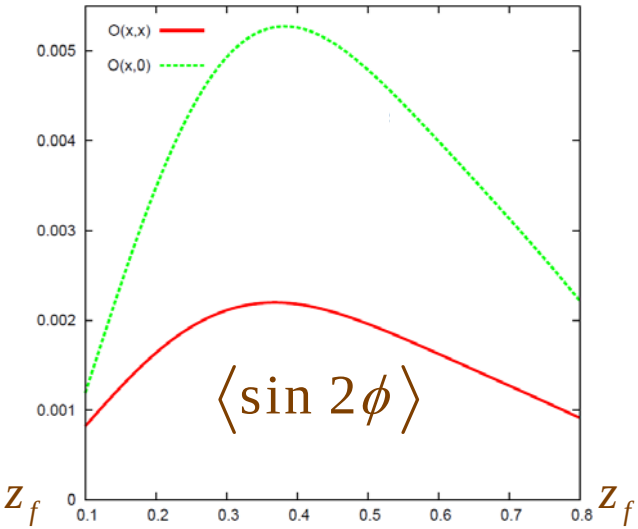
$Q = 2.5\ \text{GeV}$

$x_{bj} = 0.01$

$P_{h\perp} = 2\ \text{GeV}$







$$\frac{d^5\sigma_{\text{twist-3}}}{dx_{bj}dQ^2dz_f dP_{h\perp}^2 d\phi_h} = \frac{\alpha_{em}^2\alpha_s e_c^2 M_N}{8\pi x_{bj}^2 z_f^2 S_{ep}^2 Q^2}\left(\frac{-\pi}{2}\right)\sum_{k=1,\cdots,4,8,9}\mathcal{A}_k\mathcal{S}_k\int\frac{dx}{x}\int\frac{dz}{z}\delta\left(\frac{P_{h\perp}^2}{z_f^2Q^2}-\left(1-\frac{1}{\hat{x}}\right)\left(1-\frac{1}{\hat{z}}\right)+\frac{m_c^2}{\hat{z}^2Q^2}\right)$$

$$\times\sum_{a=c,\bar{c}}D_a(z)\left[\delta_a\left\{\left(\frac{dO(x,x)}{dx}-\frac{2O(x,x)}{x}\right)\Delta\hat{\sigma}_k^1+\left(\frac{dO(x,0)}{dx}-\frac{2O(x,0)}{x}\right)\Delta\hat{\sigma}_k^2+\frac{O(x,x)}{x}\Delta\hat{\sigma}_k^3+\frac{O(x,0)}{x}\Delta\hat{\sigma}_k^4\right\}\right.$$

$$\left.+\left\{\left(\frac{dN(x,x)}{dx}-\frac{2N(x,x)}{x}\right)\Delta\hat{\sigma}_k^1-\left(\frac{dN(x,0)}{dx}-\frac{2N(x,0)}{x}\right)\Delta\hat{\sigma}_k^2+\frac{N(x,x)}{x}\Delta\hat{\sigma}_k^3-\frac{N(x,0)}{x}\Delta\hat{\sigma}_k^4\right\}\right]$$

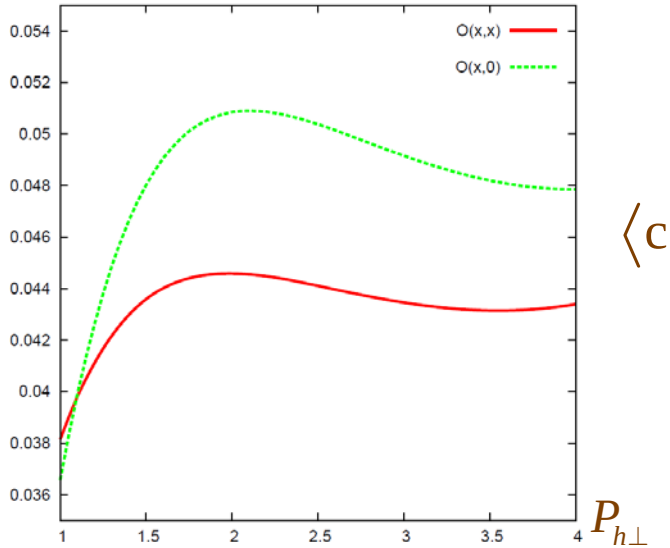
$S_{ep} = 1000\ \text{GeV}^2$

$Q = 2.5\ \text{GeV}$

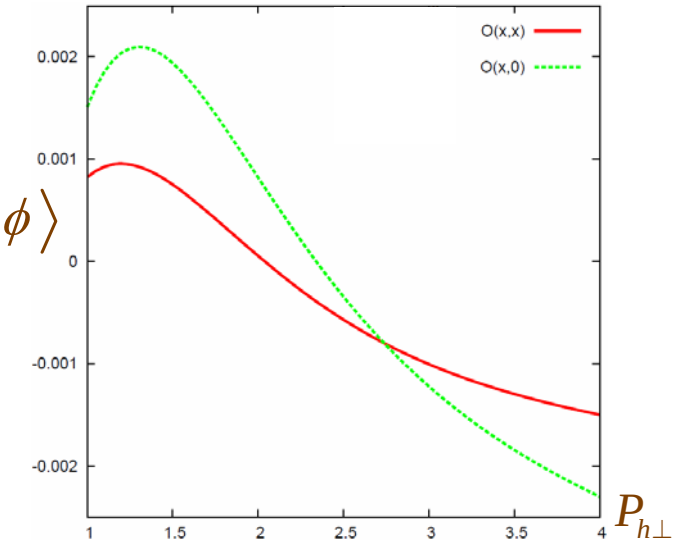
$x_{bj} = 0.01$

$z_f = 0.3$

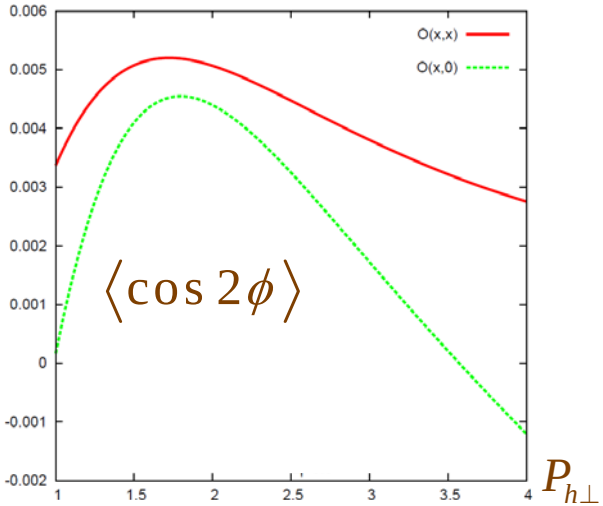
$\langle 1 \rangle$



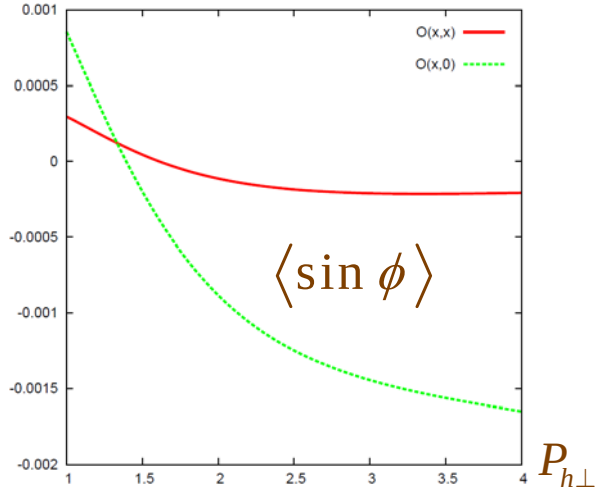
$\langle \cos\phi \rangle$



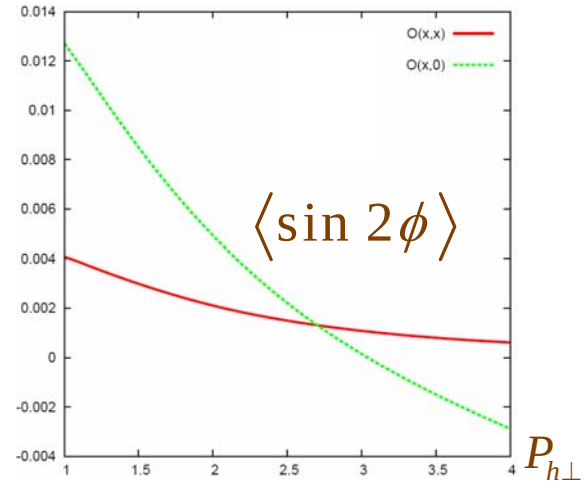
$\langle \cos 2\phi \rangle$



$\langle \sin\phi \rangle$

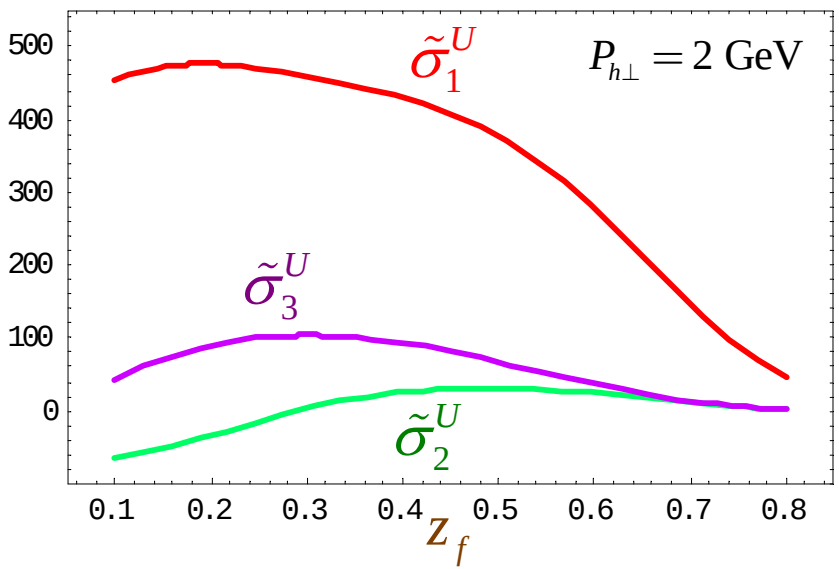


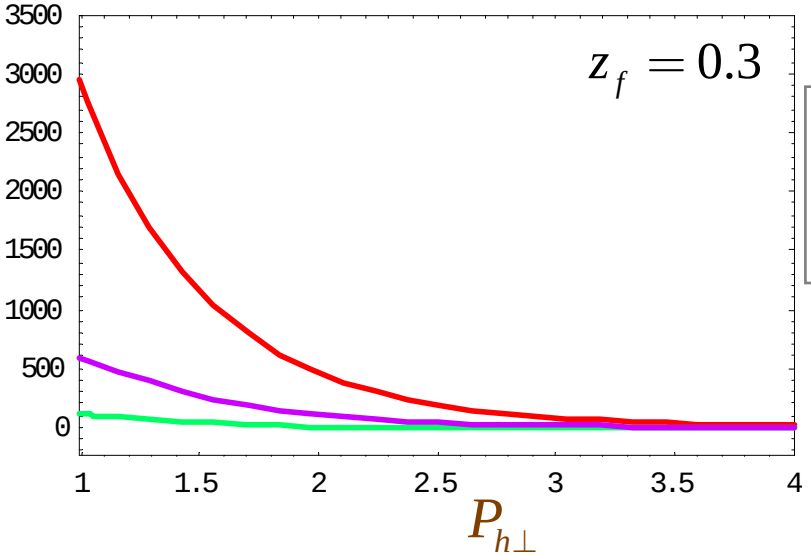
$\langle \sin 2\phi \rangle$



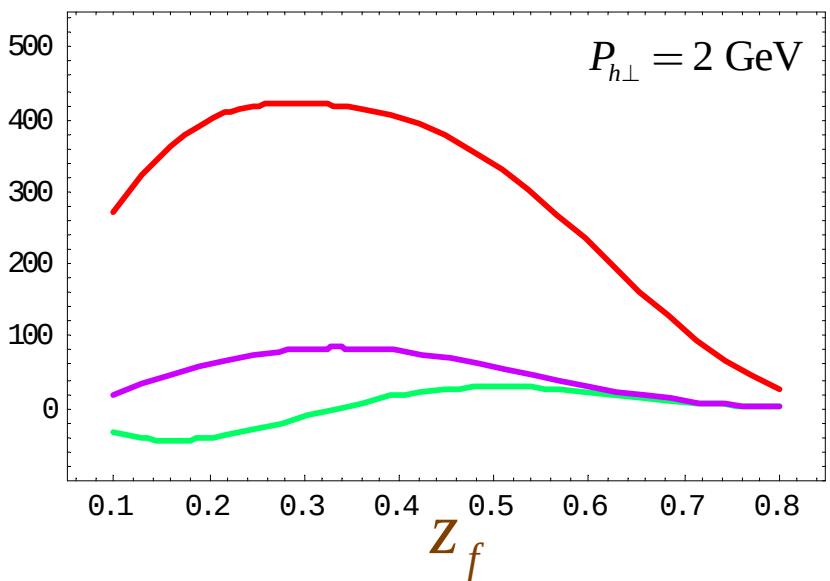
$$\frac{d^5\sigma^{\text{unpol}}_{\text{twist-2}}}{dx_{bj}\,dy\,dz_f\,dP^2_{h\perp}\,d\phi_h} = \tilde{\sigma}^U_1 + \tilde{\sigma}^U_2 \cos\phi_h + \tilde{\sigma}^U_3 \cos 2\phi_h$$

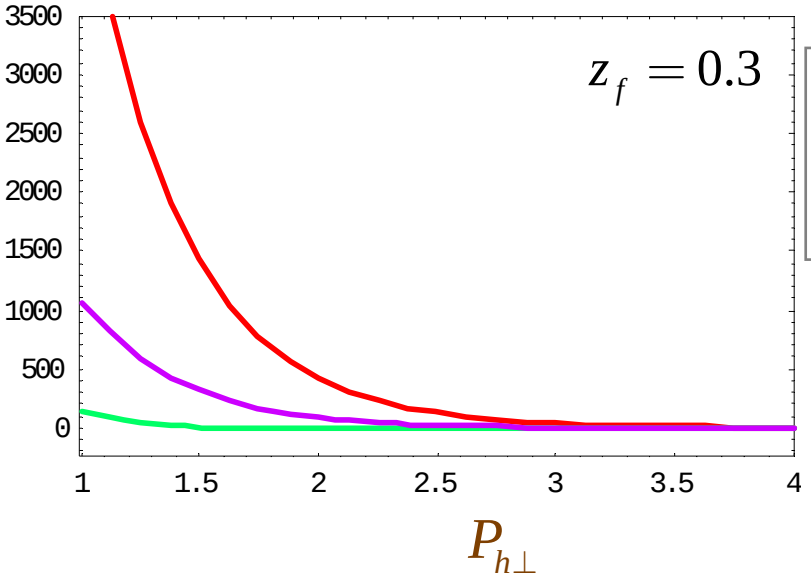
$[\text{pb}/\text{GeV}^2]$





$S_{ep} = 2500$ GeV^2
$Q = 4$ GeV
$x_{bj} = 0.01$





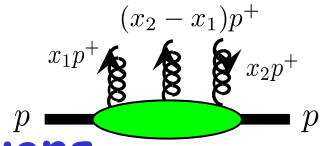
$S_{ep} = 1000$ GeV^2
$Q = 2.5$ GeV
$x_{bj} = 0.01$

Summary

SSA in SIDIS $ep^{\uparrow} \to eDX$ at $P_{h\perp} \gg \Lambda_{\rm QCD}$

Twist-3 mechanism from three-gluon correlation inside the nucleon

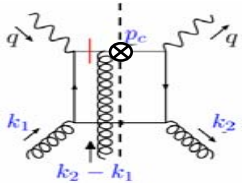
photon-gluon fusion



Factorization formula for twist-3 SSA

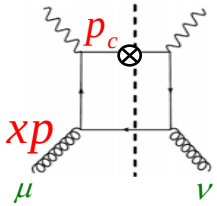
gauge invariance permutation symmetry for three gluons

All partonic hard parts are derived as the SGP contributions arising in 3→2 subprocess



convoluted with $\left\{ \begin{array}{l} N(x,x),\ O(x,x),\ \dfrac{dN(x,x)}{dx},\ \dfrac{dO(x,x)}{dx} \\ N(x,0),\ O(x,0),\ \dfrac{dN(x,0)}{dx},\ \dfrac{dO(x,0)}{dx} \end{array} \right.$

5 different azimuthal dependences



Master formula from 2→2 Born subprocess

exact, universal structure behind SGP

Numerical estimates of A_N: good chance to access multi-gluon effects at EIC $\langle 1 \rangle$, $\langle \sin 2\phi \rangle$

Our results are immediately applicable to other processes

$pp^{\uparrow} \to DX$ (⬅ Yoshida's talk) , $m_c \to 0$: $ep^{\uparrow} \to e\pi X$, Drell-Yan, direct γ