Nucleon Tomography

Feng Yuan

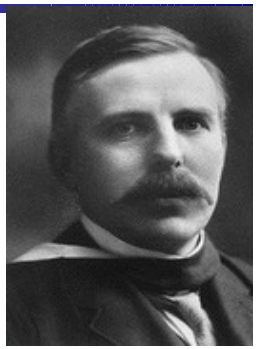
Lawrence Berkeley National Laboratory

RBRC, Brookhaven National Laboratory

Thanks Kresimir Kumerick, Dieter Mueller, Alexei Prokudin for providing nice plots, and authors of TMD-charpter of the EIC Write-up

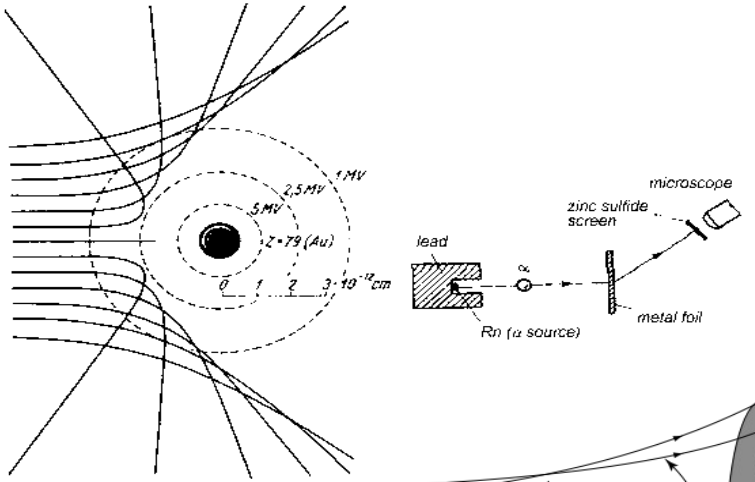
Exploring the nucleon: Of fundamental importance in science

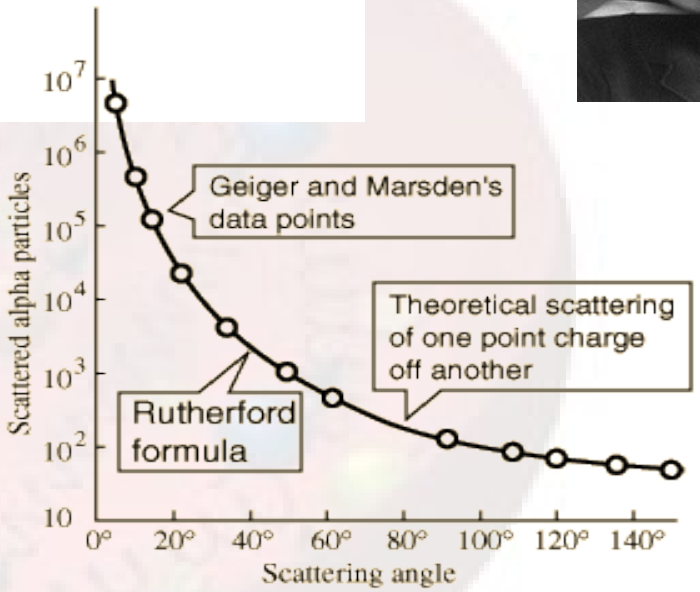
Rutherford scattering

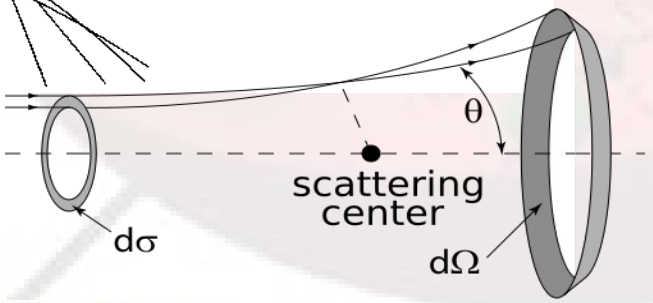


The Scattering of α and β Particles by Matter and the Structure of the Atom

E. Rutherford, F.R.S.*
Philosophical Magazine
Series 6, vol. 21
May 1911, p. 669-688

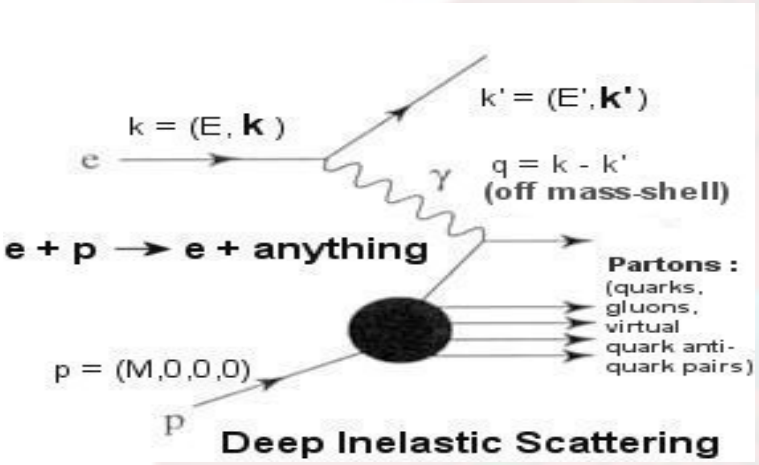




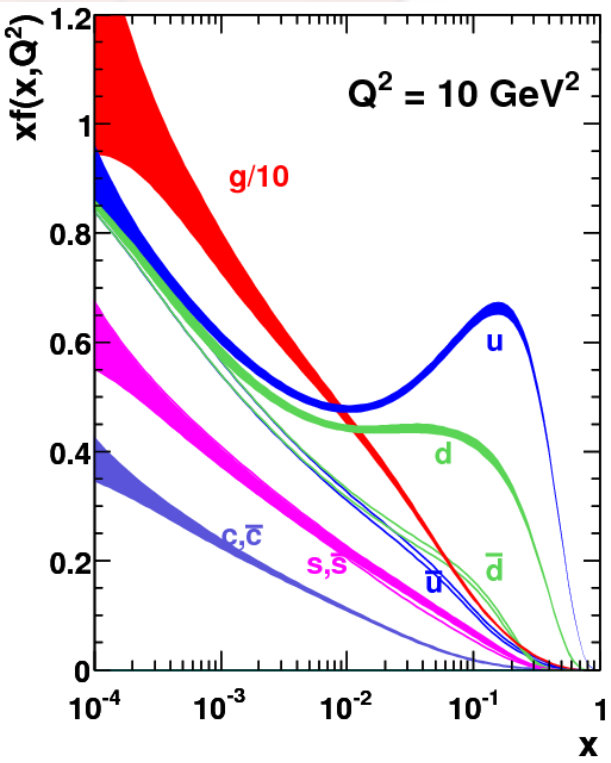


$$\frac{d\sigma}{d\Omega} = \left(\frac{\alpha\hbar c}{2mv_0^2}\right)^2 \frac{1}{\sin^4(\theta/2)}$$

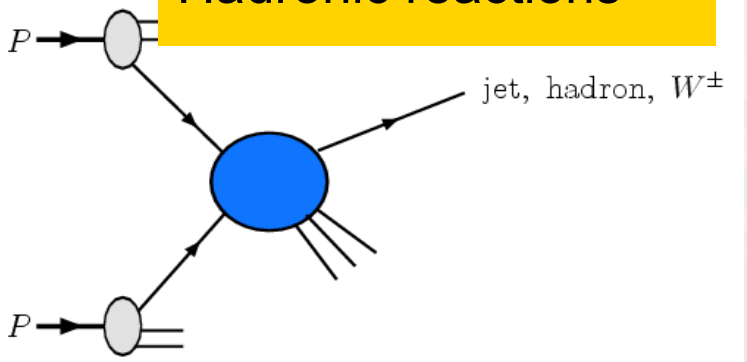
Feynman Parton: one-dimension



- Inclusive cross sections probe the momentum (longitudinal) distributions of partons inside nucleon



Hadronic reactions



Extension to transverse direction…

- Semi-inclusive measurements
  - Transverse momentum dependent (TMD) parton distributions
- Deeply Virtual Compton Scattering and Exclusive processes
  - Generalized parton distributions (GPD)
- Recently, there have been very exciting developments in both fields

Hope to highlight

- Fruitful imaging from the GPDs and TMDs measurements
  - Try to answer more detailed questions as Rutherford was doing 100 years ago
- QCD dynamics involved in these processes
  - In particular for the TMD part: universality, factorization, …

Wigner function

- Define as

Wigner 1933

$$W(x,p) = \int \psi^*(x-\eta/2)\psi(x+\eta/2)e^{ip\eta}d\eta \ ,$$

  - When integrated over x (p), one gets the momentum (probability) density
  - Not positive definite in general, but is in classical limit
  - Any dynamical variable can be calculated as

$$\langle O(x,p)\rangle = \int dxdpO(x,p)W(x,p)$$

Wigner distribution for the quark

- The quark operator

Ji: PRL91,062001(2003)

$$\hat{\mathcal{W}}_\Gamma(\vec{r},k) = \int \overline{\Psi}(\vec{r}-\eta/2)\Gamma\Psi(\vec{r}+\eta/2)e^{ik\cdot\eta}d^4\eta$$

- Wigner distributions

$$\begin{aligned} W_\Gamma(\vec{r},k) &= \frac{1}{2M_N}\int\frac{d^3\vec{q}}{(2\pi)^3}\left\langle \vec{q}/2\left|\hat{\mathcal{W}}_\Gamma(\vec{r},k)\right|-\vec{q}/2\right\rangle \\ &= \frac{1}{2M_N}\int\frac{d^3\vec{q}}{(2\pi)^3}e^{-i\vec{q}\cdot\vec{r}}\left\langle \vec{q}/2\left|\hat{\mathcal{W}}_\Gamma(0,k)\right|-\vec{q}/2\right\rangle \end{aligned}$$

After integrating over r, one gets TMD

After integrating over k, one gets Fourier transform of GPDs

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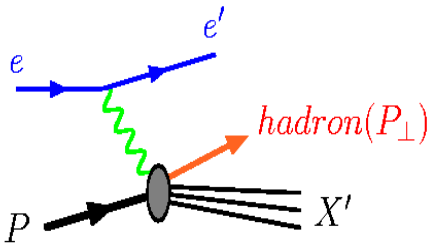
RIKEN BNL Research Center

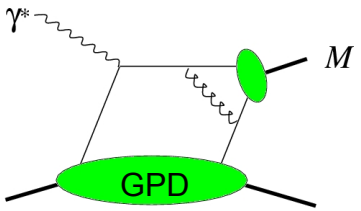
Wigner Distribution
$W(x,r,k_t)$



d^3r

$d^2k_t dz$

F.T.



Transverse Momentum Dependent PDF $f(x,k_t)$

Generalized Parton Distr.
$H(x,\xi,t)$

d^2k_t

dx

PDF $f(x)$

Form Factors
$F_1(Q), F_2(Q)$

TMD Parton Distributions

- The definition contains explicitly the gauge links

Collins-Soper 1981, Collins 2002, Belitsky-Ji-Yuan 2002

$$f(x,k_\perp) = \frac{1}{2}\int \frac{d\xi^- d^2\xi_\perp}{(2\pi)^3} e^{-i(\xi^- k^+ - \vec{\xi}_\perp\cdot\vec{k}_\perp)} \times \langle PS|\overline{\psi}(\xi^-,\xi_\perp)L^\dagger_{\xi_\perp}(\xi^-)\gamma^+ L_0(0)\psi(0)|PS\rangle$$

- The polarization and kt dependence provide rich structure in the quark and gluon distributions
  - Mulders-Tangerman 95, Boer-Mulders 98

Generalized Parton Distributions

Mueller, et al. 1994; Ji, 1996, Radyushkin 1996

- Off-diagonal matrix elements of the quark operator (along light-cone)

$$F_q(x,\xi,t) = \frac{1}{2}\int \frac{d\lambda}{2\pi} e^{i\lambda x} \left\langle P' \left| \overline{\psi}_q\left(-\frac{\lambda}{2}n\right) \not{n} \mathcal{P} e^{-ig\int_{\lambda/2}^{-\lambda/2} d\alpha\ n\cdot A(\alpha n)} \psi_q\left(\frac{\lambda}{2}n\right) \right| P \right\rangle$$
$$= H_q(x,\xi,t)\ \frac{1}{2}\overline{U}(P')\not{n}U(P) + E_q(x,\xi,t)\ \frac{1}{2}\overline{U}(P')\frac{i\sigma^{\mu\nu}n_\mu\Delta_\nu}{2M}U(P)\ .$$

- It depends on quark momentum fraction x and skewness ξ, and nucleon momentum transfer t

$$\xi = -n\cdot(P'-P)/2$$
$$t = \Delta^2 = (P-P')^2$$

Impact parameter dependent parton distributions

Soper 1977
Burkardt 2000,2003

- Quark operator depends on b_t

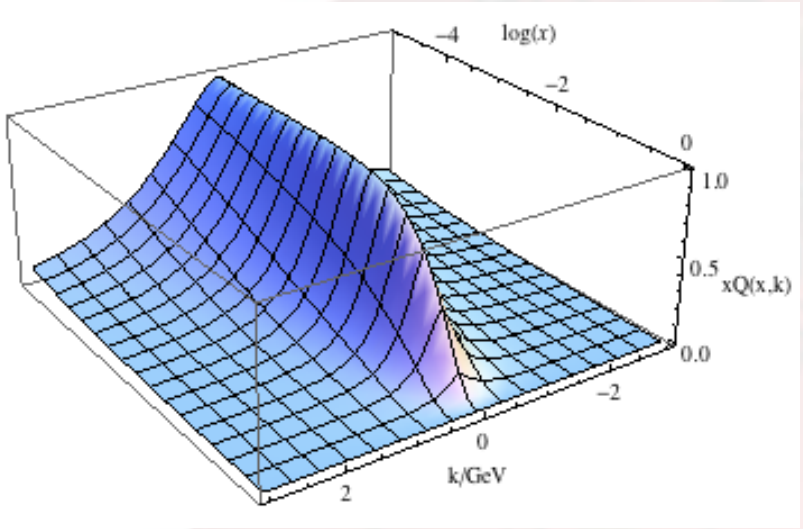
$$\hat{O}_q(x,\mathbf{b}_\perp) = \int \frac{dx^-}{4\pi}\bar{q}\left(-\frac{x^-}{2},\mathbf{b}_\perp\right)\gamma^+ q\left(\frac{x^-}{2},\mathbf{b}_\perp\right)e^{ixp^+x^-}$$

- It is the F.T. of the GPD at $\xi = 0$

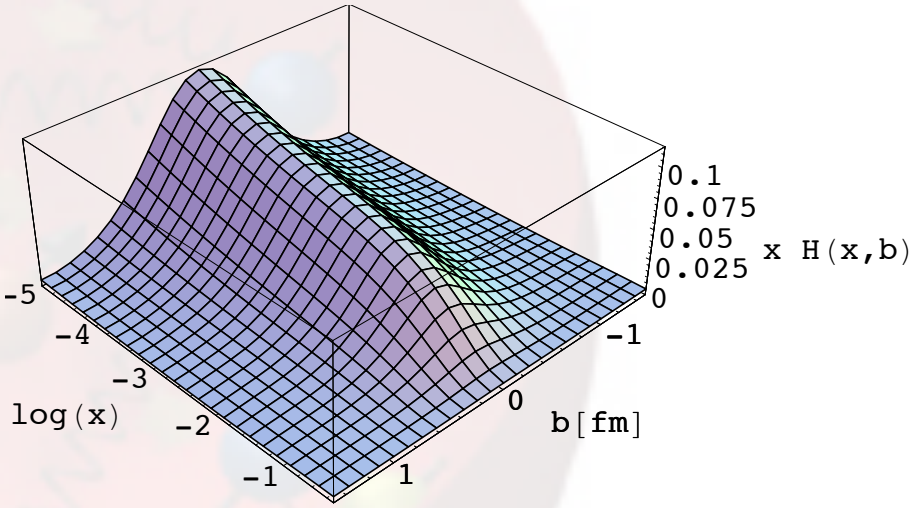
$$q(x,\mathbf{b}_\perp) \equiv \left\langle p^+,\mathbf{R}_\perp=\mathbf{0}_\perp,\lambda\right|\hat{O}_q(x,\mathbf{b}_\perp)\left|p^+,\mathbf{R}_\perp=\mathbf{0}_\perp,\lambda\right\rangle$$

$$= \int \frac{d^2\boldsymbol{\Delta}_\perp}{(2\pi)^2}H_q(x,-\boldsymbol{\Delta}_\perp^2)e^{-i\mathbf{b}_\perp\cdot\boldsymbol{\Delta}_\perp}$$

Transverse profile for the quark distribution: k_t vs b_t

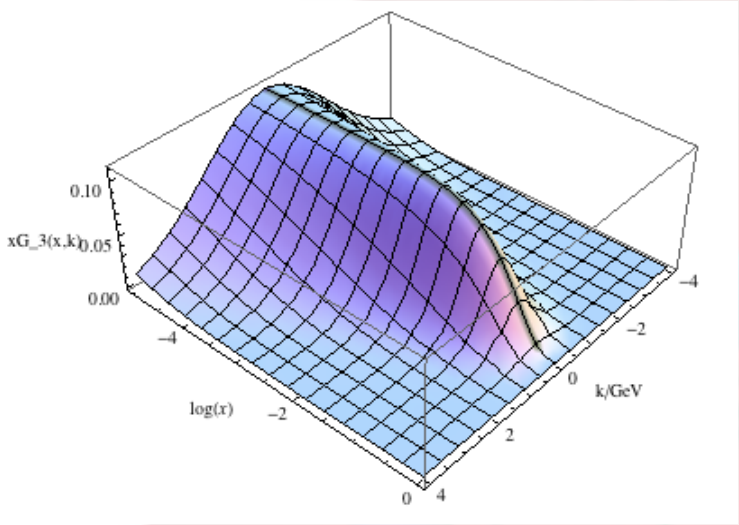


Quark distribution calculated from a saturation-inspired model A.Mueller 99, McLerran-Venugopalan 99

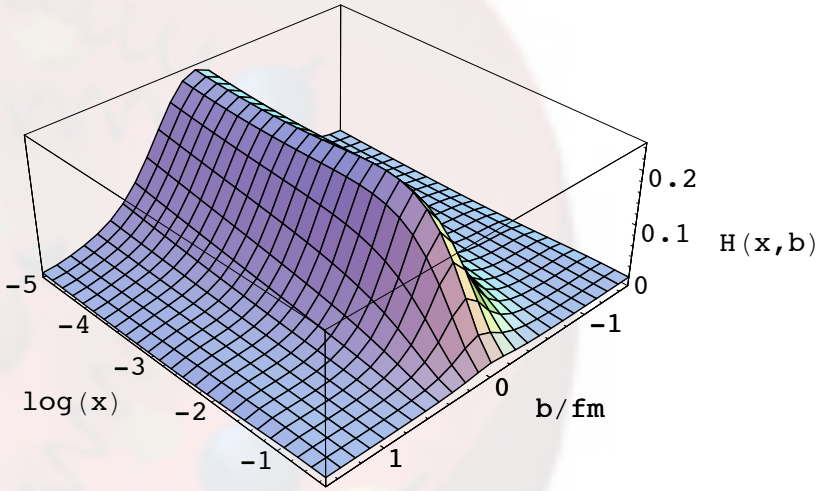


GPD fit to the DVCS data from HERA, Kumerick-D.Mueller, 09,10

Gluon distribution



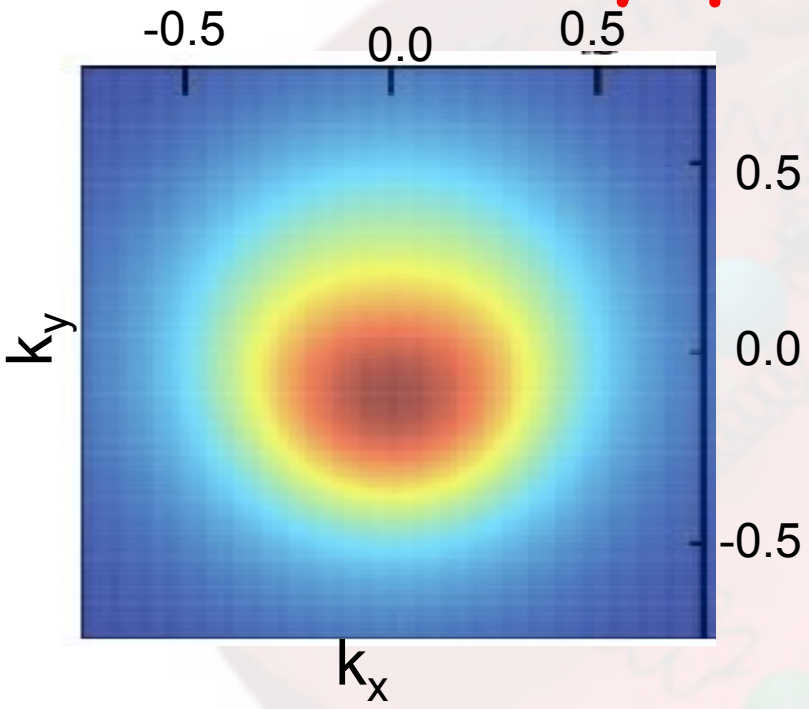
One of the TMD gluon distributions at small-x



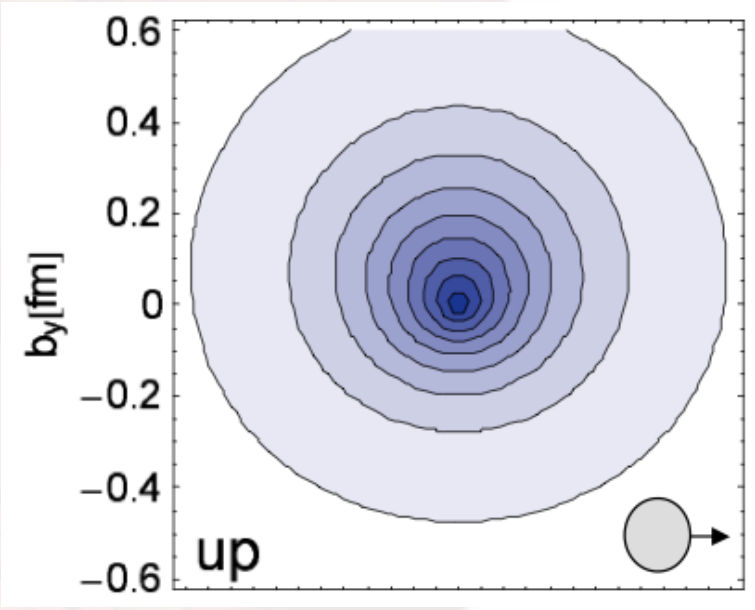
GPD fit to the DVCS data from HERA, Kumerick-Mueller, 09,10

Deformation when nucleon is transversely polarized

Burkardt 2002



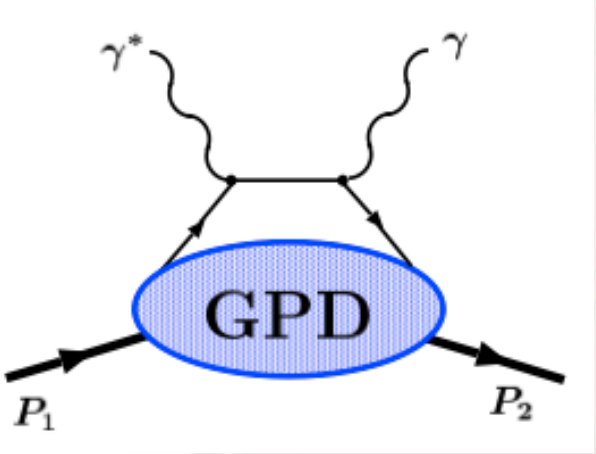
Quark Sivers function fit to the SIDIS Data, Anselmino, et al. 20009

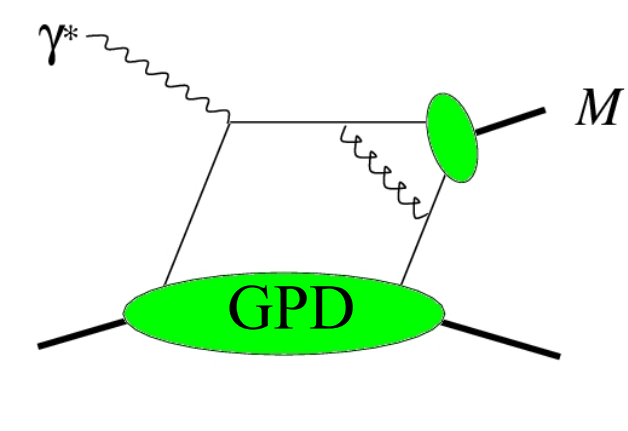


Lattice Calculation of the IP density of Up quark, QCDSF/UKQCD Coll., 2006

Access the GPDs

- Deeply virtual Compton Scattering (DVCS) and deeply virtual exclusive meson production (DVEM)

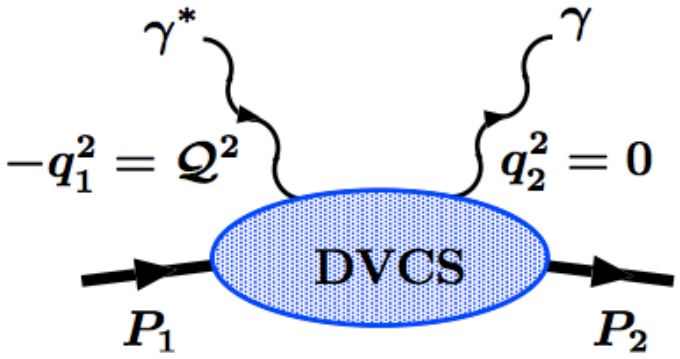




In the Bjorken limit: $Q^2 >> (-t), \Lambda^2_{QCD}, M^2$

DVCS kinematics

$$t = (P_2 - P_1)^2\,, \quad q = (q_1 + q_2)/2$$



Generalized Bjorken limit:

$$-q^2 \simeq \mathcal{Q}^2/2 \to \infty$$

$$\xi = \frac{-q^2}{2P \cdot q} \to \text{const}$$

Experiments measure the Compton Form Factors (CFF)

$$\mathcal{H}(\xi, t, \mathcal{Q}^2),\ \mathcal{E}(\xi, t, \mathcal{Q}^2),\ \tilde{\mathcal{H}}(\xi, t, \mathcal{Q}^2),\ \tilde{\mathcal{E}}(\xi, t, \mathcal{Q}^2)$$

Extract the GPDs

- The theoretical framework has been well established
  - Perturbative QCD corrections at NLO, some at NNLO
- However, GPDs depend on x, ξ, t, it is much more difficult than PDFs (only depends on x)
  - There will be model dependence at the beginning

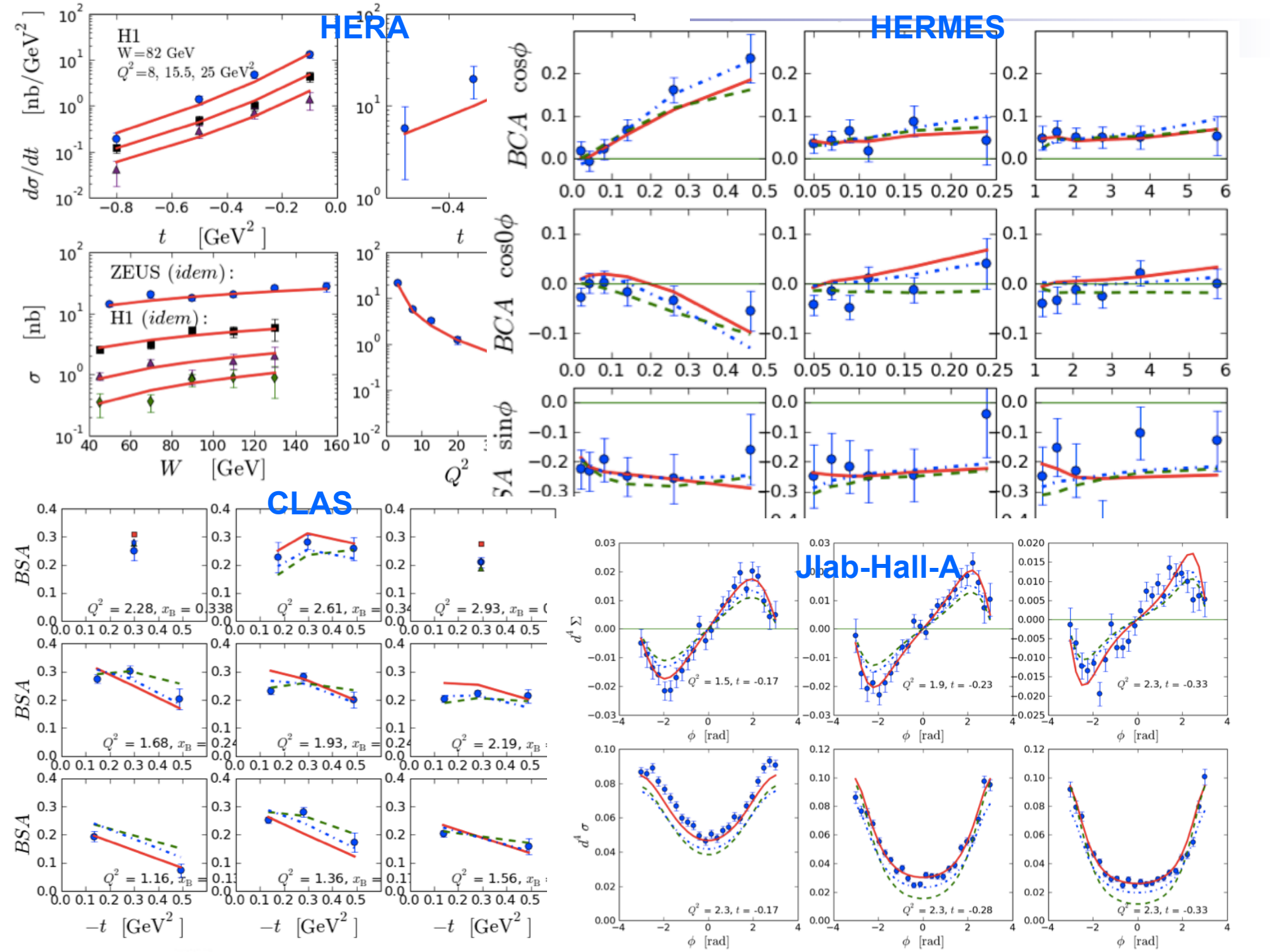
Great efforts have been made

- Several groups have been working on extraction of the GPDs
  - See some of these in the parallel sessions
- We do have a glimpse of the GPDs
- Decisive answer shall be possible with future JLab 12 GeV and COMPASS, and the planed EIC

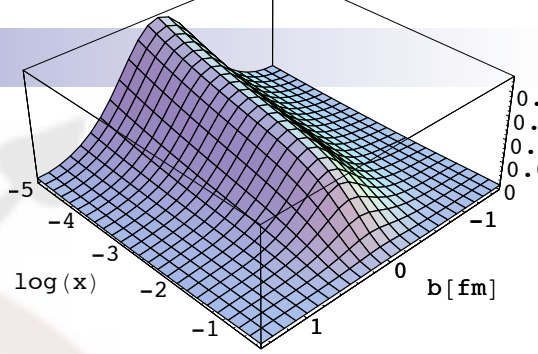
Highlights from Mueller et al. 09, 10

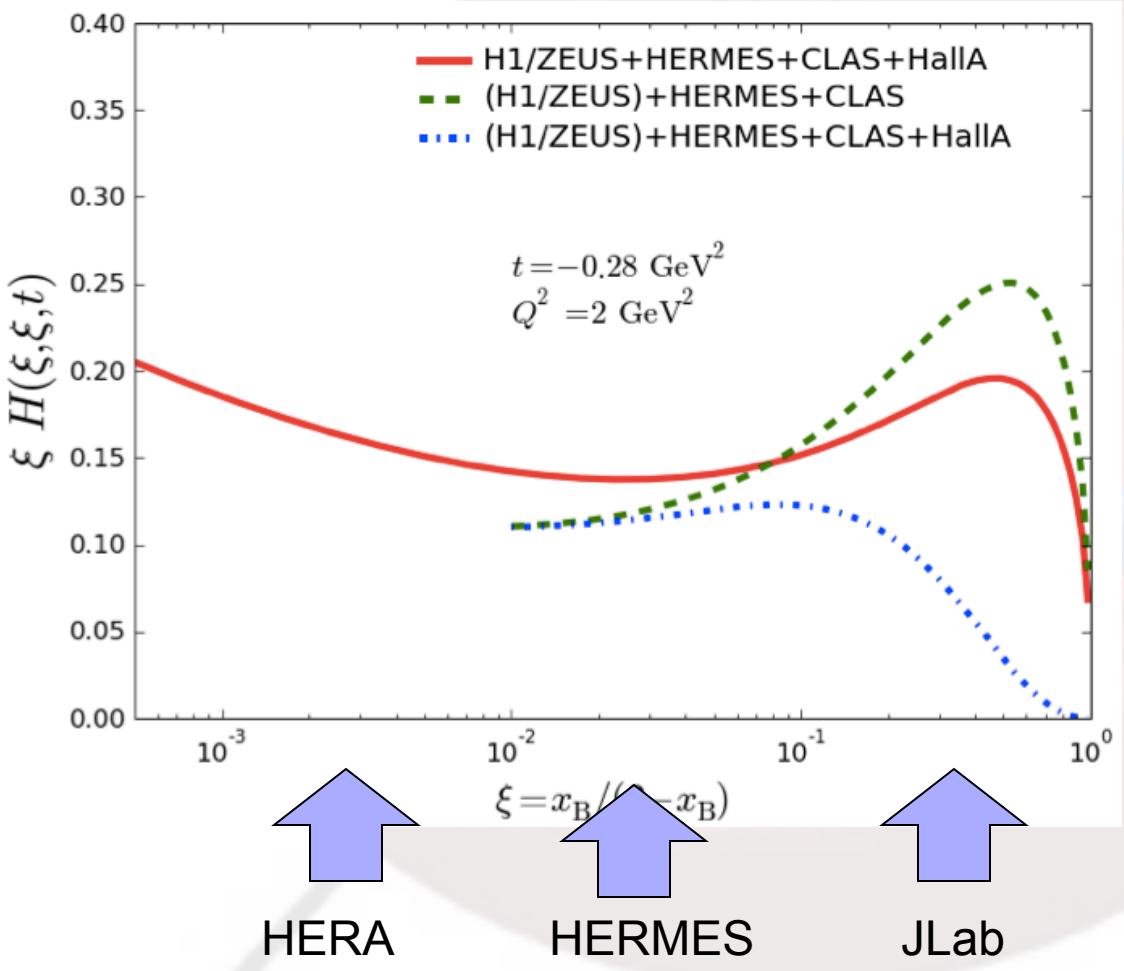
- Sea quark and gluon GPDs parameterized in the conformal moment space, t-channel partial wave expansion
- Valence quark GPDs parameterized on the η=x trajectory, dispersion relation to construct the real part

Muller's parallel session talk


HERA
HERMES
CLAS
Jlab-Hall-A
H1
W=82 GeV
Q^2=8, 15.5, 25 GeV^2
$d\sigma/dt$ [nb/GeV^2]
t [GeV^2]
ZEUS (idem):
H1 (idem):
σ [nb]
W [GeV]
Q^2
BCA $\cos\phi$
BCA $\cos0\phi$
SA $\sin\phi$
BSA
Q^2 = 2.28, x_B = 0.338
Q^2 = 2.61, x_B = 0.34
Q^2 = 2.93, x_B = 0
Q^2 = 1.68, x_B = 0.24
Q^2 = 1.93, x_B = 0.24
Q^2 = 2.19, x_B = 0.
Q^2 = 1.16, x_B = 0.1
Q^2 = 1.36, x_B = 0.1
Q^2 = 1.56, x_B = 0.
$-t$ [GeV^2]
$d^4\Sigma$
$d^4\sigma$
Q^2 = 1.5, t = -0.17
Q^2 = 1.9, t = -0.23
Q^2 = 2.3, t = -0.33
Q^2 = 2.3, t = -0.17
Q^2 = 2.3, t = -0.28
Q^2 = 2.3, t = -0.33
ϕ [rad]

One example: H(x,x,t)





D. Mueller, et al, 09, 10

Small-x range constrained by HERA, uncertainties at large-x shall be very much reduced with Jlab 12 GeV COMPASS, and the planed EIC

Of course, there are also other GPDs, in particular, the GPD E

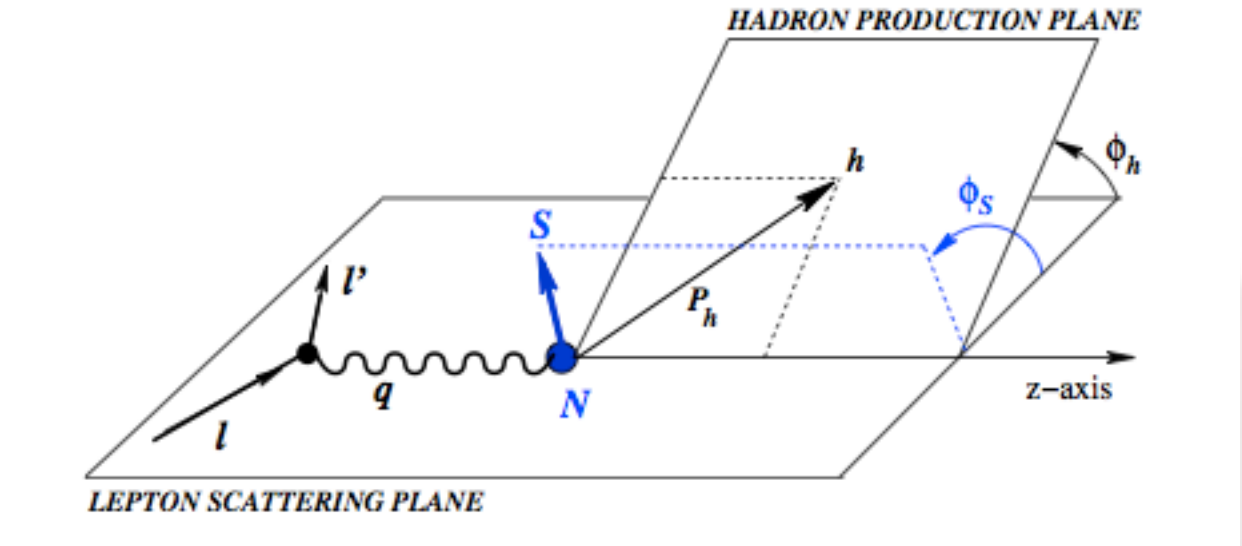
Transverse momentum dependent parton distribution

- Straightforward extension
  - Spin average, helicity, and transversity distributions
- Transverse momentum-spin correlations
  - Nontrivial distributions, $S_T X P_T$
  - In quark model, depends on S- and P-wave interference

Where can we learn TMDs

- Semi-inclusive hadron production in deep inelastic scattering (SIDIS)
- Drell-Yan lepton pair, photon pair productions in pp scattering
- Dijet correlation in DIS
- Relevant e+e- annihilation processes
- …

Semi-inclusive DIS



- Novel Single Spin Asymmetries

$$A_{UT}^{\sin(\phi+\phi_S)} \propto S_\perp \frac{\sum_{q,\bar{q}} e_q^2 \delta q(x)\, H_1^\perp(z)}{\sum_{q,\bar{q}} e_q^2 q(x)\, D_1(z)}$$

$$z \overset{lab}{=} \frac{E_h}{\nu}$$

$$A_{UT}^{\sin(\phi-\phi_S)} \propto S_\perp \frac{\sum_{q,\bar{q}} e_q^2 f_{1T}^{\perp,q}(x) \cdot D_1(z)}{\sum_{q,\bar{q}} e_q^2 q(x)\, D_1(z)}$$

U: unpolarized beam
T: transversely polarized target

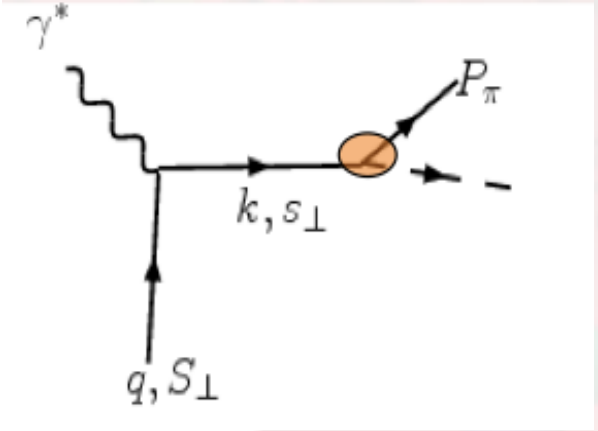
Two major contributions

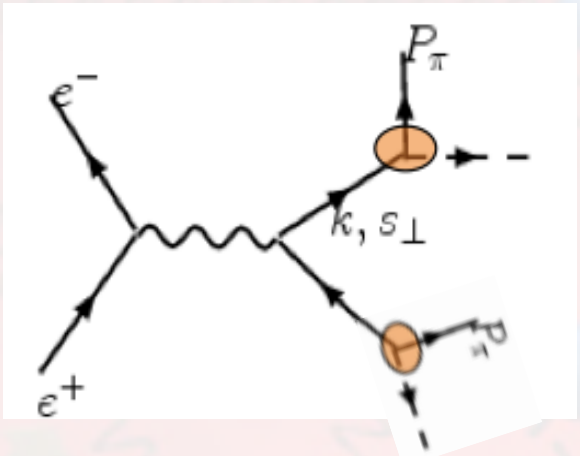
- Sivers effect in the distribution

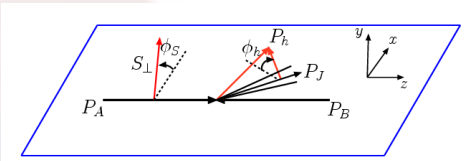
- Collins effect in the fragmentation

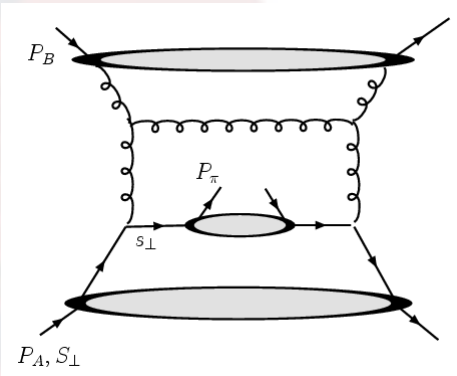
- Other contributions…

Universality of the Collins Fragmentation









ep--> e Pi X

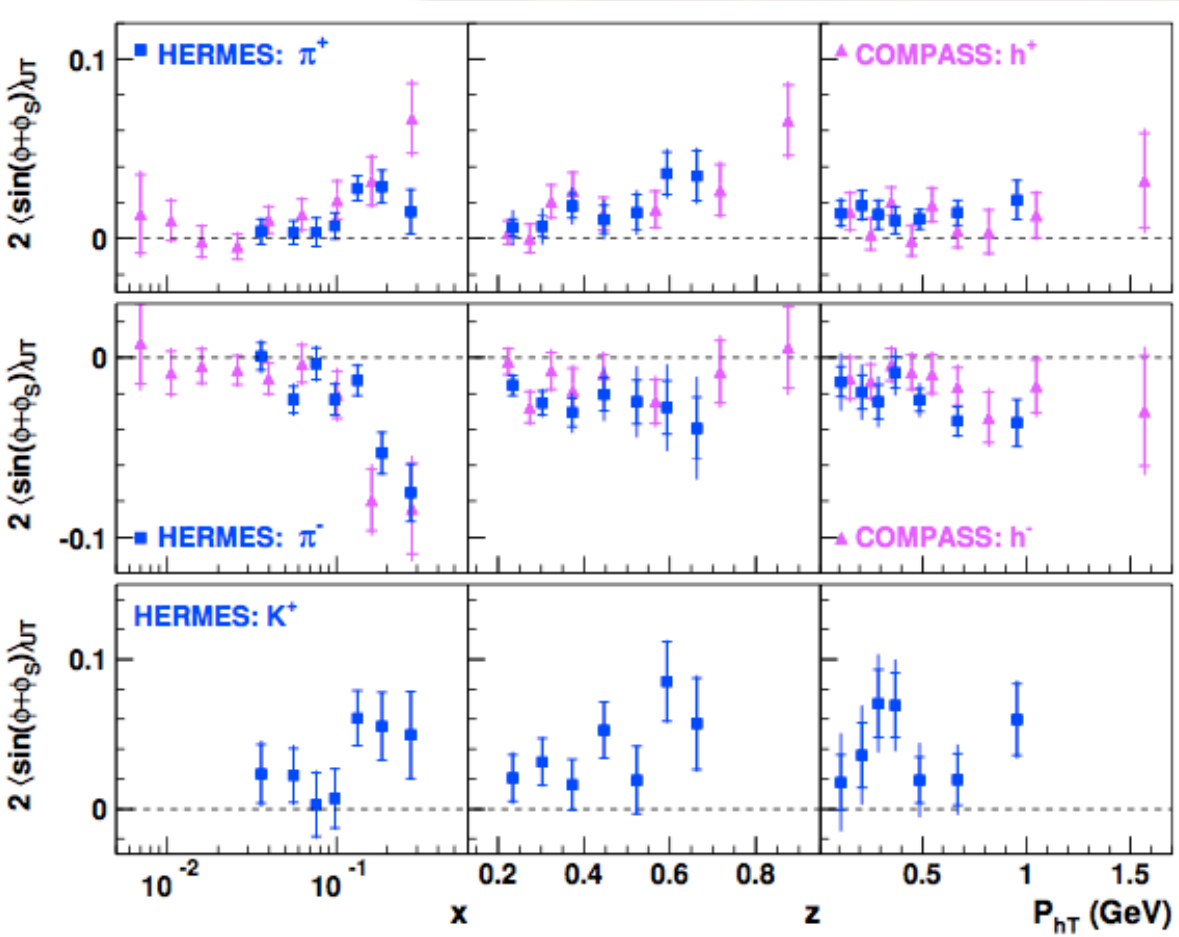
e^+e^---> Pi Pi X

pp--> jet(->Pi) X

Metz 02, Collins-Metz 02,
Yuan 07,
Gamberg-Mukherjee-Mulders 08,10
Meissner-Metz 0812.3783
Yuan-Zhou, 0903.4680

Exps: BELLE, HERMES, STAR, parallel talks

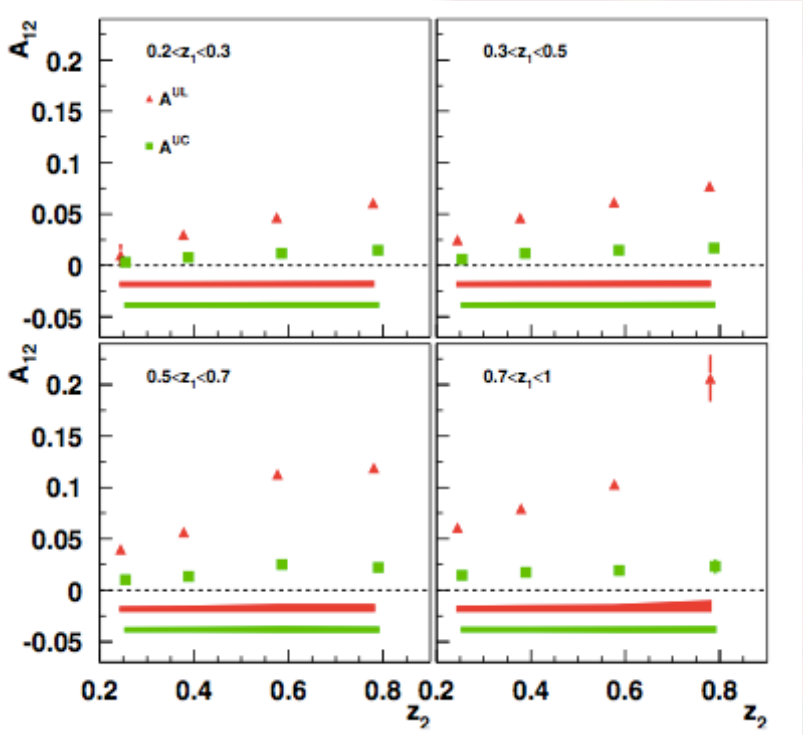
Collins asymmetries in SIDIS



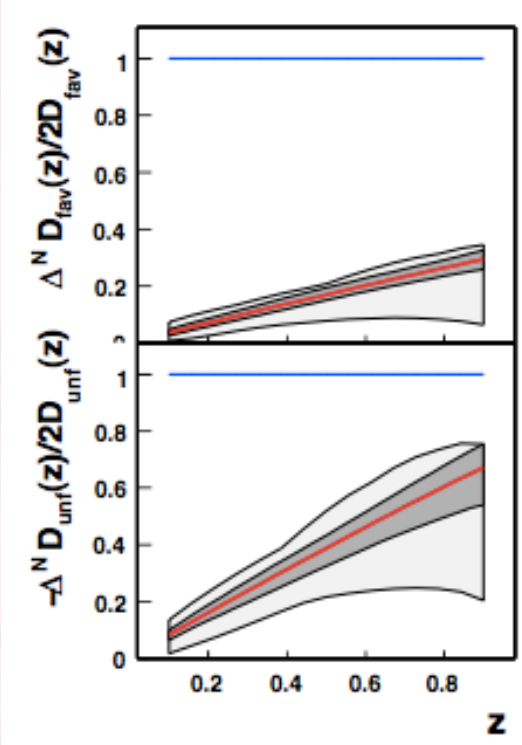
Summarized in the EIC Write-up

Details see the parallel Session talks

Collins effects in e^+e^-



BELLE Coll., 2008

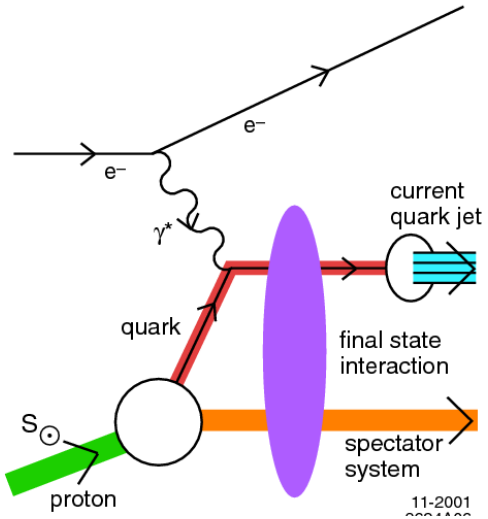


Collins functions extracted from the Data, Anselmino, et al., 2009

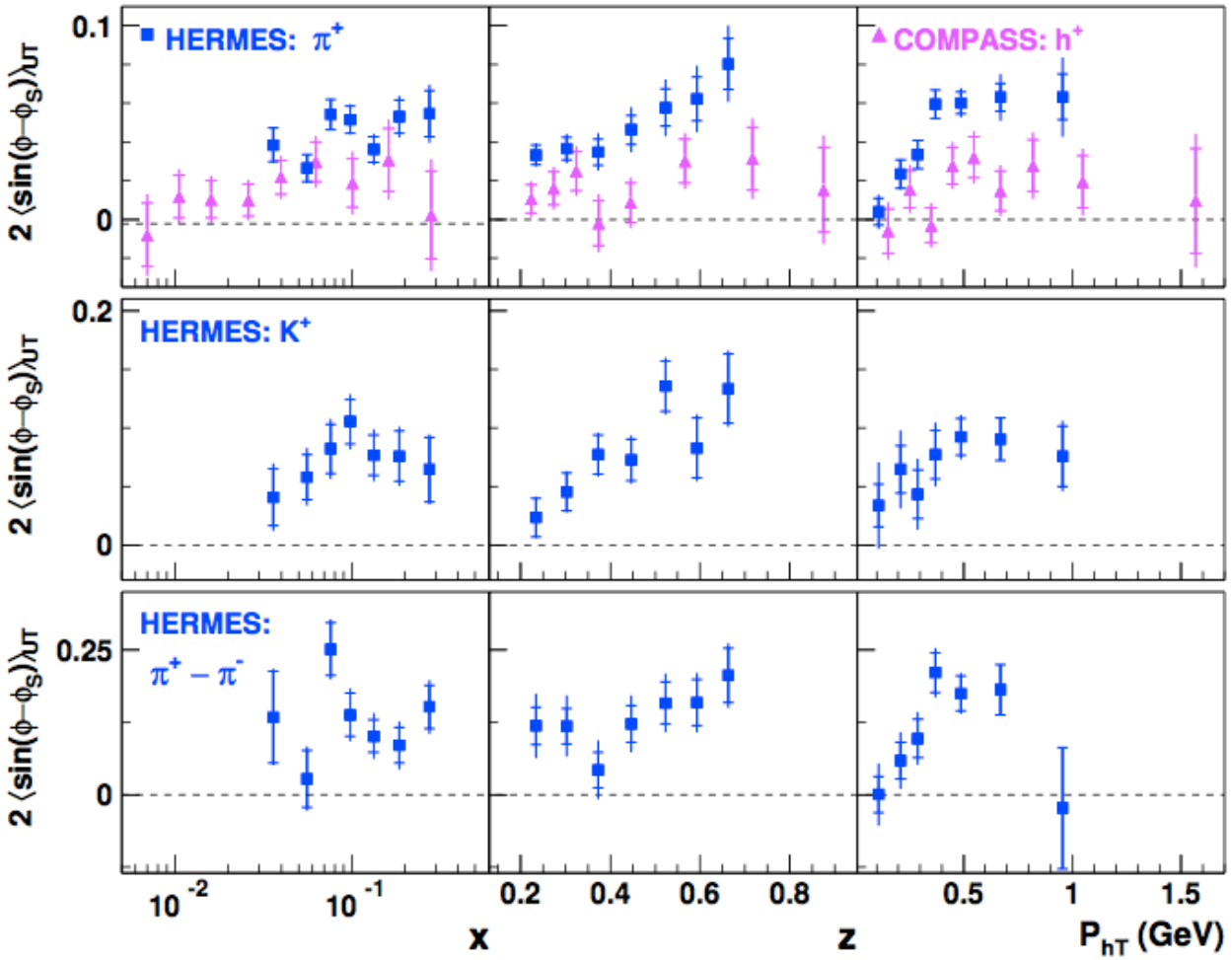
Sivers effect is different

- It is the final state interaction providing the phase to a nonzero SSA
- Non-universality in general
- Only in special case, we have "Special Universality"

Brodsky,Hwang,Schmidt 02
Collins, 02;
Ji,Yuan,02;
Belitsky,Ji,Yuan,02



Sivers asymmetries in SIDIS

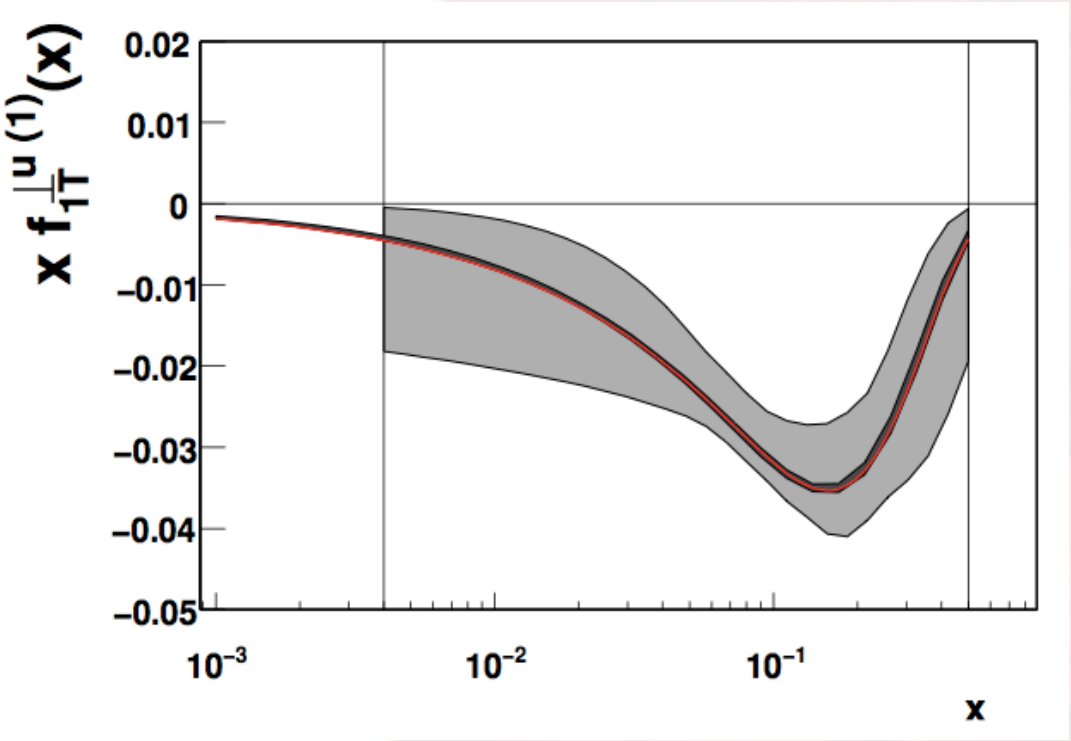


Jlab Hall A ^{3}He data to appear

Non-zero Sivers effects Observed in SIDIS

Parallel sessions

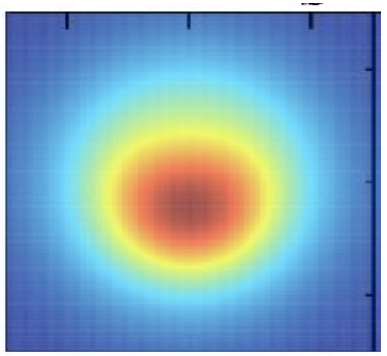
Quark Sivers function extracted from the data



Leading order fit, simple Gaussian assumption for the Sivers function

There are still theoretical uncertainti
In the fit: scale dependence, high order corrections, …

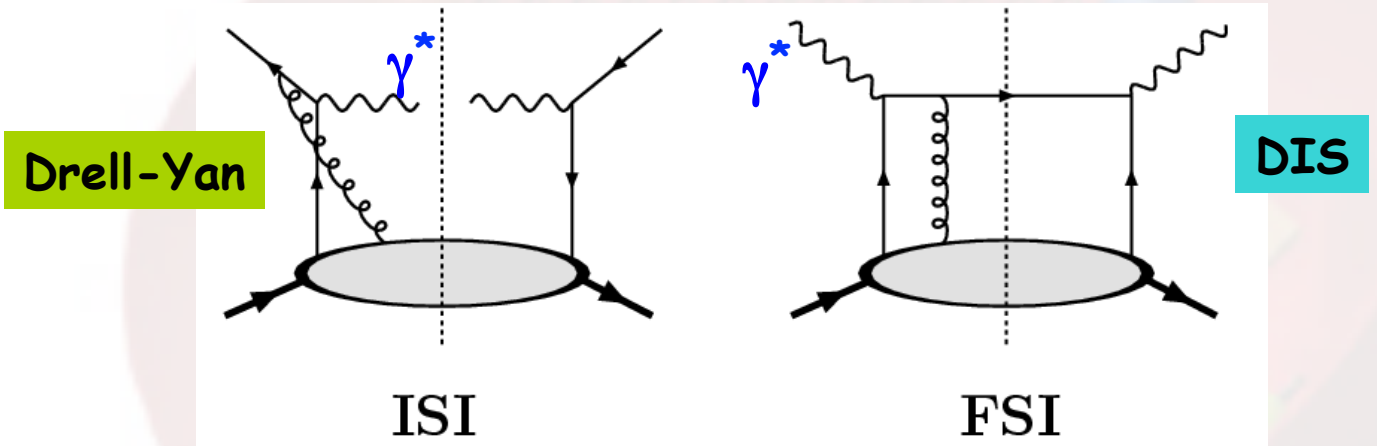
Inner band is the impact from the planed EIC kiematics



Alexei Prokudin, et al.

DIS and Drell-Yan

- Initial state vs. final state interactions

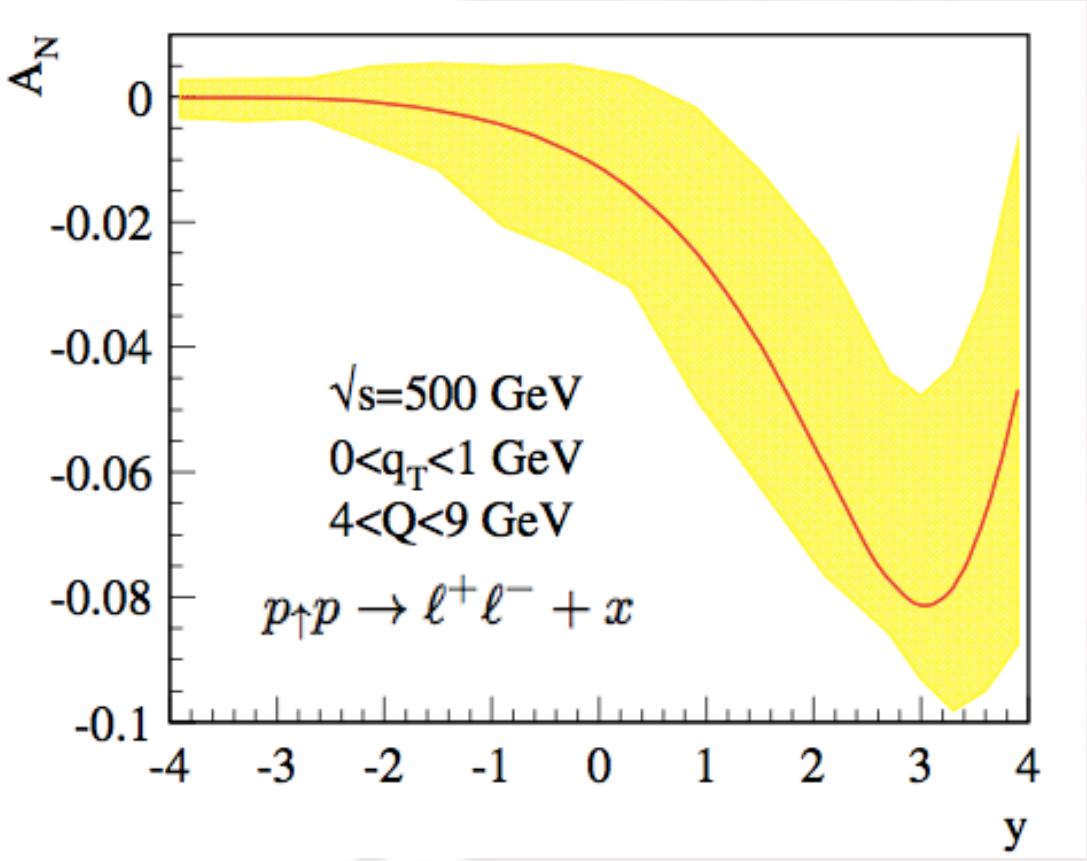


$$\text{Sivers}|_{\text{DY}} = -\text{Sivers}|_{\text{DIS}}$$

HERMES

- "Universality": QCD prediction

RHIC predictions



Kang, Qiu, 2008

There have been proposals to Do this measurement at RHIC

http://spin.riken.bnl.gov/rsc/

Collider or fixed target modes

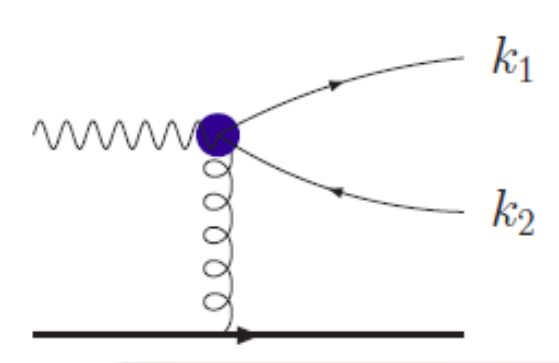
There is also a COMPASS Proposal in the near future

It is very important to test the sign change of the quark Sivers function

TMD gluon distributions

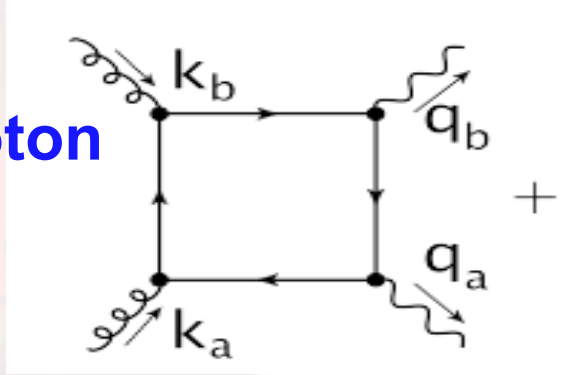
- It is not easy, because gluon does not couple to photon directly
- Can be studied in two-particle processes

Dijet In DIS



Vogelsang-Yuan, 2007
Dominguez-Xiao-Yuan, 2010
Boer-Brodsky-Mulders-Pisano, 2010

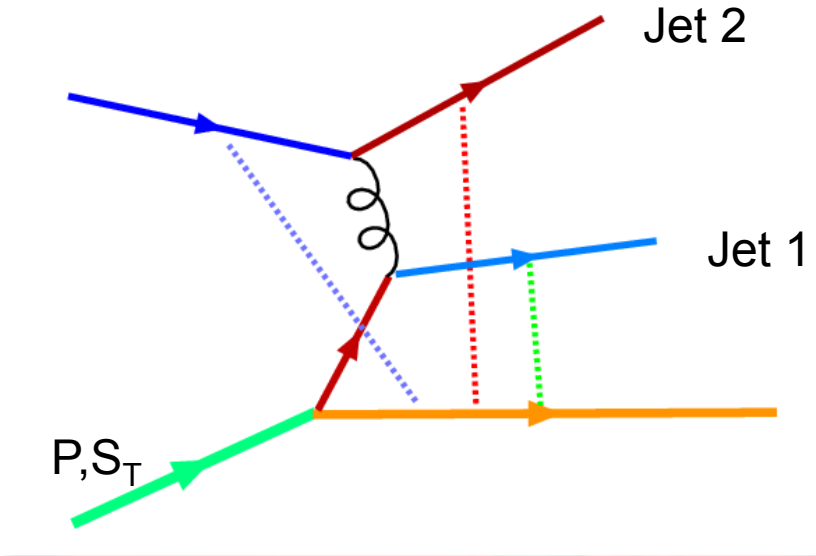
Di-photon In pp

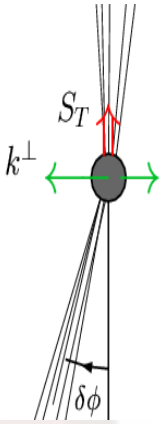


Qiu-Schlegel-Vogelsang, 2011

Dijet-correlation at RHIC

- Initial state and/or final state interactions





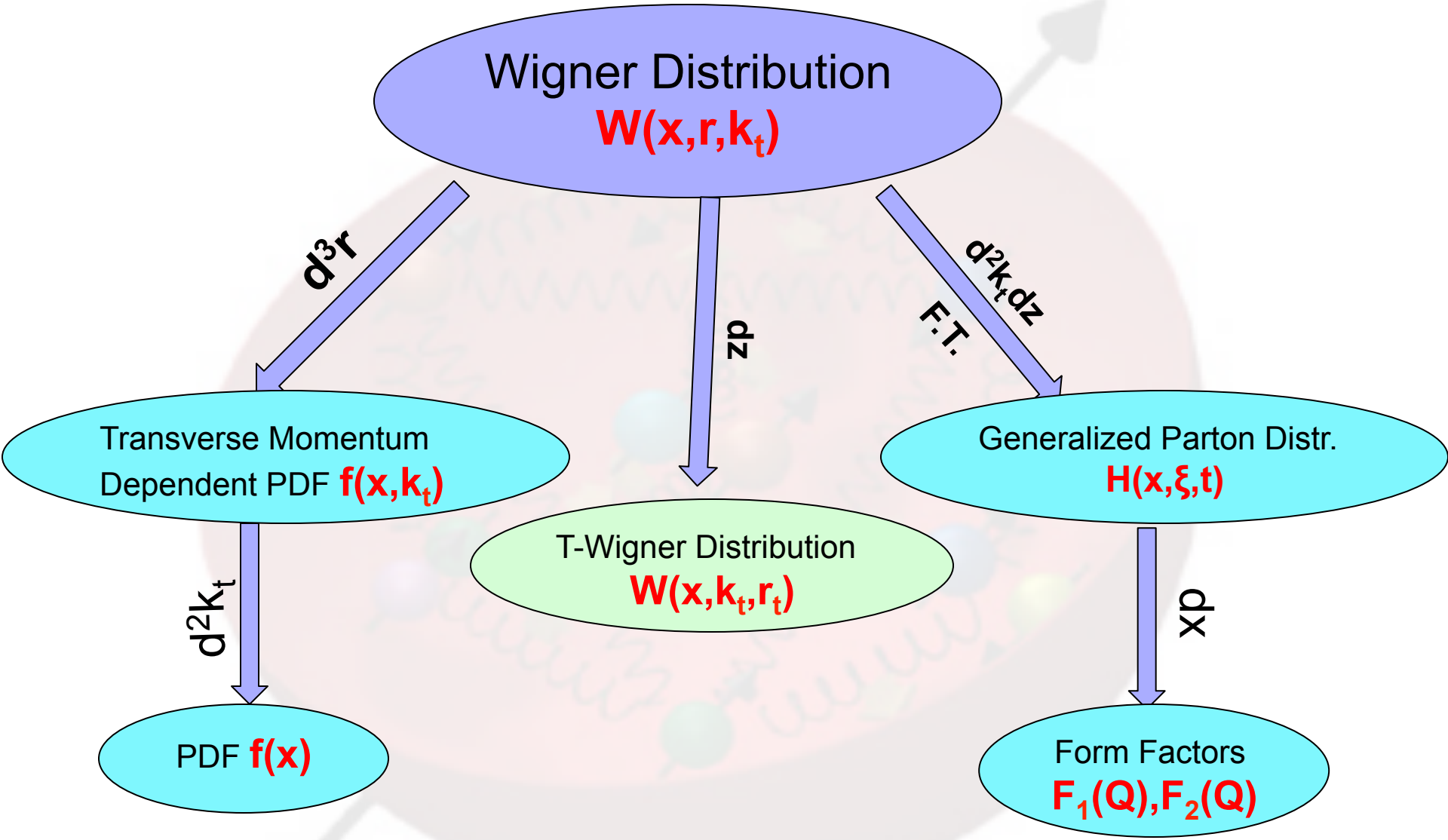
Boer-Vogelsang 03

Standard (naïve) Factorization breaks!

Dominguez and Rogers' talks in parallel sessions

Theoretical challenges in the TMD

- Q^2 dependence and soft gluon resummation, in particular, for the SSA
  - Parallel session talks by
- Global study at the Next-to-leading order
- Relation to the Orbital angular momentum
- Unified picture for GPDs and TMDs
- …


Wigner Distribution
W(x,r,k_t)
d^3r
dp
d^2k_t dz
F.T.
Transverse Momentum Dependent PDF f(x,k_t)
T-Wigner Distribution
W(x,k_t,r_t)
Generalized Parton Distr.
H(x,ξ,t)
d^2k_t
dx
PDF f(x)
Form Factors
F_1(Q),F_2(Q)

Transverse Wigner Distributions

- Integrate out z in the Wigner function

$$W_T(x, k_\perp, b_\perp = r_\perp) = \int dr_z W(x, \vec{r}, k_\perp)$$

  - Depends on x, k_t, b_t
  - Also referred as GTMD in the literature
    - See for example, Metz, et al., 09; Pasquini, 10,11
- It has close connection to the small-x parton distributions in large nuclei

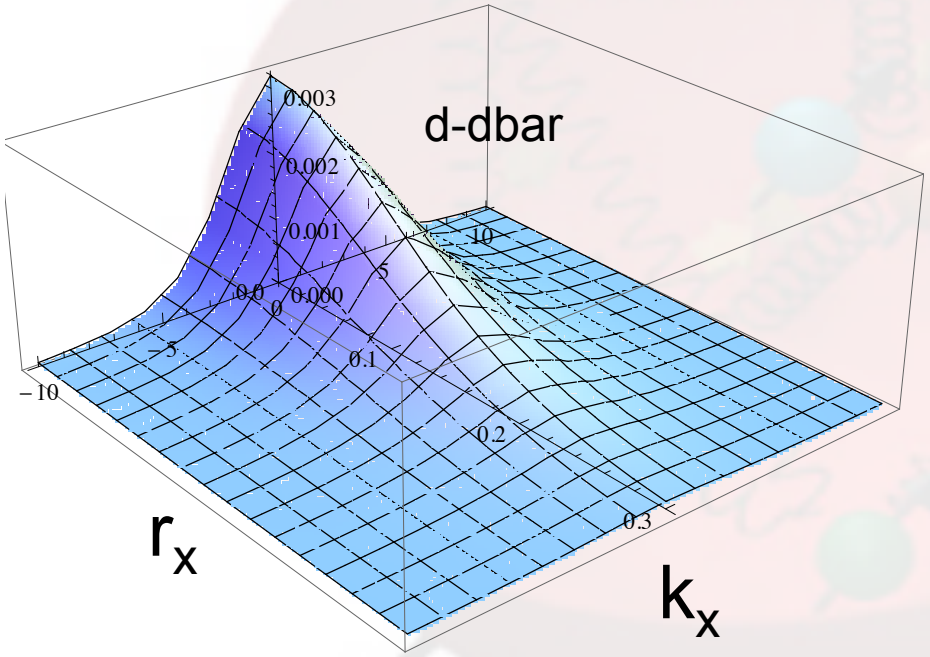
e.g., gluon number distr.
Mueller, NPB 1999

$$\frac{dN}{d^2b d^2\ell} = \frac{N_c^2 - 1}{4\pi^3 \alpha N_c} \int_1^\infty \frac{dt}{t} e^{-t\ell^2/Q_s^2}$$

Albacete's talk this afternoon

Further integrate out x

$$\mathcal{H}(\vec{r}_\perp, \vec{k}_\perp) = \int dx dz W(\vec{r}, x, k_\perp)$$



Quark model calculation: Xiong, et al.

AMO Exp.

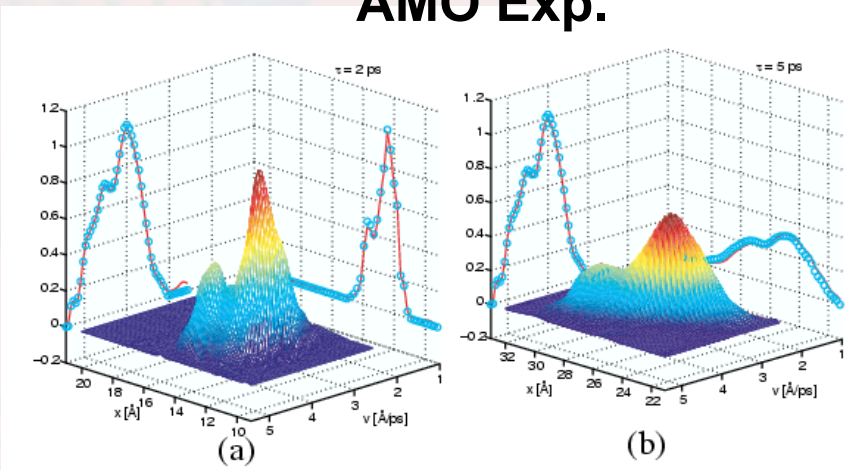


FIG. 2 (color online). Three-dimensional surface plots showing the phase space distribution function $W(x, p)$ (Wigner function) at two different times after the pump pulse dissociates the I_2 molecule [(a) $\tau = 2$ ps, and (b) $\tau = 5$ ps]. The marginal position and momentum distributions of the dissociated molecules are shown on the side panels: The full curves are the reconstructed distributions (integrals over the Wigner function with respect to momentum and position, respectively), and the open circles are the measured distributions. To provide a more intuitive view the momentum variable is presented in velocity units.

Skovsen et al. (Denmark) PRL91, 090604

- There is no known process can be used to measure the T-Wigner distribution
- We have to either use a model (constrained by the GPD and TMDs) to calculate this function
  - Muller and Pasquini's talks in parallel sessions
- Or parameterize them and fit to GPDs and TMDs simultaneously

Summary

- There have been great progresses in both experiment and theory for GPD and TMD physics
- Challenges are still in both fields
- Future Jlab 12 GeV upgrade, COMPASS, RHIC and the planed EIC experiments, will lead us a complete 3D tomography of the nucleon