Partonic Transverse Motion in Unpolarized SIDIS Processes

XIX International Workshop on Deep-Inelastic Scattering and Related Subjects

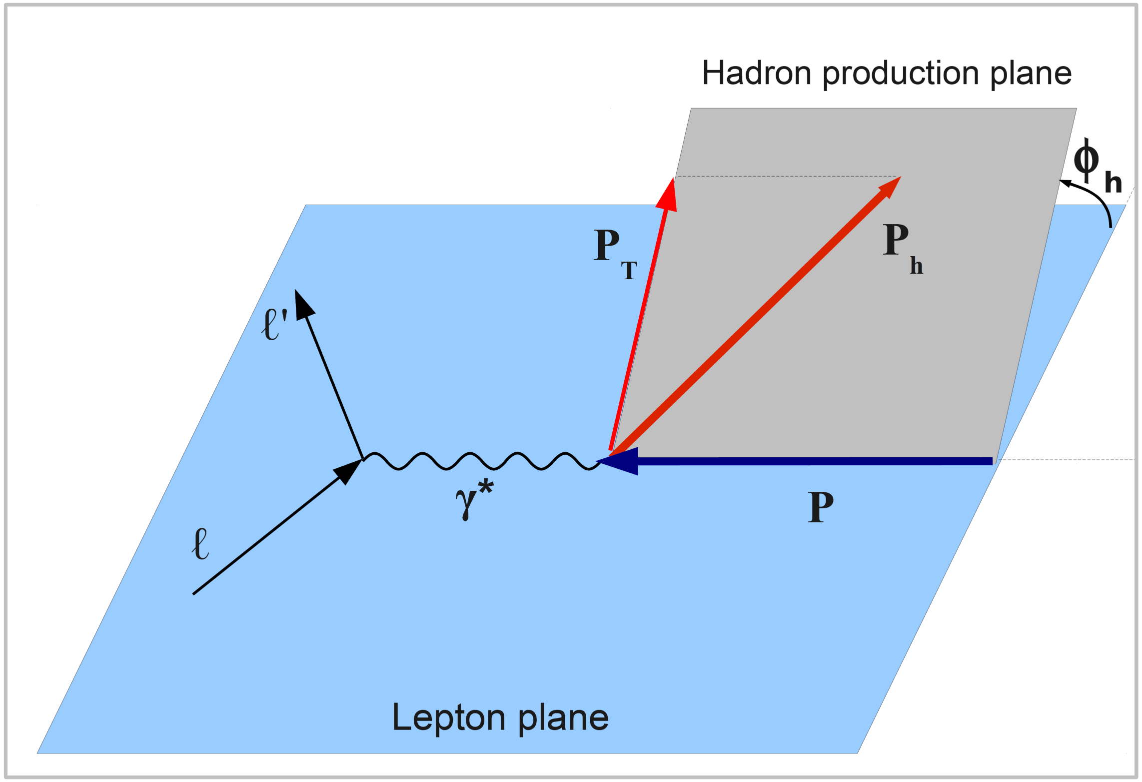
Mariaelena Boglione

INFN Torino

UNIVERSITÀ DEGLI STUDI DI TORINO
ALMA UNIVERSITAS TAURINENSIS

In collaboration with S. Melis and A. Prokudin

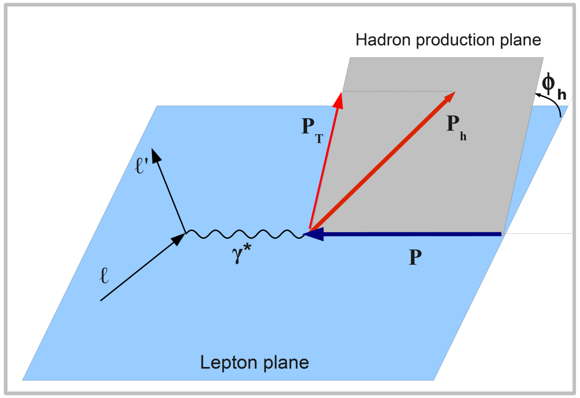
SIDIS kinematics and Cross Section



$$\frac{d\sigma^{\ell+p\to\ell' hX}}{dx_B\, dy\, dz_h\, d^2\boldsymbol{P}_T} = \frac{4\pi\,\alpha^2}{x_B\, s y^2}\left\{\frac{1+(1-y)^2}{2} F_{UU} + (2-y)\sqrt{1-y}\,\cos\phi_h F_{UU}^{\cos\phi_h} + (1-y)\,\cos 2\phi_h F_{UU}^{\cos 2\phi_h}\right\}$$

SIDIS kinematics and Cross Section

$$\frac{d\sigma^{\ell+p\to\ell' hX}}{dx_B\,dy\,dz_h\,d^2\boldsymbol{P}_T} = \frac{4\pi\,\alpha^2}{x_B\,sy^2}\left\{\frac{1+(1-y)^2}{2}F_{UU} + (2-y)\sqrt{1-y}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} + (1-y)\,\cos 2\phi_h\,F_{UU}^{\cos 2\phi_h}\right\}$$



Azim. indep. unp. contrib.

$$F_{UU} \propto f_{q/p}(x,k_\perp) \otimes D_{h/q}(z,p_\perp)\,,$$

Cahn effect **Boer-Mulders effect**

$$F_{UU}^{\cos\phi_h} \propto \frac{k_\perp}{Q}\left[\, f_{q/p}(x,k_\perp) \otimes D_{h/q}(z,p_\perp) + \Delta f_{q^\uparrow/p}(x,k_\perp) \otimes \Delta^N D_{h/q^\uparrow}(z,p_\perp)\right]$$

cos ϕ_h

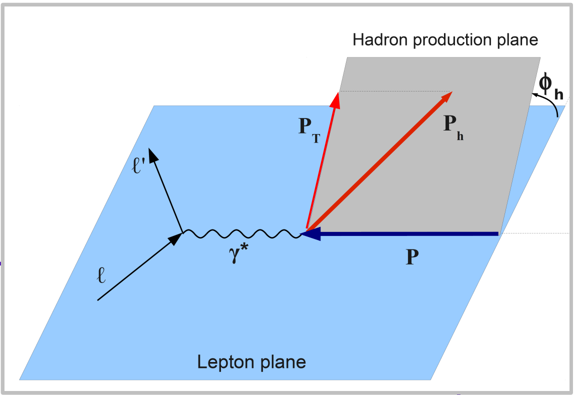
Boer-Mulders effect

$$F_{UU}^{\cos 2\phi_h} \propto \Delta f_{q^\uparrow/p}(x,k_\perp) \otimes \Delta^N D_{h/q^\uparrow}(z,p_\perp)\,.$$

cos 2ϕ_h

SIDIS kinematics and Cross Section

$$\frac{d\sigma^{\ell+p\to\ell' hX}}{dx_B\, dy\, dz_h\, d^2\boldsymbol{P}_T} = \frac{4\pi\,\alpha^2}{x_B\, s y^2}\left\{\frac{1+(1-y)^2}{2} F_{UU} + (2-y)\sqrt{1-y}\,\cos\phi_h\, F_{UU}^{\cos\phi_h} + (1-y)\,\cos 2\phi_h\, F_{UU}^{\cos 2\phi_h}\right\}$$



$$F_{UU} = \sum_q e_q^2 \int d^2\boldsymbol{k}_\perp\, \boxed{f_{q/p}(x,k_\perp)\, D_{h/q}(z,p_\perp)\,,}$$

$$F_{UU}^{\cos\phi_h} = 2\sum_q e_q^2 \int d^2\boldsymbol{k}_\perp\, \frac{k_\perp}{Q}\Big[(\hat{\boldsymbol{P}}_T\cdot\hat{\boldsymbol{k}}_\perp)\boxed{f_{q/p}(x,k_\perp)\, D_{h/q}(z,p_\perp)}$$

$$+\frac{P_T - z_h k_\perp\,(\hat{\boldsymbol{P}}_T\cdot\hat{\boldsymbol{k}}_\perp)}{2p_\perp}\boxed{\Delta f_{q^\uparrow/p}(x,k_\perp)\,\Delta^N D_{h/q^\uparrow}(z,p_\perp)}\Big]\,,$$

$$F_{UU}^{\cos 2\phi_h} = -\sum_q e_q^2 \int d^2\boldsymbol{k}_\perp \left[\frac{P_T\,(\hat{\boldsymbol{P}}_T\cdot\hat{\boldsymbol{k}}_\perp) - 2z_h k_\perp(\hat{\boldsymbol{P}}_T\cdot\hat{\boldsymbol{k}}_\perp)^2 + z_h k_\perp}{2p_\perp}\right]\boxed{\Delta f_{q^\uparrow/p}(x,k_\perp)\,\Delta^N D_{h/q^\uparrow}(z,p_\perp)}$$

Parametrization of Distribution and Fragmentation TMDs

Unpolarized TMDs

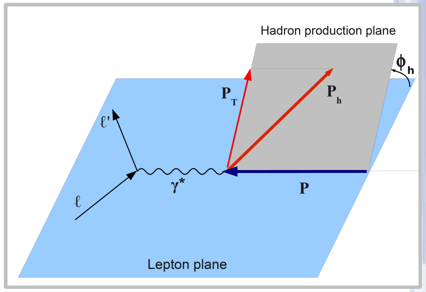
$$f_{q/p}(x,k_\perp) = f_{q/p}(x)\,\frac{1}{\pi\langle k_\perp^2\rangle}\,e^{-k_\perp^2/\langle k_\perp^2\rangle}$$

$$D_{h/q}(z,p_\perp) = D_{h/q}(z)\,\frac{1}{\pi\langle p_\perp^2\rangle}\,e^{-p_\perp^2/\langle p_\perp^2\rangle}$$

$<k_\perp^2> = 0.25\ \mathrm{GeV}^2$

$< p_\perp^2 > = 0.20\ \mathrm{GeV}^2$

Extracted by fitting SIDIS data in M. Anselmino et al, Phys. Rev. D71, 074006 (2005), hep-ph/0501196



Boer-Mulders distribution and Collins fragmentation functions

$$\Delta f_{q^\uparrow/p}(x,k_\perp) \;=\; \Delta f_{q^\uparrow/p}(x)\,\sqrt{2e}\,\frac{k_\perp}{M_{BM}}\,\frac{e^{-k_\perp^2/\langle k_\perp^2\rangle_{BM}}}{\pi\langle k_\perp^2\rangle},$$

$$\Delta^N D_{h/q^\uparrow}(z,p_\perp) \;=\; \Delta^N D_{h/q^\uparrow}(z)\,\sqrt{2e}\,\frac{p_\perp}{M_h}\,\frac{e^{-p_\perp^2/\langle p_\perp^2\rangle_C}}{\pi\langle p_\perp^2\rangle}$$

Gaussian smearings

Gaussian ansatz supported, for example, by Metz,Schweitzer,Teckentrup, Phys. Rev. D81: 094019, 2010

SIDIS kinematics and Cross Section

$$\frac{d\sigma^{\ell+p\to\ell' hX}}{dx_B\,dy\,dz_h\,d^2\boldsymbol{P}_T} = \frac{4\pi\,\alpha^2}{x_B\,s y^2}\left\{\frac{1+(1-y)^2}{2}F_{UU} + (2-y)\sqrt{1-y}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} + (1-y)\,\cos 2\phi_h\,F_{UU}^{\cos 2\phi_h}\right\}$$

After an analytical $k_\perp$ integration over the full range [0,∞], one obtains

$$F_{UU} = \sum_q e_q^2\, f_{q/p}(x_B)\, D_{h/q}(z_h)\,\frac{e^{-P_T^2/\langle P_T^2\rangle_G}}{\pi\langle P_T^2\rangle_G}$$

$$F_{UU}^{\cos 2\phi_h} = -e\,P_T^2 \sum_q e_q^2\,\frac{\Delta f_{q^\uparrow/p}(x_B)}{M_{BM}}\,\frac{\Delta^N D_{h/q^\uparrow}(z_h)}{M_h}\,\frac{e^{-P_T^2/\langle P_T^2\rangle_{BM}}}{\pi\langle P_T^2\rangle_{BM}^3}\,\frac{z_h\,\langle k_\perp^2\rangle_{BM}^2\,\langle p_\perp^2\rangle_C^2}{\langle k_\perp^2\rangle\langle p_\perp^2\rangle}$$

$$F_{UU}^{\cos\phi_h} = -2\frac{P_T}{Q}\sum_q e_q^2\, f_{q/p}(x_B)\, D_{h/q}(z_h)\,\frac{e^{-P_T^2/\langle P_T^2\rangle_G}}{\pi\langle P_T^2\rangle_G^2}\,z_h\langle k_\perp^2\rangle$$

$$+2e\frac{P_T}{Q}\sum_q e_q^2\,\frac{\Delta f_{q^\uparrow/p}(x_B)}{M_{BM}}\,\frac{\Delta^N D_{h/q^\uparrow}(z_h)}{M_h}\,\frac{e^{-P_T^2/\langle P_T^2\rangle_{BM}}}{\pi\langle P_T^2\rangle_{BM}^4}$$

$$\times\frac{\langle k_\perp^2\rangle_{BM}^2\,\langle p_\perp^2\rangle_C^2}{\langle k_\perp^2\rangle\langle p_\perp^2\rangle}\left[z_h^2\langle k_\perp^2\rangle_{BM}\left(P_T^2-\langle P_T^2\rangle_{BM}\right)+\langle p_\perp^2\rangle_C\langle P_T^2\rangle_{BM}\right]$$

Gaussian smearings

$$\langle P_T^2\rangle_G = \langle p_\perp^2\rangle + z_h^2\langle k_\perp^2\rangle\,,$$
$$\langle P_T^2\rangle_{BM} = \langle p_\perp^2\rangle_C + z_h^2\langle k_\perp^2\rangle_{BM}\,.$$

$\langle k_\perp^2\rangle = 0.25\ \mathrm{GeV}^2$

$\langle p_\perp^2\rangle = 0.20\ \mathrm{GeV}^2$

Analytical $k_\perp$ integration

It is important to stress that the relations

$$\boxed{\begin{aligned} \langle P_T^2\rangle_G &= \langle p_\perp^2\rangle + z_h^2\langle k_\perp^2\rangle\,, \\ \langle P_T^2\rangle_{BM} &= \langle p_\perp^2\rangle_C + z_h^2\langle k_\perp^2\rangle_{BM}\,. \end{aligned}}$$

occur as a direct consequence of the following choices:

- We have assumed a gaussian $k_\perp$ ($p_\perp$) distribution of the TMDs
- We have performed an analytical integration over $k_\perp$
- We have integrated $k_\perp$ over the full range $[0,\infty]$

$$\boxed{\int d^2\boldsymbol{k}_\perp \Rightarrow \int_0^\infty dk_\perp\, k_\perp \int_0^{2\pi} d\varphi}$$

- At present experimental facilities (JLAB, HERMES and COMPASS) **the average Q^2 is not so large (~ 2 GeV^2).**
- Therefore, in some kinematical ranges of x and Q^2 , it could happen that **the accessed values of $(k_\perp/Q)$ and $(k_\perp/Q)^2$ are not necessarily small**.
- One immediately visible signal of this is the **Cahn effect** in both azimuthal moments, at twist-3 in <$\cos\phi_h$> and at twist-4 in <$\cos 2\phi_h$>, which is directly proportional to the ratios $(k_\perp/Q)$ and $(k_\perp/Q)^2$ respectively, and is found to be (phenomenologically) large.

Physical partonic cuts on $k_\perp$

- The integration over $k_\perp$ from zero to infinity is clearly a crude assumption.
- The parton model provides kinematical limits on the transverse momentum size.

By requiring the energy of the parton to be smaller than the energy of its parent hadron, we have

$$\boxed{k_\perp^2 \leq (2 - x_B)(1 - x_B)Q^2}$$

By requiring the parton not to move backward with respect to its parent hadron, we find

$$\boxed{\boldsymbol{k}_\perp^2 \leq \frac{x_B(1 - x_B)}{(2x_B - 1)^2} Q^2}$$

- If we apply these additional constraints on the partonic kinematics and our results are not affected, it means that we are in a kinematical region where corrections of order **$(k_\perp/Q)^2$** can be safely neglected and where the parton model holds.

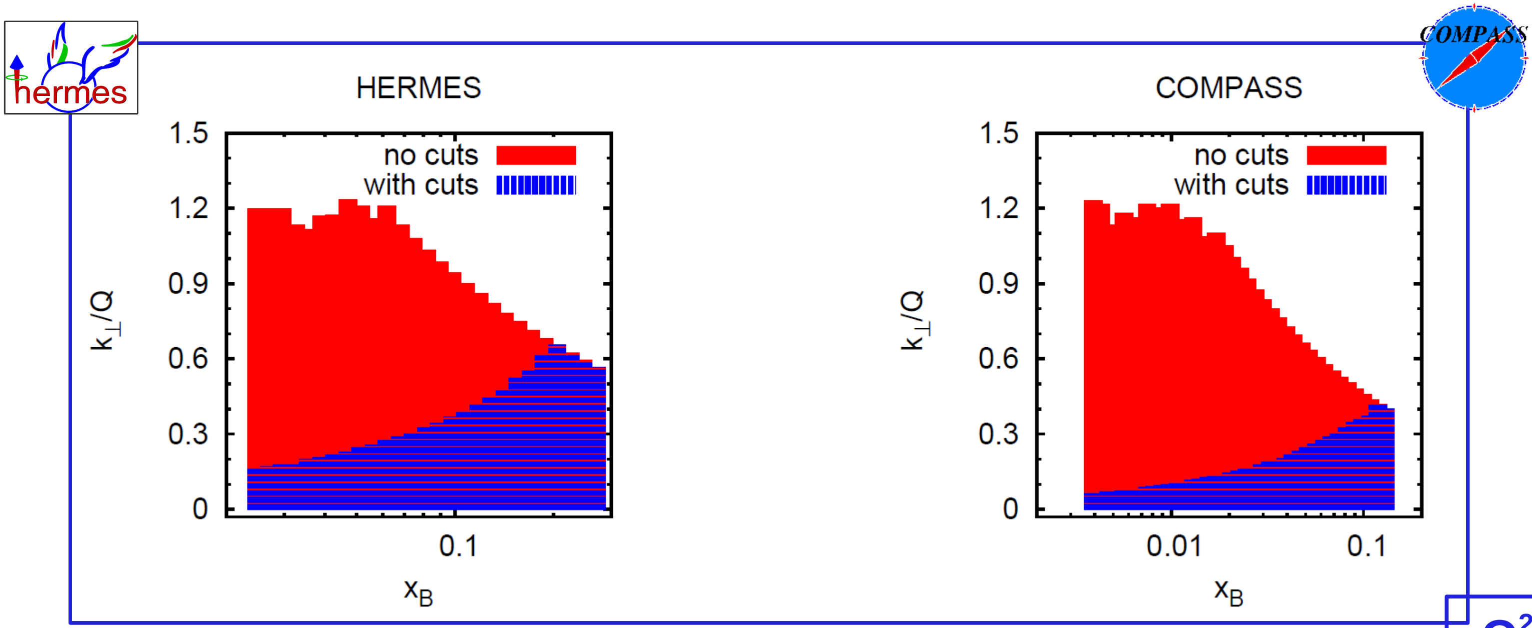
Impact of the physical partonic cuts on the SIDIS cross section and on $<P_T^2>$

The average transverse momentum of the final detected hadron is defined as

$$\boxed{\langle P_T^2 \rangle = \frac{\int d^2\boldsymbol{P}_T\, P_T^2\, d\sigma}{\int d^2\boldsymbol{P}_T\, d\sigma}}$$

- If the integral is performed over the range [0,∞], then $<P_T> \equiv <P_T>_G$.
- The experimental P_T range, however, usually span a finite region between P_T^{min} and P_T^{max} therefore in any experimental analysis one inevitably has $<P_T> \neq <P_T>_G$ even without considering the physical cuts introduced above.
- Consequently, the relation $<P_T^2> = <p_\perp^2> + z^2 <k_\perp^2>$ holds only approximatively.

Impact of the physical partonic cuts on the SIDIS $k_\perp/Q$ phase space



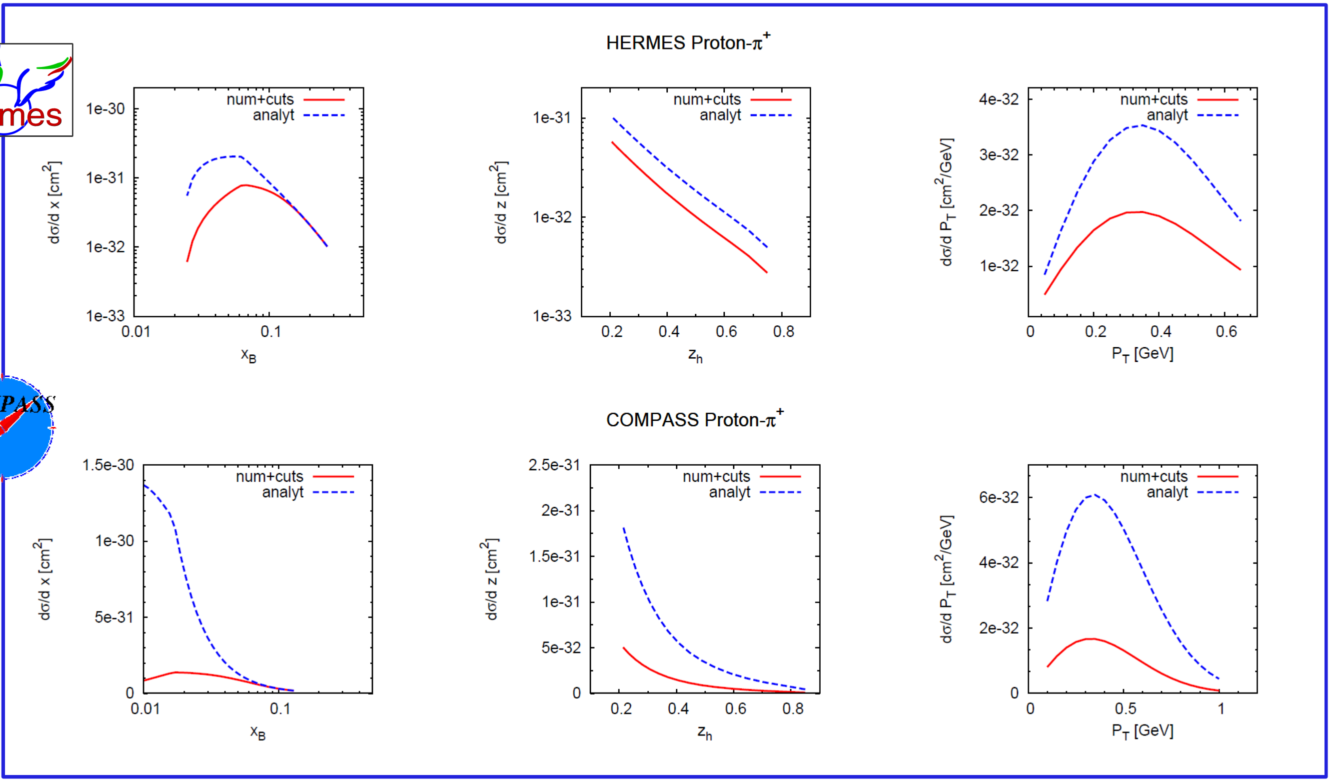
$Q^2 = x_B\, y\, s$

($k_\perp/Q$) phase space as a function of x

Red area: $k_\perp$ integration with partonic $k_\perp$ cuts

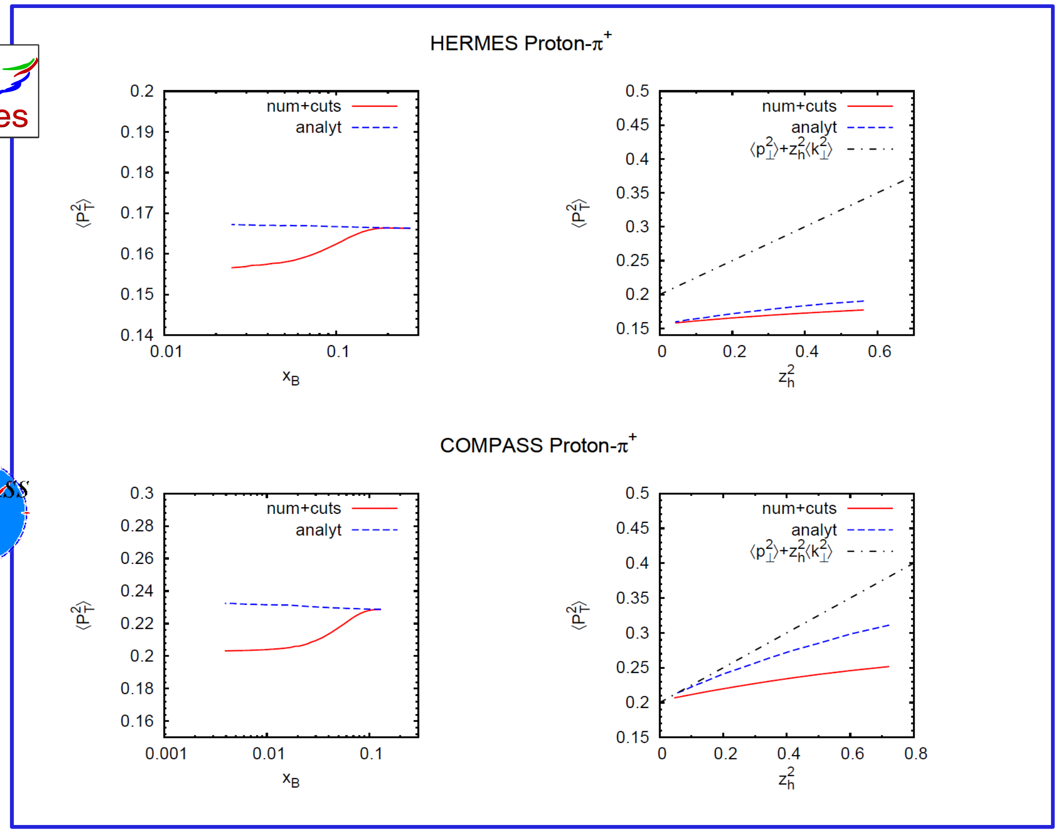
Blue area: analytical $k_\perp$ integration

Impact of the physical partonic cuts on the SIDIS cross section



Unpolarized cross section for HERMES and COMPASS pion production.
Red line: $k_\perp$ integration with partonic $k_\perp$ cuts
Blue line: analytical $k_\perp$ integration

Impact of the physical partonic cuts on $<P_T^2>$



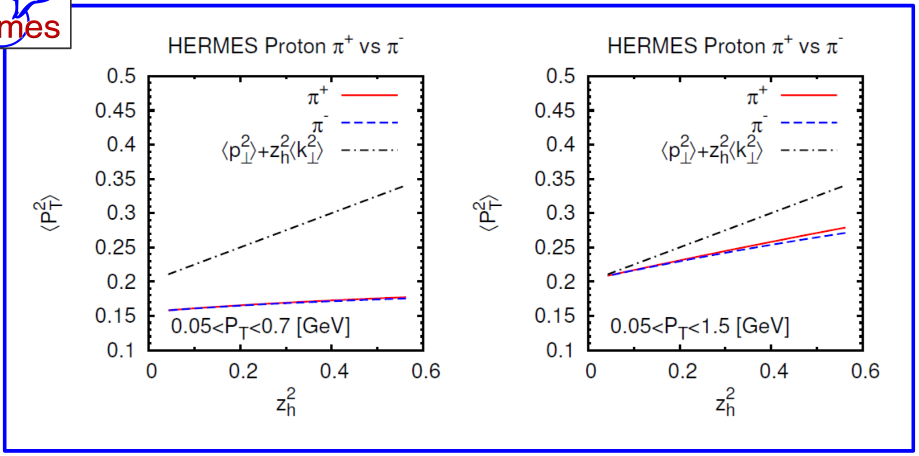
Calculated $<P_T^2>$ for HERMES and COMPASS pion production.

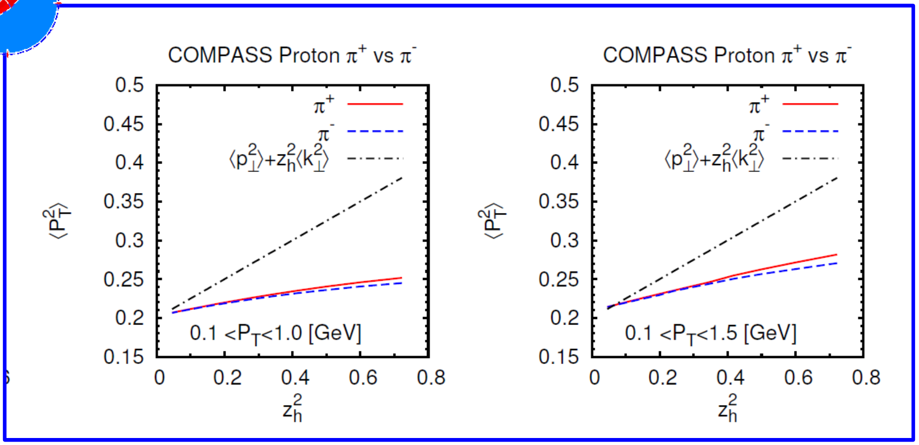
Red line: $k_\perp$ integration with partonic $k_\perp$ cuts

Blue line: analytical $k_\perp$ integration with experimental P_T cuts

Black line: $<P_T^2> = <p_\perp^2> + z^2 <k_\perp^2>$

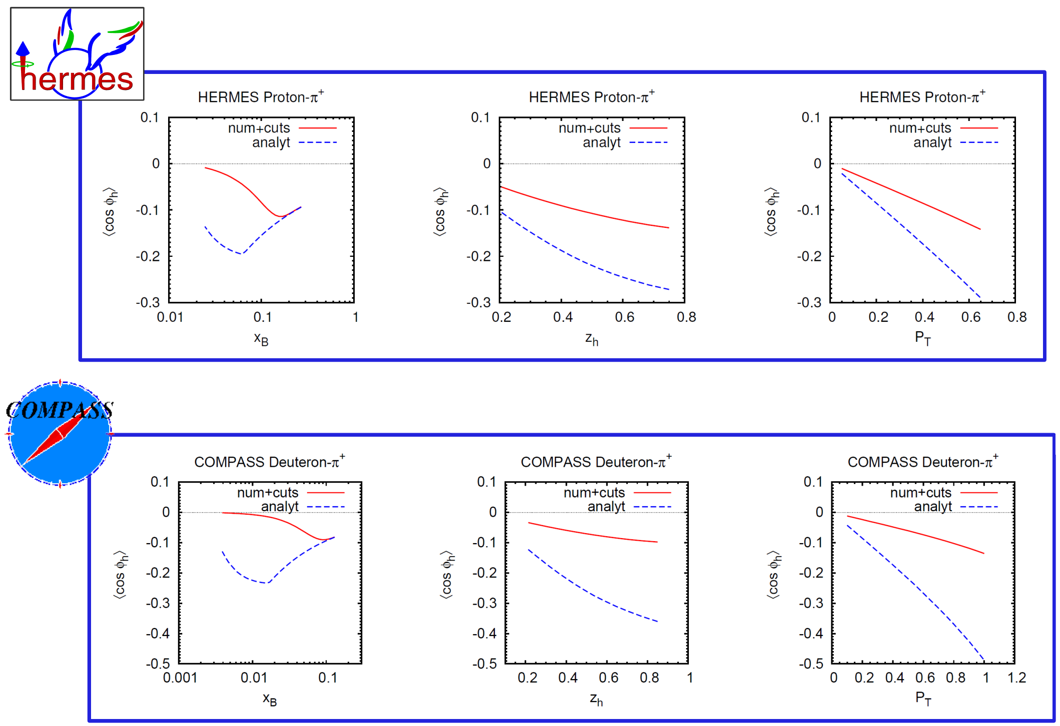
Impact of the physical partonic cuts on $<P_T^2>$





Deviations of $<P_T^{\,2}>_{obs}$ from $<P_T^{\,2}>_G$ are very sensitive to the cuts we require on P_T

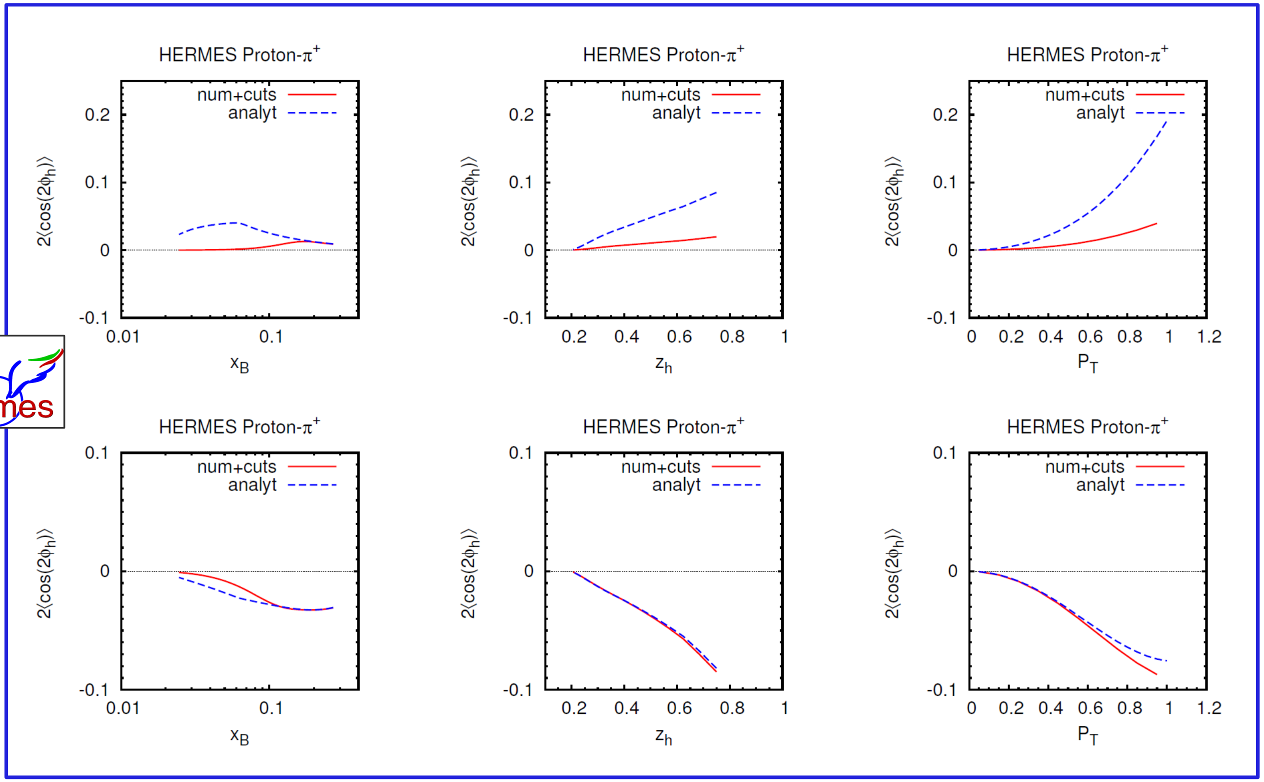
Impact of the physical partonic cuts on the azimuthal moment $<\cos\phi_h>$



$$\propto \frac{k_\perp}{Q} \left[f_{q/p}(x, k_\perp) \otimes D_{h/q}(z, p_\perp) \right]$$

Twist-3 Cahn contribution to the $<cos\phi_h>$ azimuthal moment.
The Boer-Mulders contribution is tiny, we do not show it here.

Impact of the physical partonic cuts on the azimuthal moment <cos $2\phi_h$>



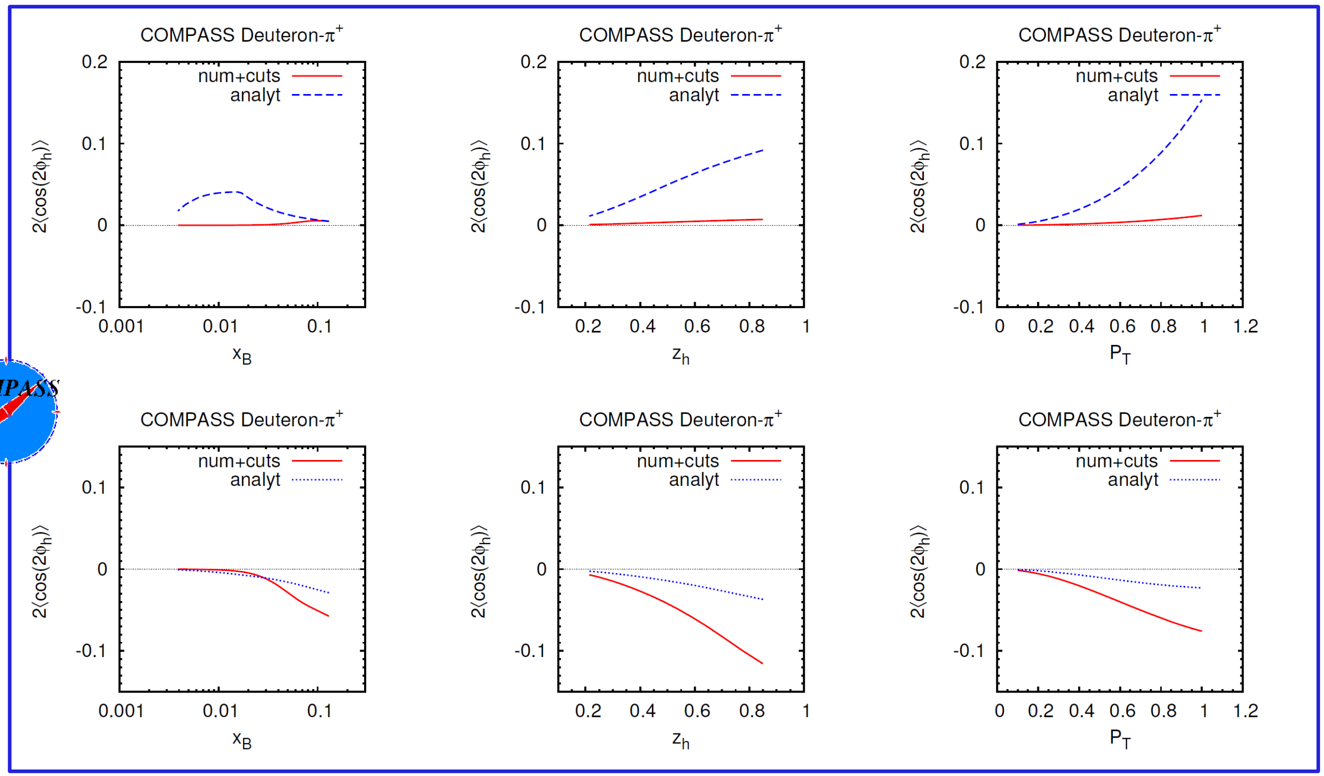
$$\propto \frac{k_\perp^2}{Q^2}\left[\, f_{q/p}(x,k_\perp) \otimes D_{h/q}(z,p_\perp)\right]$$

$$\propto \Delta f_{q^\uparrow/p}(x,k_\perp) \otimes \Delta^N D_{h/q^\uparrow}(z,p_\perp)$$

Twist-4 Cahn contribution to the <*cos2*ϕ_h> azimuthal moment (upper panel)
Twist-2 Boer-Mulders contribution to the <*cos2*ϕ_h> azimuthal moment (lower panel)

The Boer-Mulders TMD is taken from Barone, Melis, Prokudin, Phys.Rev.D81:114026,2010.

Impact of the physical partonic cuts on the azimuthal moment <cos 2ϕ_h>



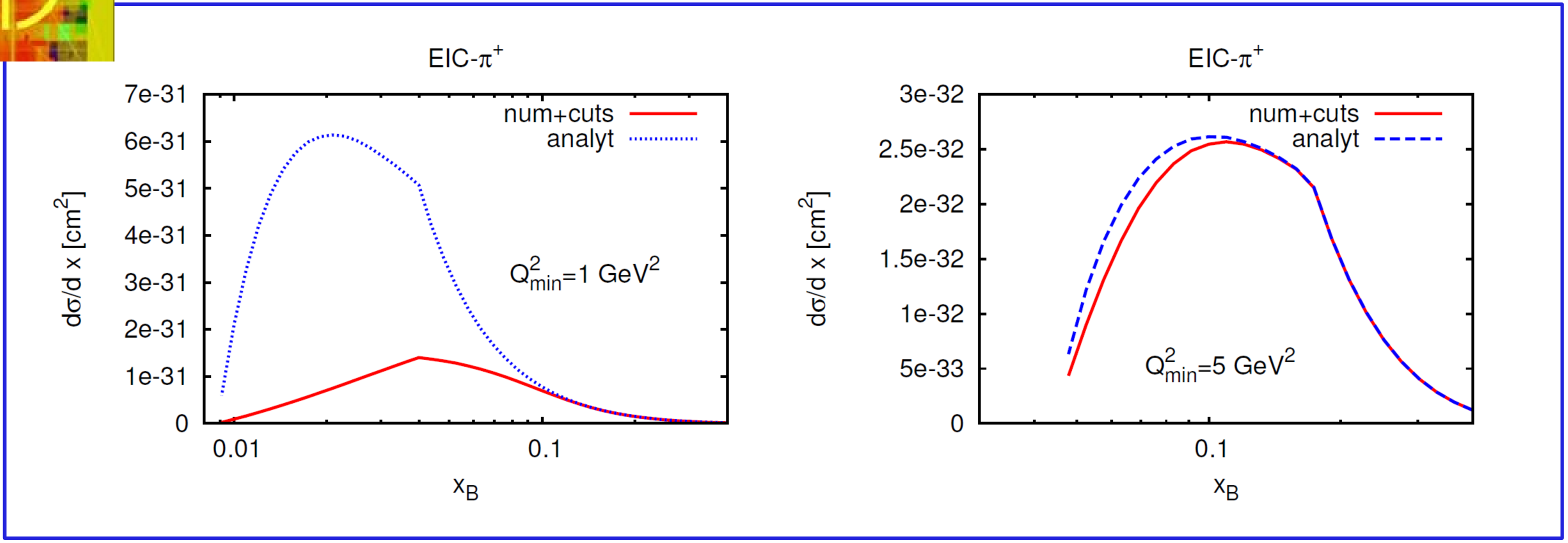
$$\propto \frac{k_\perp^2}{Q^2}\left[\, f_{q/p}(x,k_\perp) \otimes D_{h/q}(z,p_\perp)\right]$$

$$\propto \Delta f_{q^\uparrow/p}(x,k_\perp) \otimes \Delta^N D_{h/q^\uparrow}(z,p_\perp)$$

Twist-4 Cahn contribution to the <*cos2ϕ_h*> azimuthal moment (upper panel)
Twist-2 Boer-Mulders contribution to the <*cos2ϕ_h*> azimuthal moment (lower panel)

The Boer-Mulders TMD is taken from Barone, Melis, Prokudin, Phys.Rev.D81:114026,2010.

Impact of the physical partonic cuts on the SIDIS cross section at EIC



EIC kinematical span will allow to cut to relatively large values of Q^2 (Q^2 >5 GeV^2), which guarantee an almost complete insensitivity to the $k_\perp$ cuts.

Conclusions and future prospects

- One of the basic assumptions of TMD factorization is **$k_\perp/Q \ll 1$**
- In the kinematics of present experimental facilities (JLab, HERMES, COMPASS …) this condition is not always fulfilled, due to their low Q^2 cuts.
- Phenomenological calculations of the unpolarized cross section and of $\langle P_T^2 \rangle$ in the low-x and low-Q^2 regions are affected by the choice of $k_\perp$- cuts.
- EIC kinematics (with $Q^2 > 5$ GeV^2) would guarantee **small** $k_\perp/Q$ values, for which the parton model applies and the effect of the choice of $k_\perp$- cuts is negligible.
- In this context, TMD evolution equations are of vital importance.