C F N S

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- Modifying DGLAP equations, ...
- Theoretical arguments
- Program implementation
- cfns massless comparison

Motivations

- I am not happy with present situation
- In nature discontinuities are common: faults, freezing points ,liquid flow…
  but physicist will do not enter in the game
- Schemes induce better and better modeling of the observables but they induce also discontinuities which may lead to doubts on the whole methodology
- Heavy quarks do exist before they get a pdf

Outline of actions to be taken

- Have always six flavors but with heavy quarks contributions increasing with Q^2
- Modify splitting functions … to satisfy heavy quarks kinematic constraints and continuity
- Do all that coherently, satisfying sum rules
- Solve equations
- Write evolution code (In QCDFIT program)

DGLAP equations

- The electron proton reaction is :
  e(*l*) +p(*p*) -> e(*l-q*)+ X(*l+q*)
  Quadrimomenta are within ().
  Q^2=Sxy =-q^2 x=Q^2=/(2*pq*) y=*pq*/*pl* S=(l+p)^2
  $W^2 = (p+q)^2 = M^2 + Q^2(1/x-1)$
- Parton m_o may be kicked out of the target if $W>2m_o$ where m_0 is the parton *o* mass. x has a kinematical limit $l_o = (1+4\ m_0{}^2/Q^2\ )^{-1}$
- Light partons fullfill always the condition
- But heavy ones do only for $Q^2 \to \infty$

- Parton distributions, functions of x and Q^2 ,are noted by the name of the parton species
  $p=g,d,\bar{d},u,\bar{u},s,\bar{s},c,\bar{c},b,\bar{b},t,\bar{t}$
  … and for the quarks $d^{\pm}= d \pm\bar{d}$, …
- DGLAP equations are:
  $\partial o/\partial \ln(Q^2) = \Sigma_i P_{io} \otimes i$
- P_{oi} (x/z) is the change to parton *o* at Bjorken x radiated by parton *i* at x/z.
- $\otimes$ stands for the convolution integral

- *i* and *o* run on the 1+2 N_f partons species.
- N_f the number of active flavors will be the main concern of this approach.
- Usually it is taken as the number of quark families with $Q^2 > m^2_0$ so changing a constraint on x to a constraint on Q^2 !!

DGLAP + subsystem

$$\partial g/\partial \ln(Q^2)=P_{gg}\otimes g+\Sigma_r P_{gr}\otimes r^+$$
$$\partial q^+/\partial \ln(Q^2)=P^{\sim}_{qg}\otimes g+\Sigma_r P^{\sim}_{S}\otimes r^+ + P^+_{NS}\otimes q^+$$

Momentum integrals

$$\partial g/\partial \ln(Q^2)=P_{gg}\, g+\Sigma_r P_{gr}\, r^+$$
$$\partial q^+/\partial \ln(Q^2)=P^{\sim}_{qg}\, g+\Sigma_r P^{\sim}_{S}\, r^+ + P^+_{NS}\, q^+$$

Modifying DGLAP equations

- The idea is to modify the kernels in order to satisfy simultaneously the 3 kinematical constraints
  $x < l_i$, $x < x_o$, $x < \mathrm{l}_o$
  P_{oi} is the change to parton *o* at Bjorken x radiated by parton *i* at x/z. Problematic cases are for *o* heavier than *i* like for c $\rightarrow$ b

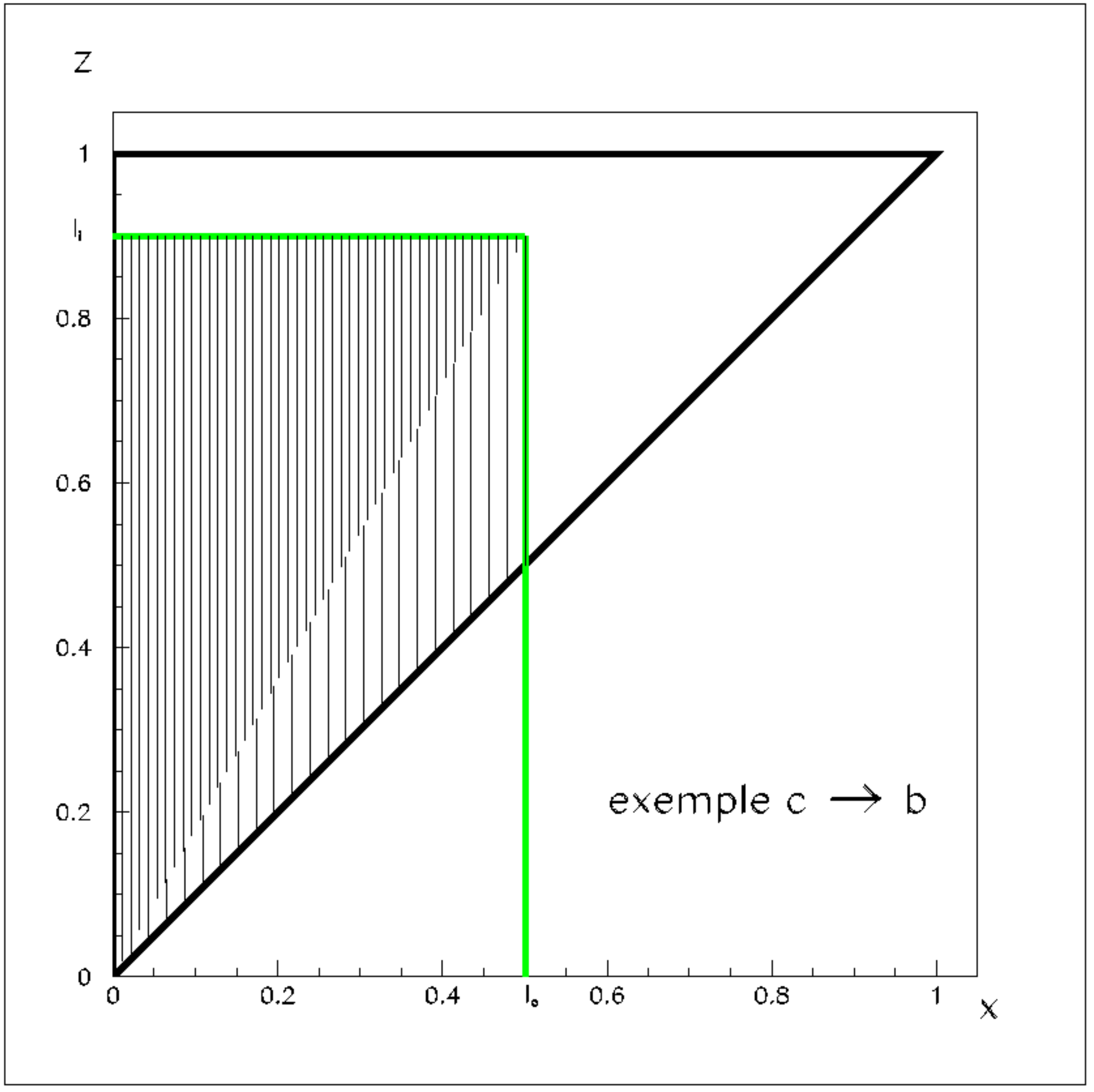
- Replacing *P* by *K* they become :

$$\int_o^1 K_{oi}(x/z)\, i(z)\, dz/z \quad : \otimes$$

- Graphical representation
- If null for $x>l_o$ then $K(u)=0$ for $u>l_o$ take $K_{oi}=\Phi_{oi}(Q^2)\, P_{oi} \otimes \delta(x-l_o)$

- Replace $K(x)$ by $P(\xi)$ with $\xi = x/l_o$

A problematic changing term



Consequences on DGLAP

- **Momentum integrals**

  $\partial g/\partial \ln(Q^2) = P_{gg}\ g + \Sigma_{gr}\ P_{gr} r^+$

  $\partial q^+/\partial \ln(Q^2) = I_q (P^{\sim}_{qg}\ g + \Sigma_r P^{\sim}_S r^+) + P^+_{NS}\ q^+$
- With $I_q = \int x\delta(x-I_q)dx \qquad \delta_q = \delta(x-I_q)$
- This leads to $N_f = 3 + I_c + I_b + I_t$
- Sum of the phase spaces of the quarks
- N_f has to be used consistently in all the P
- Also in the β function (β_0 appears in P_{gg})

Modified DGLAP equations

- **DGLAP becomes**
  $\partial g/\partial \ln(Q^2) = P_{gg} \otimes g + \Sigma_{gr} P_{gr} \otimes r^+$
  $\partial q^+/\partial \ln(Q^2) =$
  $\delta q \otimes (P\sim_{qg} \otimes g + \Sigma_r P\,\tilde{}_S \otimes r^+) + P^+_{NS} \otimes q^+$
- Sum of quark equations shows that g and Σ decouple from the others
- Non singlets cannot be defined as before but one may use Σ_{light} to define them. Light's are still decoupled and heavy's become decoupled when they are fully active.

Modified DGLAP equations

- For $l_q \to 0$ corresponding equation get decoupled (Appelquist-Carazzone theorem)
- Several equivalent linear combinations of the new equations may be used

Coefficient functions

- Structure functions are obtained by:
  $F^{a}{}_{o} = \Sigma_i \, C_{io} \otimes i$
  C are the coefficient functions
- This has exactly the same structure as the DGLAP equations and is nothing more than a change of scheme ($\overline{\text{MS}}$ to DIS for F^2) so it is natural to use the same procedure than for the DGLAP equations (and in fact it is already like that at N_f changes for the latest schemes)

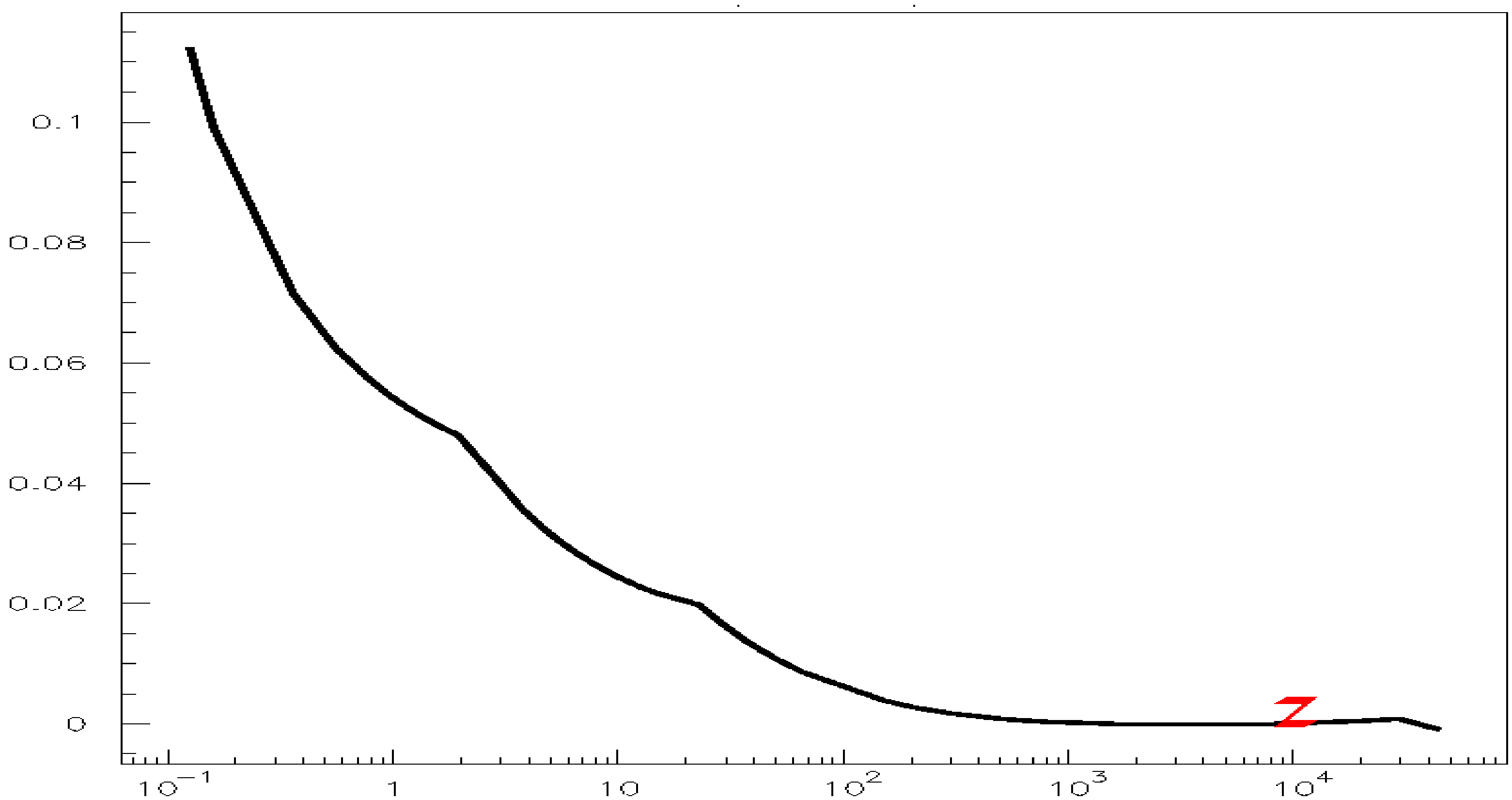
Theoretical arguments

- It is important to note moreover that the ideas presented here are not new:
- $\xi = x/l_0$ is the scaling variable used in H. Georgi and H.D. Politzer Phys Rev D 15,7 (1976) and many other papers
- H. Georgi and H.D. Politzer use anomalous dimensions variable with Q^2. Anomalous dimensions leading to splitting functions their arguments should hold here.
  For this they advocate $l_o = Q^2/(Q^2+2m^2)$

- They present a beta_o variable with Q^2. Here it is $l_o = Q^2/(Q^2+5m^2)$
- S.Brodsky et al arXiv:hep-ph/9906324 have N_f Q^2 and order dependent
- Last reference to this is D.D. Dietrich arXiv:0908.1364 [hep-th]
- As already stated the procedure leading to satisfaction of the kinematical constraints as been used lately for coefficient functions in GM-VFNS schemes
  R.S. Thorne arXiv:1006.5925 [hep-th]

- Notice also how small are the differences between massless and continuous behavior for α_s .
- Apart numerical differences (which may be eventually cured by a judicious choice of ϕ) the outcome is similar for the various l_0 used

α_s cfns massless relative difference



Cfns potential weak points

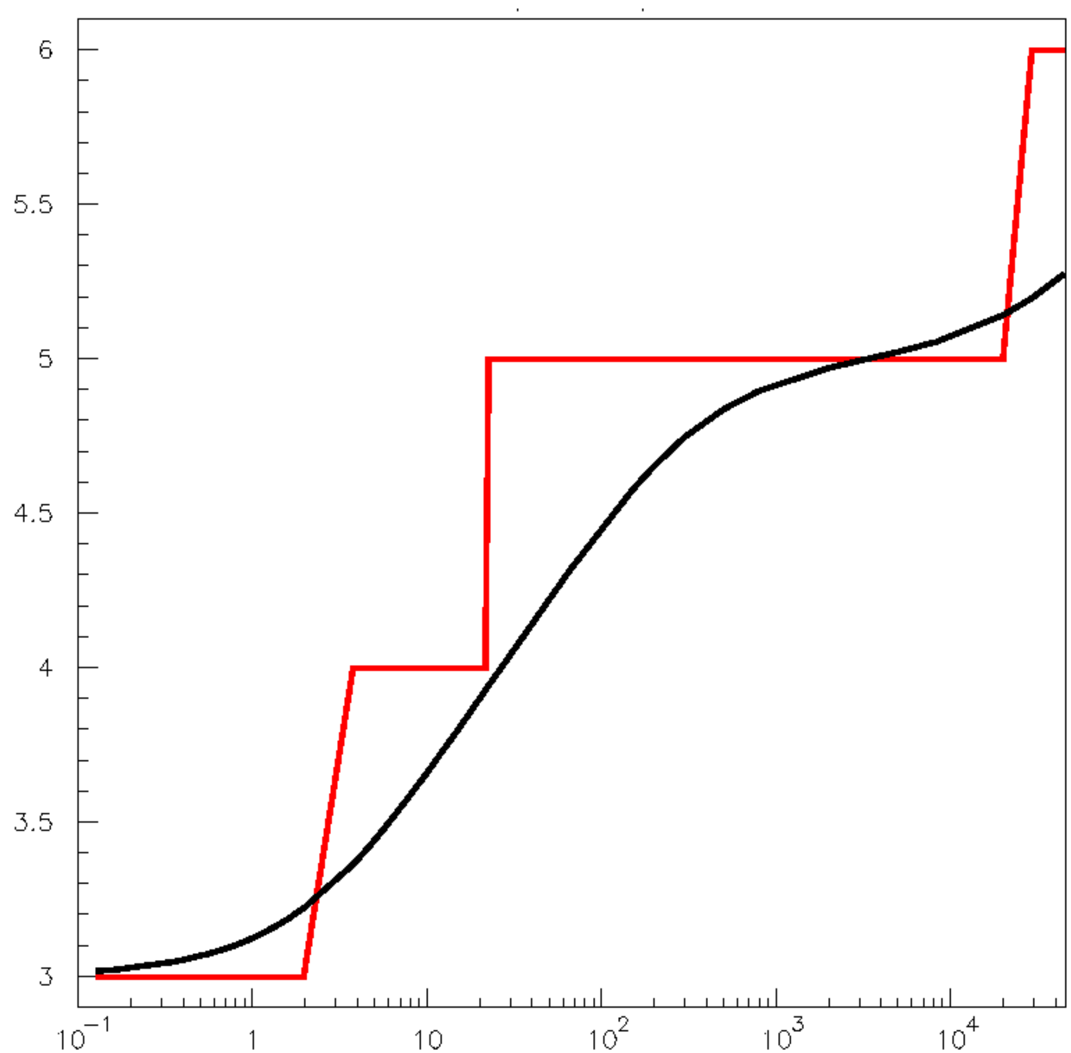
- It essentially assumes that DGLAP equations and all its components should exist even if they are calculable only when quarks are light or quasi light, so it interpolates coherently between those cases and full scale range.
- Other schemes have also their approximations
- I am not able to decide what is the best
- Renormalisation group should decide and I am not able to look into it.

Cfns strong points

- It does not mix up different α_s orders as do mixed schemes.
- Heavy quarks participate to the evolution when they start to appear, that is at the beginning and at very low x and even at leading order in α_s.
- There is no internal and external partons, only internals.

- It should be ‘better’ at the small x due to evolution
- It covers the charm-bottom region where they are both opening up which is not yet the case in usual schemes
- Heavy quarks do not apperar as generating threshold in Q^2 anymore (also a warning)

Cfns and massless N_f



Implementation (in new QCDFIT)

- It is a program which works in x space
- It includes an optimizing interface (Minuit).
- it accept a variety of input distributions.
- It has a variety of outputs: Pdfs, cross section for lepto production,Drell-Yang mecanism ...
- It pre-calculate the full evolution
- It use an x grid linearly spaced in logx and a $\log Q^2$ grid approximately in α_s
- Pdf -> one dimensional x array for a given Q^2

- Kernels have a matrix representation but due to their splitting or parton branching nature they are upper triangular band matrices with $M_{ij}=m_{i-j}$ with i≥j and so are also one dimensional arrays for a given Q^2.
- Integration of the renormalisation group equation is made numerically as its parameters are functions of N_f and so of Q^2

The transport matrices defined by

$$o(Q^2_{j+1}) = \sum_i \mathcal{T}(o,i) \otimes i(Q^2_j)$$

are obtained by integration of the subsystems using

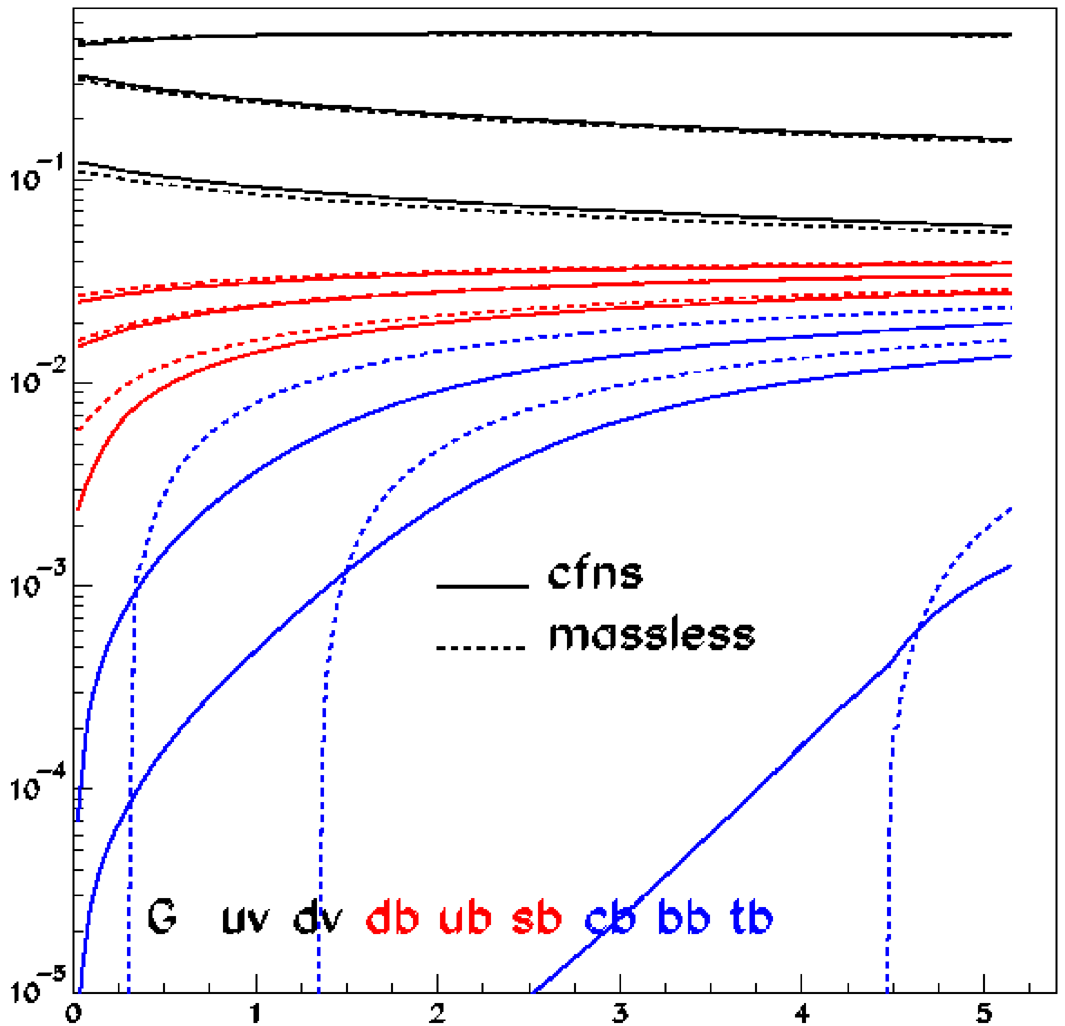
$$\mathcal{T}(o,i) = \prod_{Q^2_j}^{Q^2_{j+1}} \otimes \left(1 + \frac{\partial^2 o}{\partial i \partial ln(Q^2)} \delta ln(Q^2)\right)$$

In the product $\delta ln(Q^2)$ has to be small enough to vary only according to rounding errors when increasing the number of Q^2 nodes.

cfns massless comparison

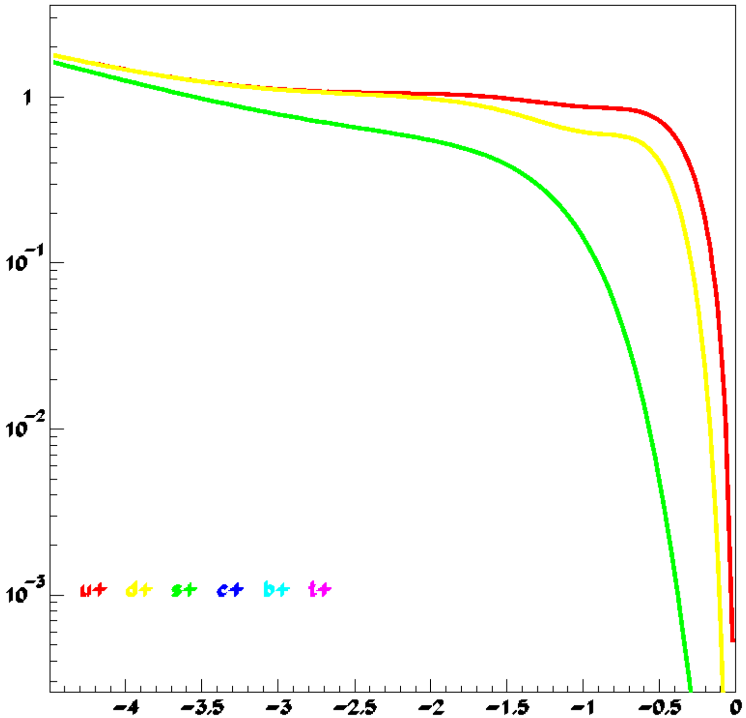
- A data sample made of about 1300 cross section measurements H1 preliminary is used to fit the input pdfs with massless scheme.
- The fitted distributions are at 1Gev^2
  g>0 U_v D_v U_{bar} D_{bar} S with
  $S(q^2 = 10) = 0.53\, D_{bar}\, S(q^2 = 10)$
- The aim of this exercise is to show what kind of new features might be seen on CFNS and how far they extend away from the transition points of the other sheme.

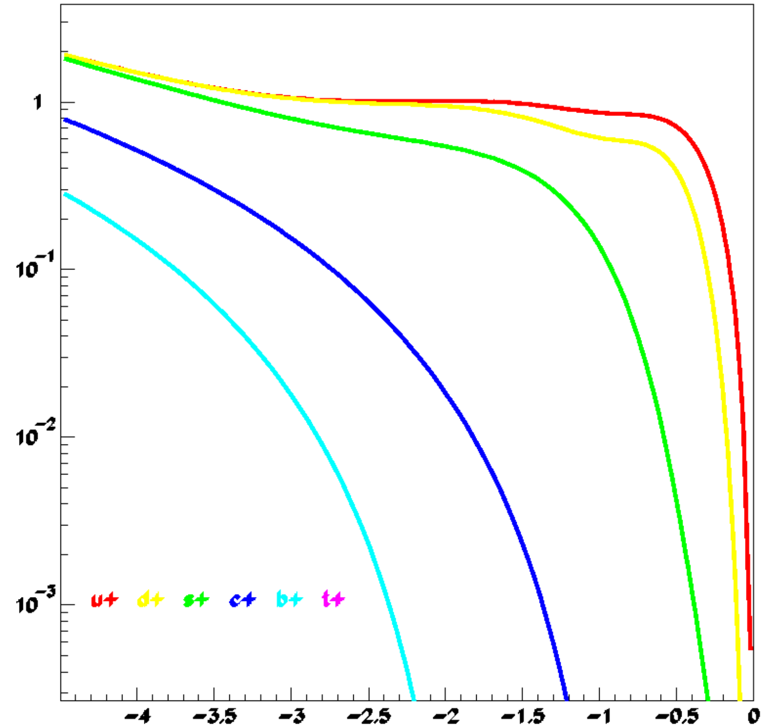
Parton momemtum fractions



Charm mass^2

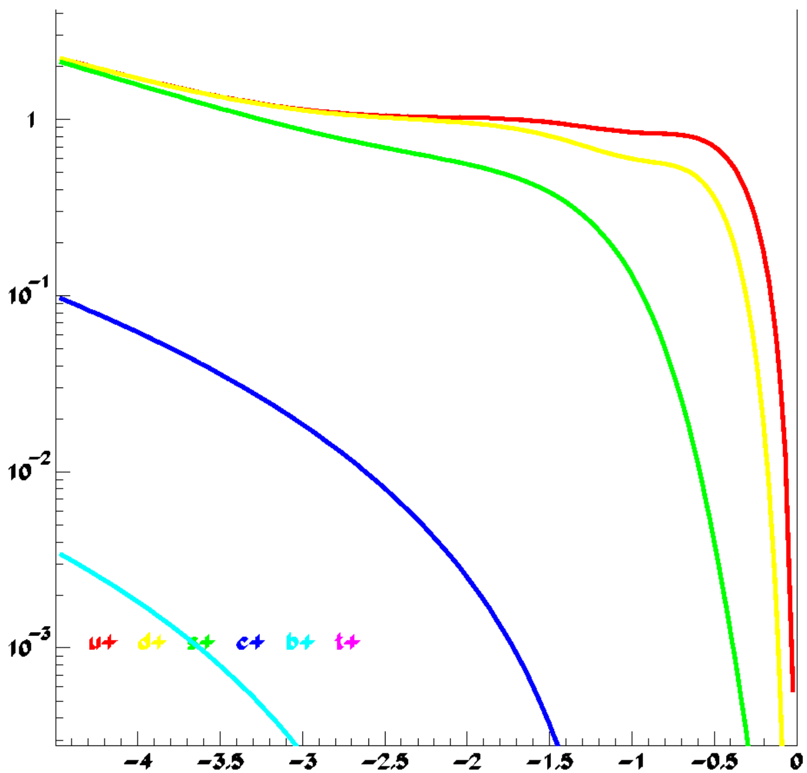
massless cfns



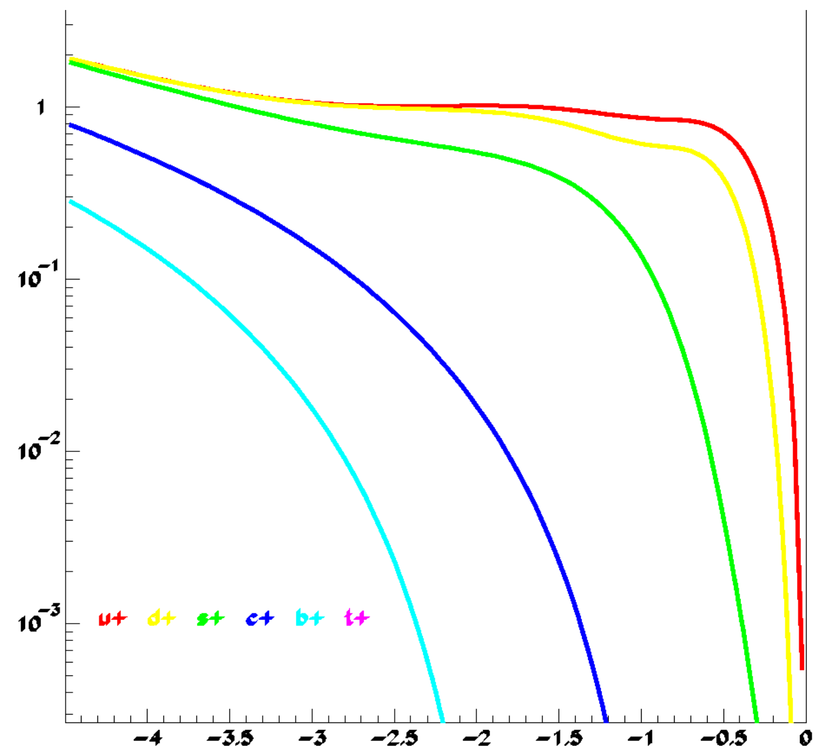


Charm mass²

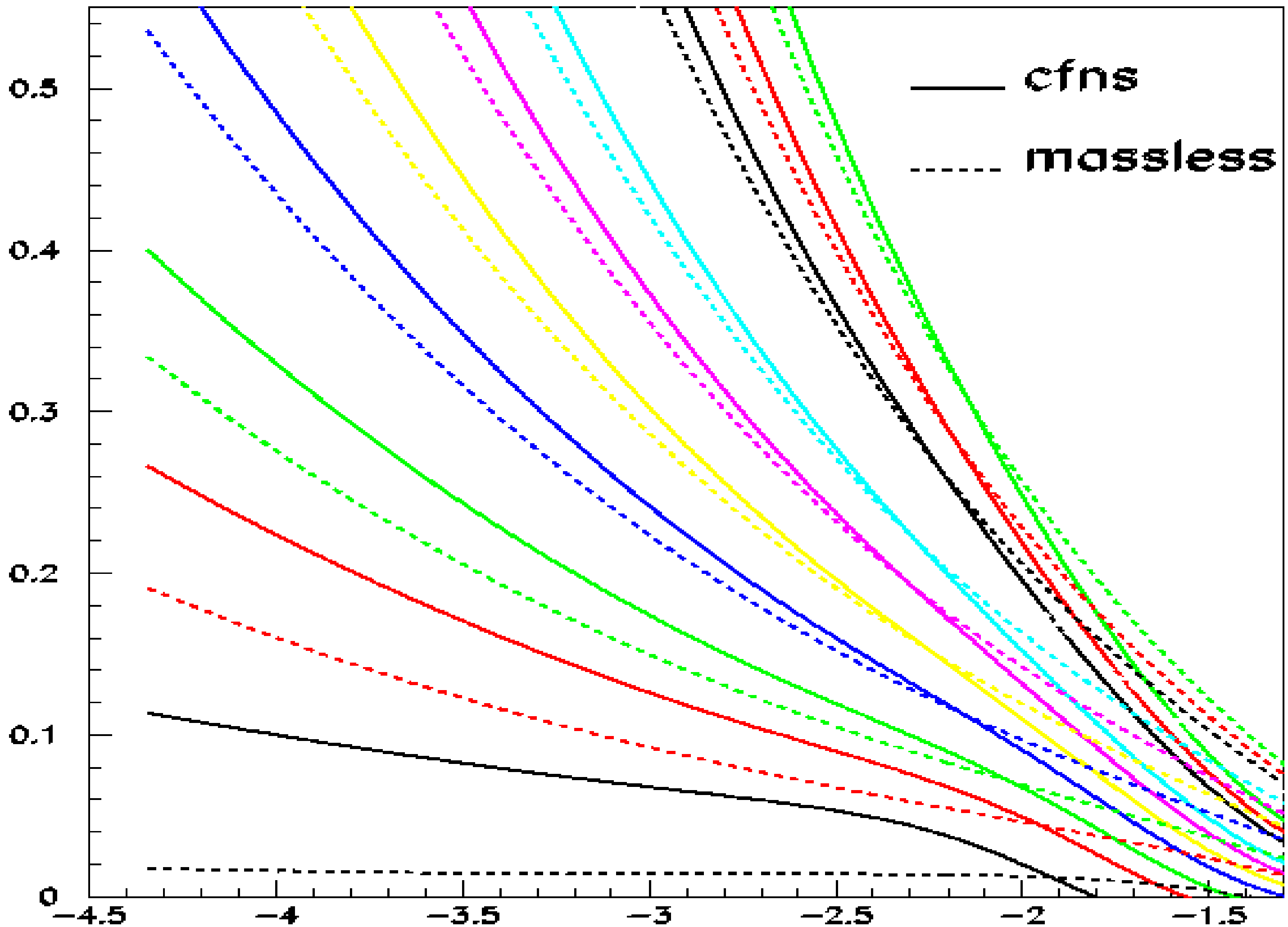
cfns2



cfns



F2cc for 2 4 6.5 12 20 35 60 ...Gev²



- END

cfns massless comparison

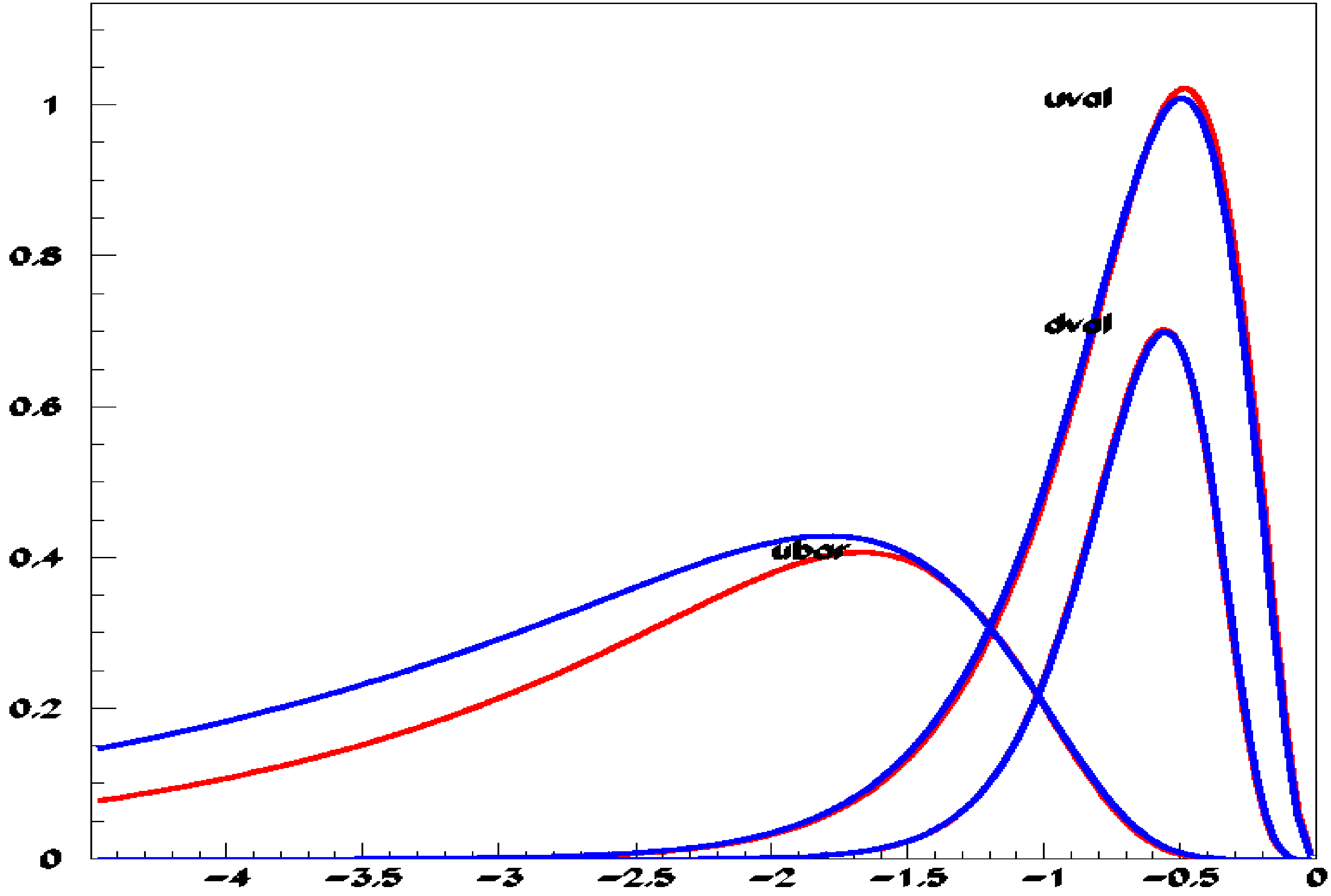
- I am of course aware that any comparison should be done at the NNLO level between this new scheme CFNS and the other outstanding schemes at least at the pdf level and probably even more, at the structure function level.
- But for that two evolution codes are needed both running at NNLO, one accepting the usual schemes and one built for the new sheme

Kinematical range used

- Very often the start of evolution Q^2_{in} is chosen just below the charm masss squared in order to define pdf inputs only for the light partons. But for CFNS heavy quarks are always present if the kinematical range permits, so in order to have only light quarks present at input, I have used Q^2_{in}=.125 low enough to justify neglecting all the heavy quarks at input. Needless to say that at so low a Q^2 predictivity is completely absent but it is a parameterless way to get a sensible charm when out of the non perturbative region.

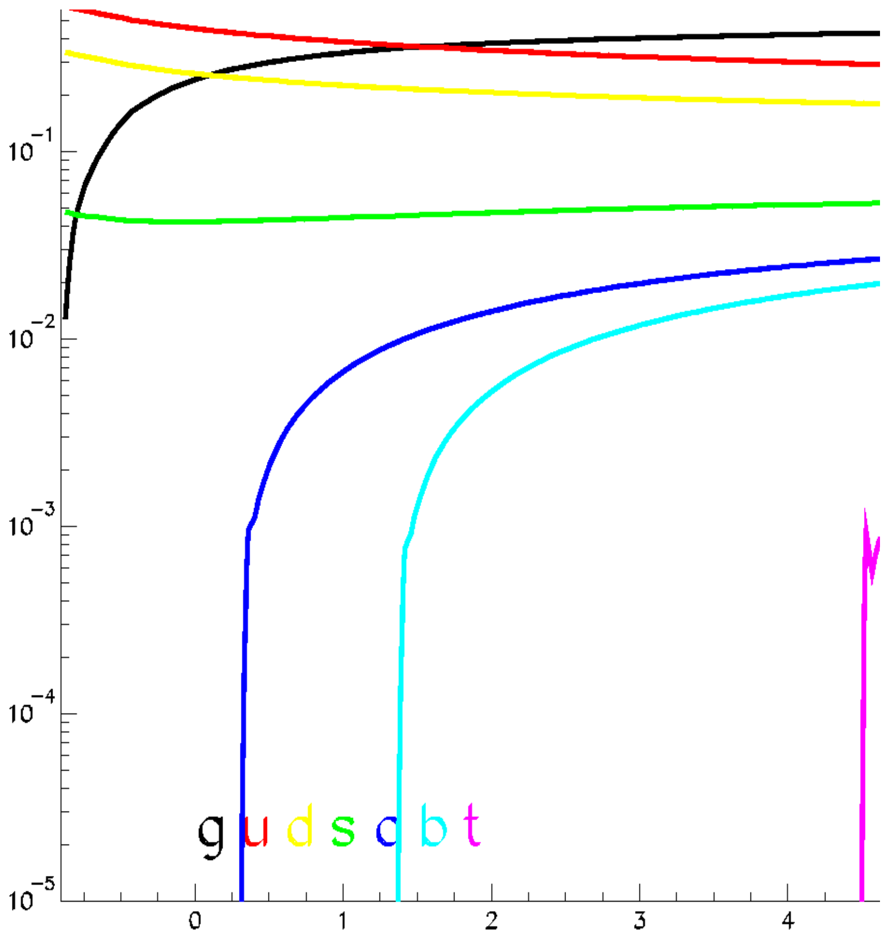
OLDS

Fitted Pdf

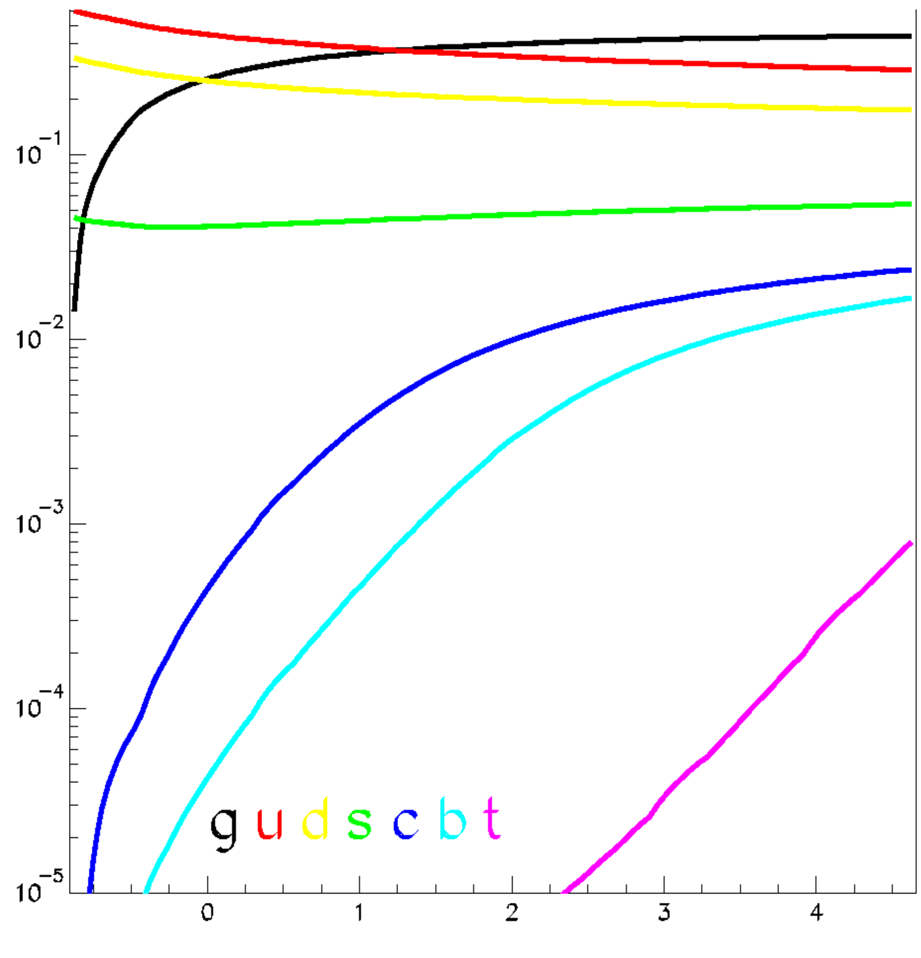


Momentum fraction

massless

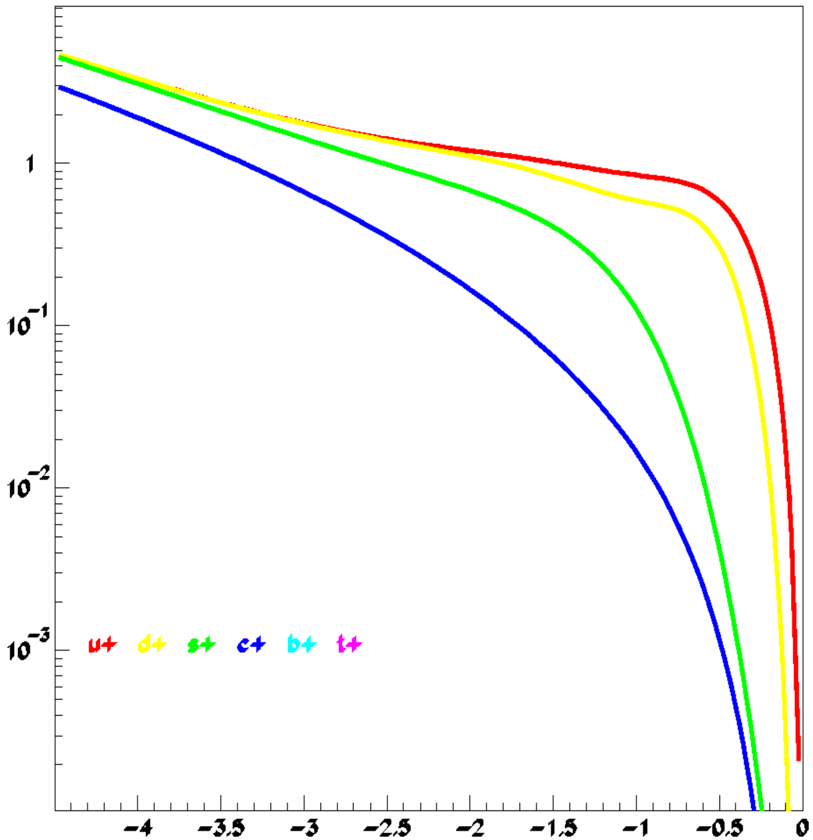


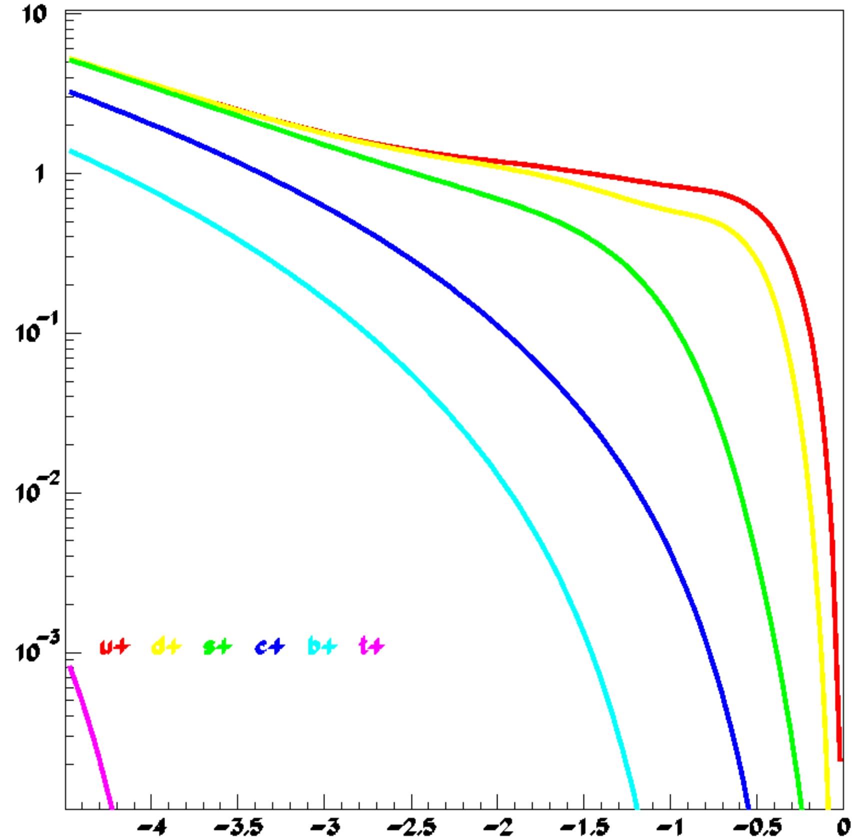
cfns



Beauty mass^2

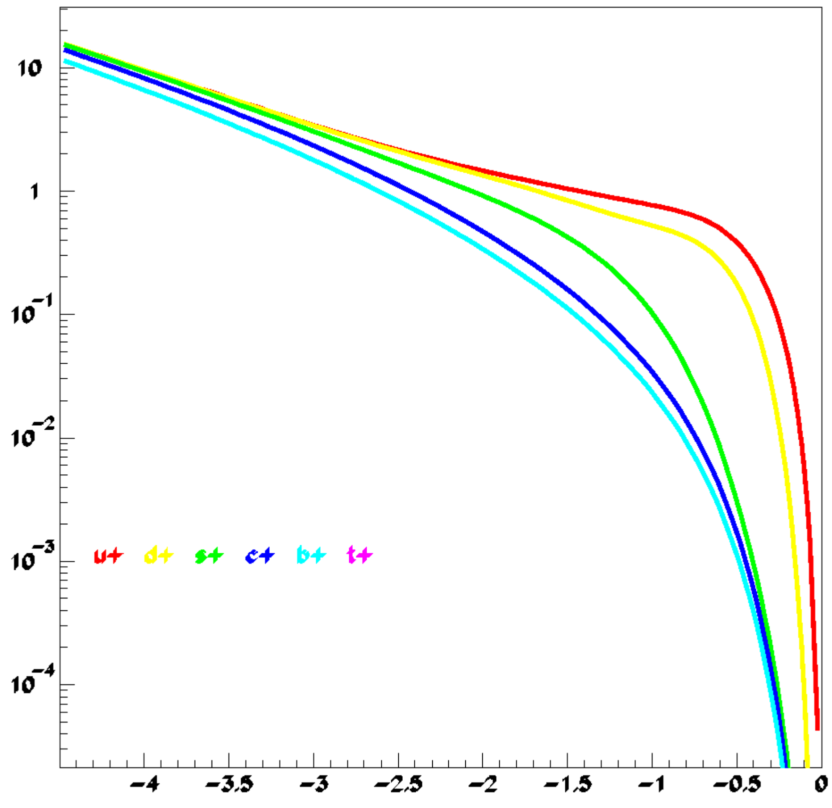
massless | cfns

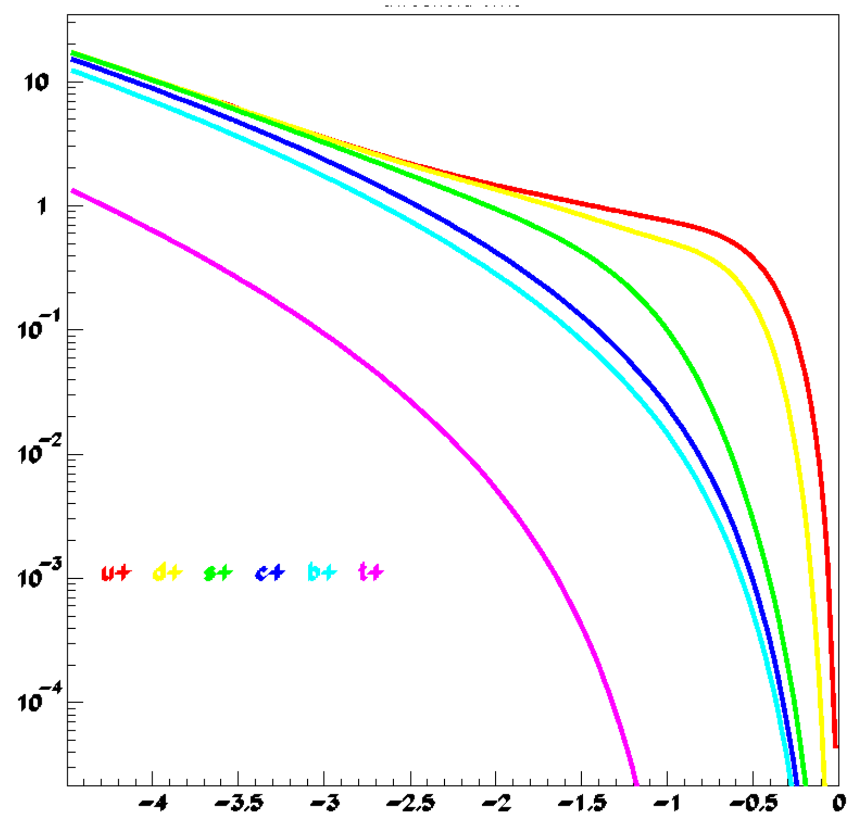




Top mass^2

massless | cfns





Parton distributions are noted by the name of the parton species $p = g,d,\bar{d},u,\bar{u},\ldots$ and for the quarks $d^{\pm} = d \pm \bar{d},\ldots$ are also introduced.

DGLAP equations read :

$$\frac{\partial o(Q^2)}{\partial ln(Q^2)} = \sum_i \mathcal{P}_{oi} \otimes i(Q^2)$$

$\mathcal{P}$ are splitting functions, i and o run on the $1 + 2\mathcal{N}_f$ partons species. $\mathcal{N}_f$ refers to the number of active flavors and is the main problem of the overall approach: Usually $\mathcal{N}_f (= N_f$ integer) is taken as the number of quark families such that $Q^2 > m_o^2$.

DGLAP equations separate into two independent subsystems when making use of q^+ and q^- ,q^+ one is:

$$\frac{\partial g(Q^2)}{\partial ln(Q^2)} = \mathcal{P}_{gg} \otimes g(Q^2) + \sum_{q=d}^{t} \mathcal{P}_{gq} \otimes q^+(Q^2)$$

$$\frac{\partial q^+(Q^2)}{\partial ln(Q^2)} = \tilde{\mathcal{P}}_{qg} \otimes g(Q^2) + \mathcal{P}^+_{NS} \otimes q^+(Q^2) + \sum_{r=d}^{t} \tilde{\mathcal{P}}^+_{S} \otimes r^+(Q^2)$$

Where

$$\mathcal{P}^{\pm}_{NS} = \mathcal{P}^V_{qq} \pm \mathcal{P}^V_{q\bar{q}},\ \mathcal{P}^{\pm}_{S} = \mathcal{P}^S_{qq} \pm \mathcal{P}^S_{q\bar{q}},\ \tilde{\mathcal{P}}_{qg} = \frac{\mathcal{P}_{qg}}{\mathcal{N}_f},\ \tilde{\mathcal{P}}^{\pm}{}_S = \frac{\mathcal{P}^{\pm}_S}{\mathcal{N}_f}.$$

Kernels $\mathcal{P}$ are polynomials in $a_s = \frac{\alpha_s}{4\pi}$ and $\mathcal{N}_f$ as follows:

$$
\begin{array}{lllllll}
\mathcal{P}_{gg} = a_s(\mathcal{P}_{gg}^{00} & +\mathcal{N}_f\mathcal{P}_{gg}^{01}) & +a_s^2(\mathcal{P}_{gg}^{10} & +\mathcal{N}_f\mathcal{P}_{gg}^{11}) & +a_s^3(\mathcal{P}_{gg}^{20} & +\mathcal{N}_f\mathcal{P}_{gg}^{21} & +{\mathcal{N}_f}^2\mathcal{P}_{gg}^{22}\ ) \\
\mathcal{P}_{qg} = a_s & \mathcal{N}_f\mathcal{P}_{qg}^{01} & +a_s^2 & \mathcal{N}_f\mathcal{P}_{qg}^{11} & +a_s^3 & (\mathcal{N}_f\mathcal{P}_{qg}^{21} & +{\mathcal{N}_f}^2\mathcal{P}_{qg}^{22}\ ) \\
\mathcal{P}_{gq} = a_s\mathcal{P}_{gq}^{00} & & +a_s^2(\mathcal{P}_{gq}^{10} & +\mathcal{N}_f\mathcal{P}_{gq}^{11}) & +a_s^3(\mathcal{P}_{gq}^{20} & +\mathcal{N}_f\mathcal{P}_{gq}^{21} & +{\mathcal{N}_f}^2\mathcal{P}_{gq}^{22}\ ) \\
 & & & & & & \\
\mathcal{P}_{qq}^{V} = a_s\mathcal{P}_{qq}^{V00} & & +a_s^2(\mathcal{P}_{qq}^{V10} & +\mathcal{N}_f\mathcal{P}_{qq}^{V11}) & +a_s^3(\mathcal{P}_{qq}^{V20} & +\mathcal{N}_f\mathcal{P}_{qq}^{V21} & +{\mathcal{N}_f}^2\mathcal{P}_{qq}^{V22}) \\
\mathcal{P}_{q\bar{q}}^{V} = & & a_s^2(\mathcal{P}_{q\bar{q}}^{V10} & +\mathcal{N}_f\mathcal{P}_{q\bar{q}}^{V11}) & +a_s^3(\mathcal{P}_{q\bar{q}}^{V20} & +\mathcal{N}_f\mathcal{P}_{q\bar{q}}^{V21} & +{\mathcal{N}_f}^2\mathcal{P}_{q\bar{q}}^{V22}) \\
\mathcal{P}_{S}^{+} = & & a_s^2(& \mathcal{N}_f\mathcal{P}_{S}^{+11}) & +a_s^3(& \mathcal{N}_f\mathcal{P}_{S}^{+21} & +{\mathcal{N}_f}^2\mathcal{P}_{S}^{+22}) \\
\mathcal{P}_{S}^{-} = & & & & a_s^3(& \mathcal{N}_f\mathcal{P}_{S}^{-21} & +{\mathcal{N}_f}^2\mathcal{P}_{S}^{-22})
\end{array}
$$

$\mathcal{P}_{gg}^{01}$ **existence comes from the** β_0 **term in** $\mathcal{P}_{gg}$.

Momentum sum rule

Sumrules

The total parton momentum is:
$\int_0^1 \Pi x dx = 1$ **with** $\Pi = g + \sum_{q=d}^{t} q^+$
This imply that derivative of $\int_0^1 \Pi x dx$ **with respect of** $ln(Q^2)$ **is null for any set of partons.**
From that derives easily the following set of properties of the kernel integrals which will be noted $\mathcal{Q} = \int_0^1 \mathcal{P} x dx$

$$\mathcal{Q}_{gg} + \mathcal{Q}_{qg} = 0$$
$$\mathcal{Q}_{gq} + \mathcal{Q}_{NS}^{+} + \mathcal{N}_f \mathcal{Q}_{S}^{+} = 0$$

Which has to be valid for any value of a_s **and** $\mathcal{N}_f$

Modifying DGLAP equations

The idea is to modify the kernels in order to satisfy simultanously the three kinematical constraints $x_o < l_o$,$x_i < l_i$, $x_o < x_i$

$\mathcal{P}_{oi}$ is the change to outcoming parton o at Bjorken x radiated by incoming parton i at Bjorken $\frac{x}{t}$. Problematic cases are when parton o is heavier than parton i like for $c \to b$ Replacing $\mathcal{P}$ by $\mathcal{K}$, the problematic changing terms are:

$$\int_x^1 \mathcal{K}_{oi}(\frac{x}{z}) i(z) \frac{dz}{z}$$

Requesting this term to be null for $x \geq l_0$ means that $\mathcal{K}(u) = 0$ for $u \geq l_o$ which is satisfied by

$$\mathcal{K}_{oi} = \mathcal{P}_{oi} \otimes \delta_{oi}$$

With the definition (for $l_o \leq 1$)

$$\delta_{oi} = \phi_{oi} \delta(x - l_o)$$

The only constraint so far on ϕ is that it goes to 0 for $Q^2 \to 0$ and to 1 for $Q^2 \to \infty$.

Note that the effect of the δ function is to replace $\mathcal{P}(x)$ by $\mathcal{P}(\xi)$ with $\xi = \frac{x}{l_o}$

With this modification the q^+ subsystem becomes:

$$\frac{\partial g(Q^2)}{\partial ln(Q^2)} = \mathcal{P}_{gg} \otimes \delta_{gg} \otimes g(Q^2) + \sum_{q=d}^{t} \mathcal{P}_{gq} \otimes \delta_{gq} \otimes q^+(Q^2)$$

$$\frac{\partial q^+(Q^2)}{\partial ln(Q^2)} = \tilde{\mathcal{P}}_{qg} \otimes \delta_{qg} \otimes g(Q^2) + \mathcal{P}^+_{NS} \otimes \delta^{NS}_{qq} \otimes q^+(Q^2) + \sum_{r=d}^{t} \tilde{\mathcal{P}}^+_{S} \otimes \delta_{qr} \otimes r^+(Q^2)$$

With the help of $\int_0^1 x\delta(x-l)dx = l$ and the use of Q relations given by the usual DGLAP, the momentum sumrule determines completly the modifications to do for heavy quark h:
$\delta_{hr} = l_h \delta(x - l_h)$ **for any** $r \in u, ..., t$
and also $N_f = \sum_{q=d}^{t} l_q$
All the others δ_{oi} do not need to exist.

Note that for $l_q \to 0$ the corresponding DGLAP equation will get decoupled and the kinematical constraint automatically verified.
In fact other solutions may be found implying correlated changes in the δ_{oi}.
An example will be given in the last part of this talk.

Modifying α_s and coefficients functions

As seen above the momentum sumrule leads to a specific non integer value of $\mathcal{N}_f$ and as a consequence also for β_0 and by extension to the full set of β governing the α_s running. It is also natural that the copling constant depends on flavor activity and not only on flavor number.
Anyhow α_s and parton evolution are linked by renormalisation group theory.

As it is the structure functions and not the parton distributions which are observable one has to find also a procedure to modify the coefficient functions.

The change parton distribution $\rightarrow$ structure function has exactly the same structure that the one of DGLAP equations:

$$\frac{\partial o(Q^2)}{\partial ln(Q^2)} \rightarrow F_o$$

$$\mathcal{P} \rightarrow \mathcal{C}$$

This change is in fact nothing more than a change of scheme, an example is going from $\bar{M}S$ to DIS for F_2.
For F_1 and F_3 the schemes are unnamed but they still exist.

From this one may infer that coefficient functions have to be modified in the same way that splitting functions.
In fact it is even the importance of kinematic constraints stressed by one of the R.Thorne papers and its use in the lastest schemes which lead me to do this work.

Charged current
Exactly the same procedure will be used, the phase space only will change using:

$$l_o^{-1} = 1 + \frac{m_o^2}{Q^2}$$

System decoupling

$$\Sigma_L = \sum_{q=d}^{s} q \qquad l_{LN} = l - \frac{\Sigma_L}{3}$$

$$\phi_q = l_q \delta(x - l_q) \qquad \phi = \sum_{q=d}^{t} \phi_q$$

Subscript LN is used do distinguich these non singlets from the usual ones which may not be used here with flavor number varying continously. With these one get the following subsystem:

$$\begin{aligned}
\frac{\partial g(Q^2)}{\partial ln(Q^2)} &= \mathcal{P}_{gg} \otimes g(Q^2) + \mathcal{P}_{gq} \otimes \Sigma(Q^2) \\
\frac{\partial \Sigma(Q^2)}{\partial ln(Q^2)} &= \phi \otimes \bar{\mathcal{P}}_{qg} \otimes g(Q^2) + (\mathcal{P}_{NS} + \phi \otimes \bar{\mathcal{P}}_S) \otimes \Sigma(Q^2) \\
\frac{\partial l_{LN}(Q^2)}{\partial ln(Q^2)} &= \mathcal{P}_{NS} \otimes l_{LN}(Q^2) \\
\frac{\partial h(Q^2)}{\partial ln(Q^2)} &= \phi_h \otimes \bar{\mathcal{P}}_{qg} \otimes g(Q^2) + \mathcal{P}_{NS} \otimes h(Q^2) + \phi_h \otimes \bar{\mathcal{P}}_S \otimes \Sigma(Q^2)
\end{aligned}$$

Evolving the seven distributions g,Σ,c,b,t,d,u the full system may be recovered using:

$$\Sigma_L = \Sigma - c - b - t \qquad d_{LN} + u_{LN} + s_{LN} = 0$$

Notice that this formulation is not unique: any linear combination of the seven equations with coefficients independent Q^2 could be used.

System decoupling

$\Sigma_L = \sum_{q=d}^{s} q$ $\quad$ $l_{LN} = l - \frac{\Sigma_L}{3}$

$\phi_q = l_q \delta(x - l_q)$ $\quad$ $\phi = \sum_{q=d}^{t} \phi_q$

Subscript LN is used do distinguich these non singlets from the usual ones which may not be used here with flavor number varying continously. With these one get the following subsystem:

$$
\begin{aligned}
\frac{\partial g(Q^2)}{\partial ln(Q^2)} &= \mathcal{P}_{gg} \otimes g(Q^2) + \mathcal{P}_{gq} \otimes \Sigma(Q^2) \\
\frac{\partial \Sigma(Q^2)}{\partial ln(Q^2)} &= \phi \otimes \bar{\mathcal{P}}_{qg} \otimes g(Q^2) + \left(\mathcal{P}_{NS} + \phi \otimes \bar{\mathcal{P}}_{S}\right) \otimes \Sigma(Q^2) \\
\frac{\partial l_{LN}(Q^2)}{\partial ln(Q^2)} &= \mathcal{P}_{NS} \otimes l_{LN}(Q^2) \\
\frac{\partial h(Q^2)}{\partial ln(Q^2)} &= \phi_h \otimes \bar{\mathcal{P}}_{qg} \otimes g(Q^2) + \mathcal{P}_{NS} \otimes h(Q^2) + \phi_h \otimes \bar{\mathcal{P}}_{S} \otimes \Sigma(Q^2)
\end{aligned}
$$

Evolving the seven pdfs g,Σ,c,b,t,d_{LN},u_{LN} the full system may be recovered using:

$\Sigma_L = \Sigma - c - b - t$ $\quad$ $d_{LN} + u_{LN} + s_{LN} = 0$

Notice that this formulation is not unique: any linear combination of the seven equations with coefficients independent Q^2 could be used.

- My first concern was also to check as soon as possible my ideas by building a transport matrix integration valid at the same time for CFNS and for constant flavor evolution

Foreword

- S as scheme: not really a scheme as vfns,acot …
- FN as flavor number: not really this number having value 3,4,5,6 changing at transitions Q^2 or at physicist will but a function of Q^2
- C: as continous

Parton distributions, functions of x and Q^2 ,are noted by the name of the parton species $p=g,d,\bar{d},u,\bar{u},s,\bar{s},c,\bar{c},b,\bar{b},t,\bar{t}$ … and for the quarks $d^{\pm} = d \pm \bar{d}$, …

DGLAP equations are:

$\partial o/\partial \ln(Q^2) = \sum_i P_{io} \otimes I$

$\otimes$ stands for the convolution integral

P are splitting kernels also functions of x and Q^2