Jet Shape in Hadron Collisions

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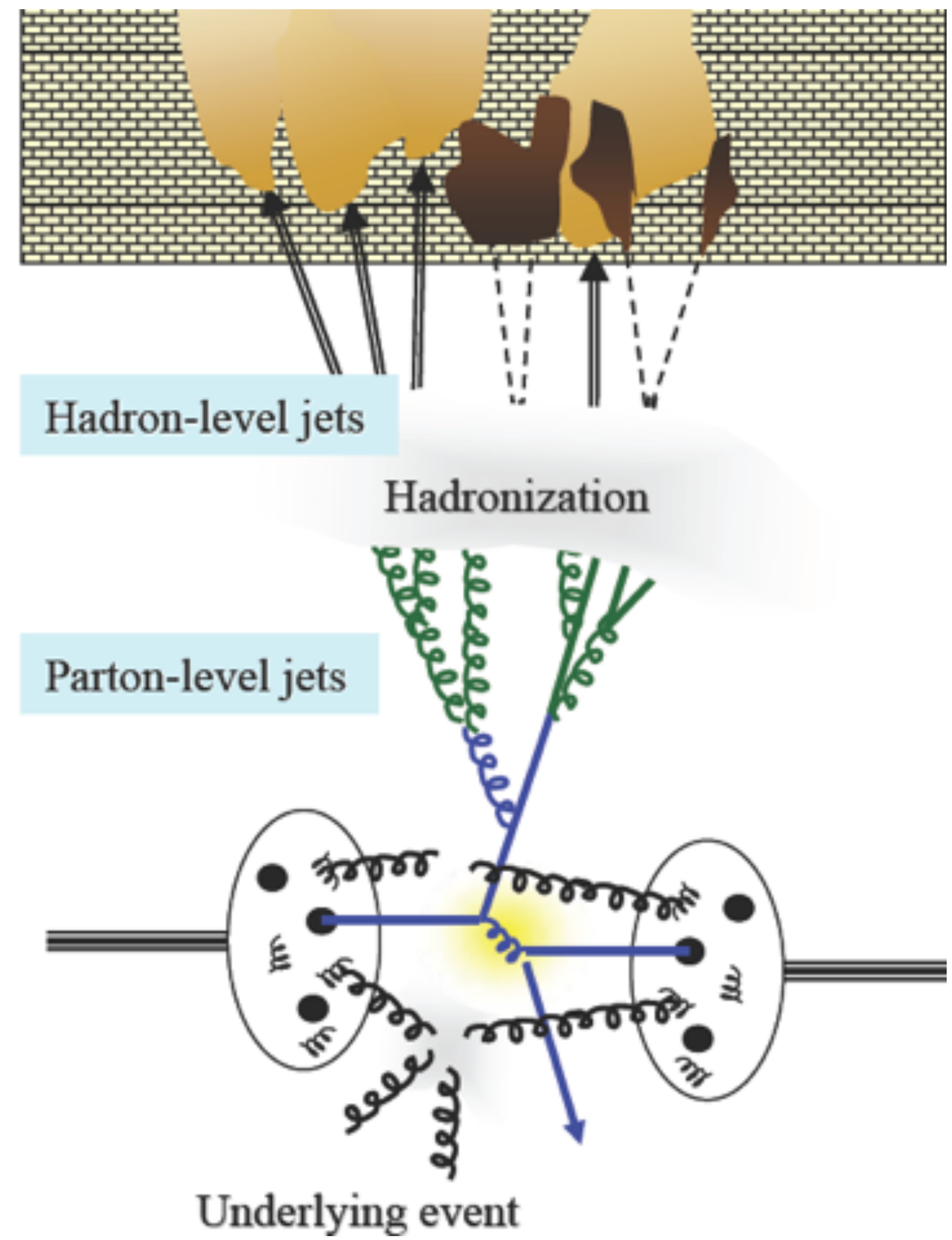
Outline

- **Motivation**
- **Jet function factorization**
- **Resummation**
- **Jet mass distribution**
- **Jet energy profile**
- **Summary**

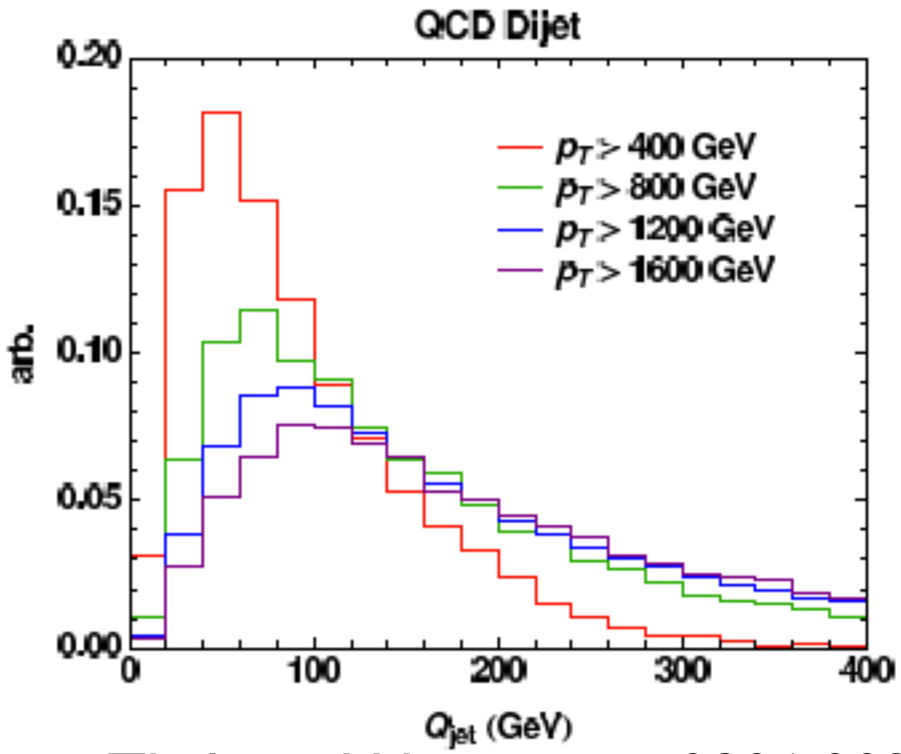
Motivation

- Copious production of jets at Hadron colliders.
- Boosted top quark (or New Physics particles) could produce single jet signal at the Large Hadron Collider (LHC).
- To discriminate QCD jets from boosted top jet (or New Physics) signal, we

need to study jet substructure

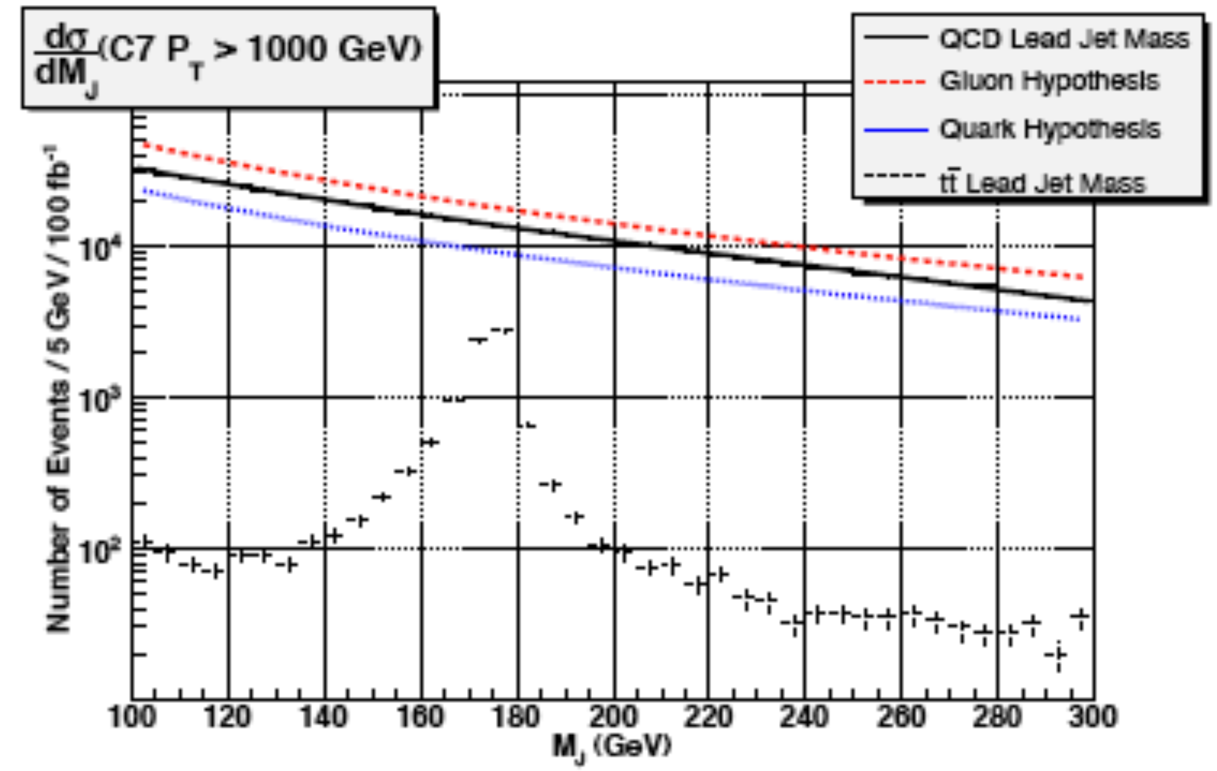


Jet substructure observables

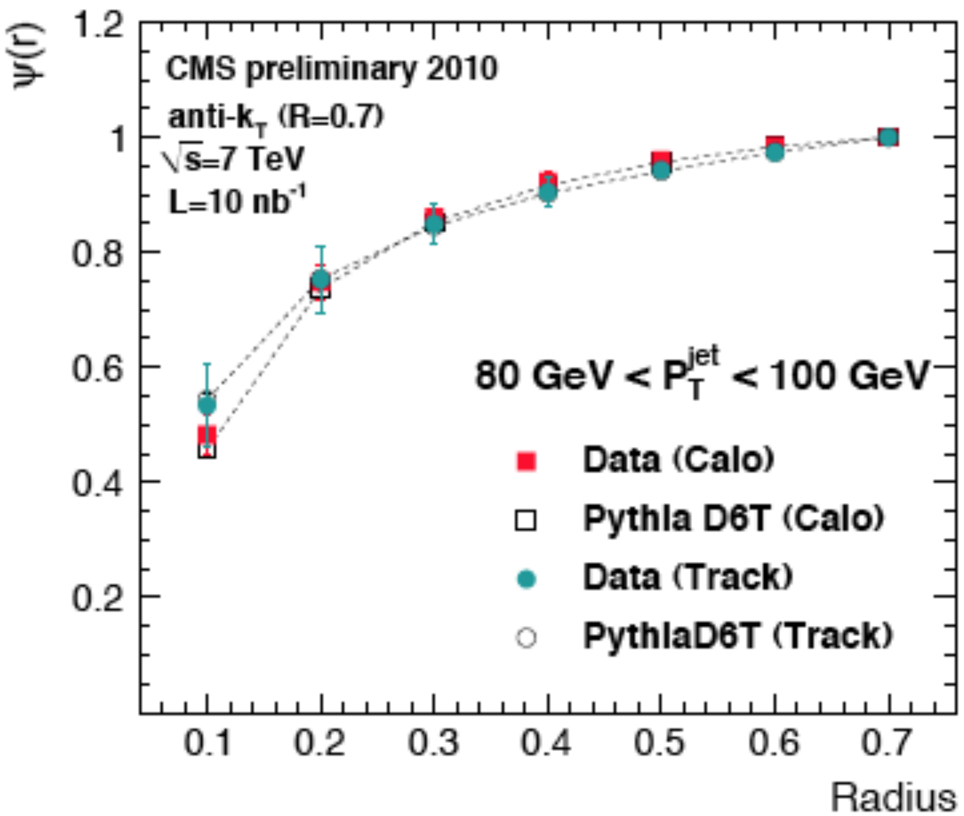


Thaler & Wang, arxiv:0806.0023

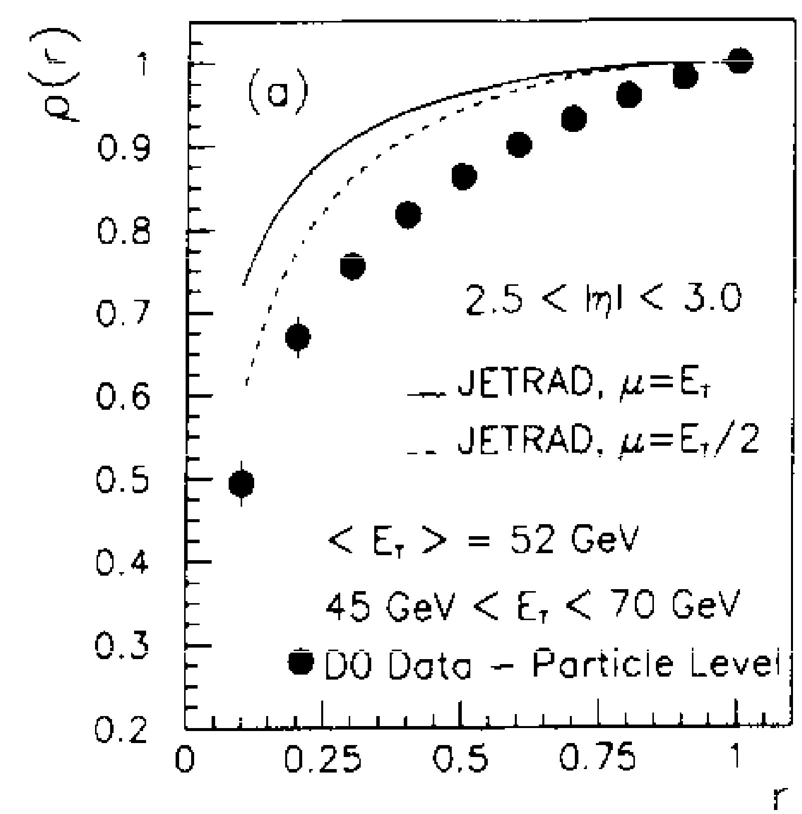
jet mass



Almeida, arxiv:0810.0934



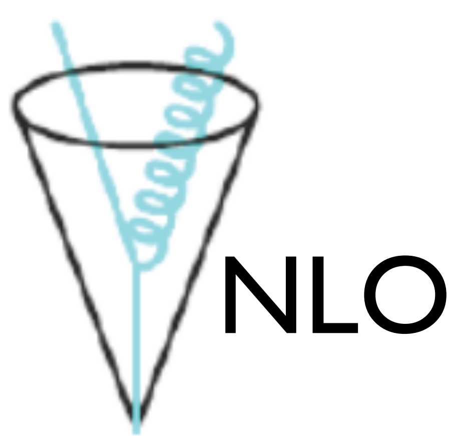
jet energy profile



DØ Collaboration / Physics Letters B 357 (1995) 500–508

Various Theoretical Predictions

- **Event Generators**: leading log radiations, hadronization, underlying events, etc.
- **Fixed order QCD calculation**: finite number of soft/collinear radiations
- **Resummation**: all order soft/collinear radiations



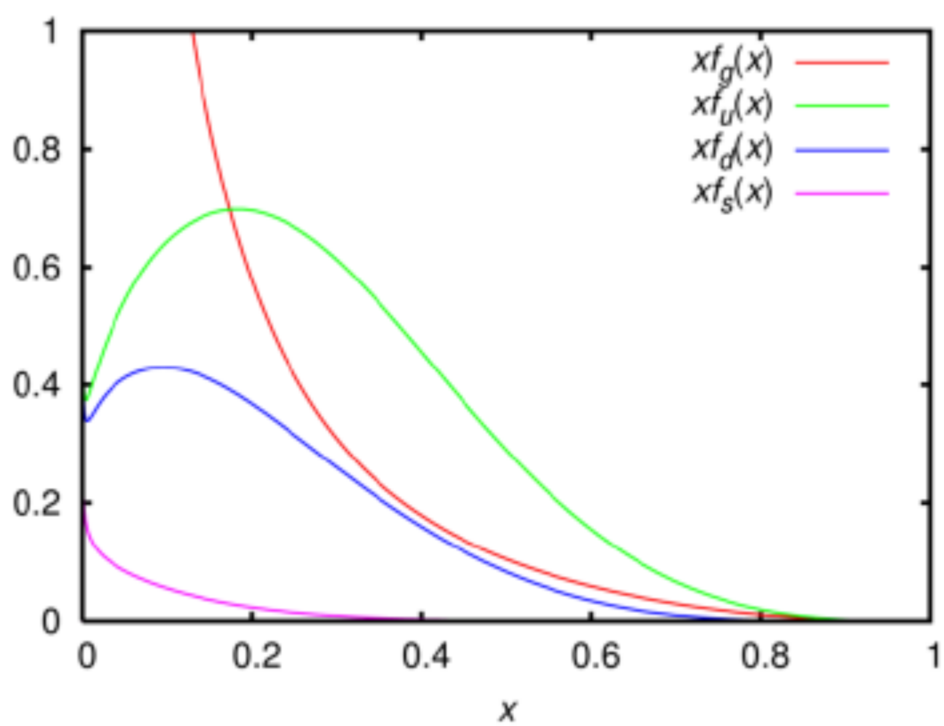
Factorization Theorem

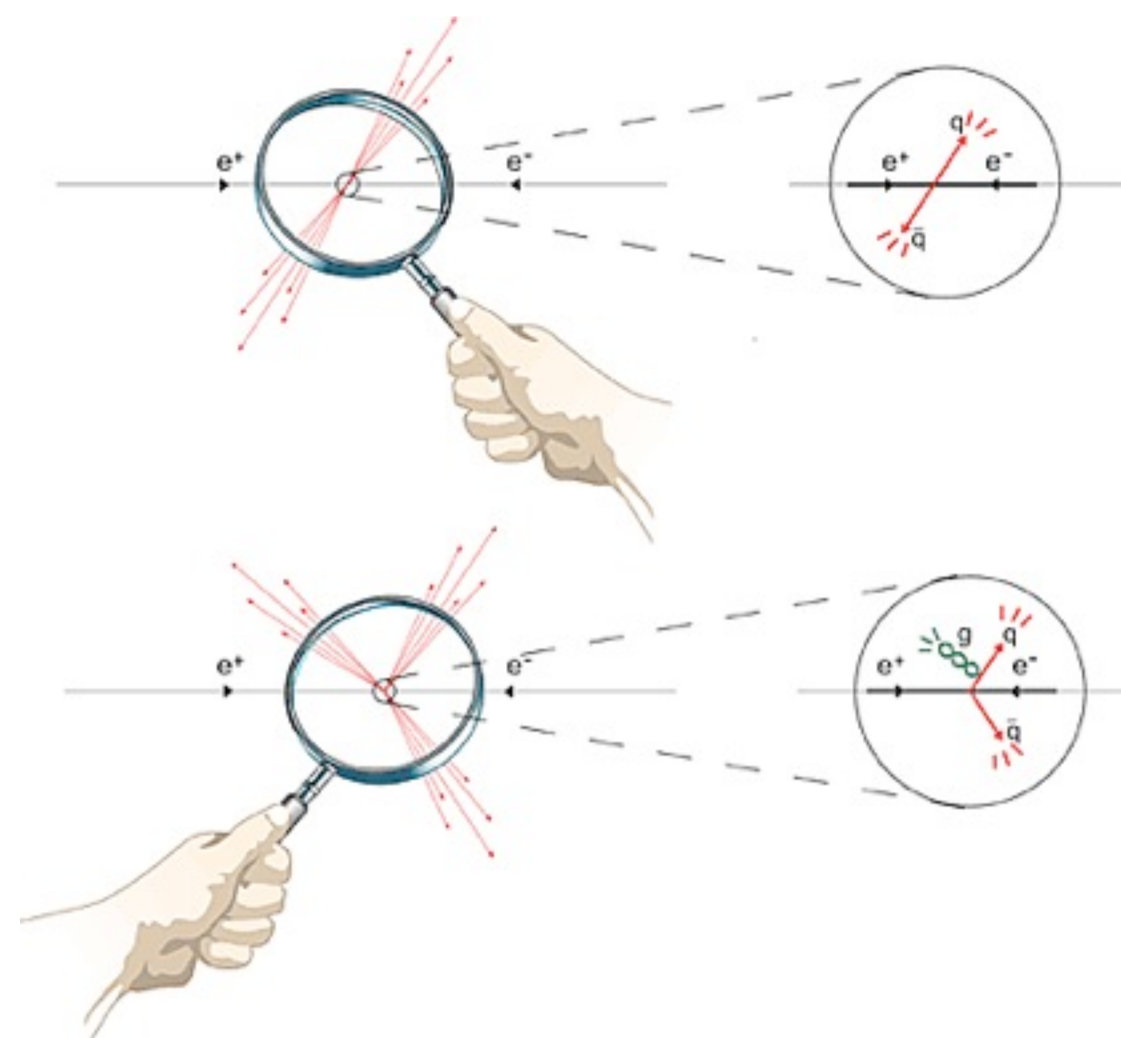
$$\sigma_{hh'} = \sum_{i,j} \int_0^1 dx_1 dx_2\, \phi_{i/h}(x, Q^2)\, H_{ij}\left(\frac{Q^2}{x_1 x_2 S}\right) \phi_{j/h'}(x_2, Q^2)$$

Nonperturbative, but universal, hence measurable

Infrared safe (IRS), calculable in pQCD

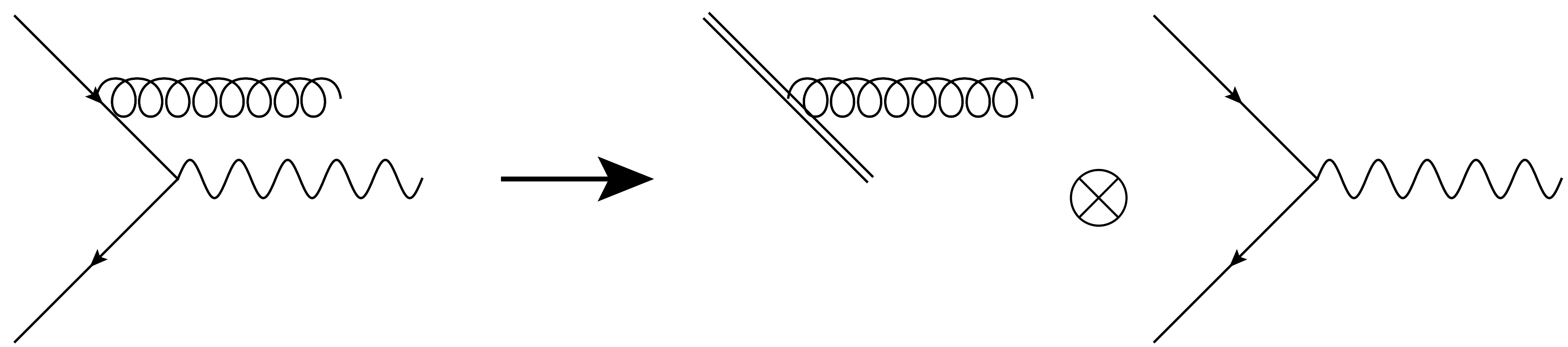
CTEQ
MSTW
NNPDF





Eikonalization

Soft/collinear radiations can be detached by eikonalization



Eikonal vertices and eikonal propagators of soft/collinear radiations can be factorized and combined into Wilson line

$$\Phi_{\xi}^{(f)}(\infty, 0; 0) = \mathcal{P}\left\{e^{-ig\int_0^\infty d\eta\, \xi\cdot A^{(f)}(\eta\, \xi^\mu)}\right\},$$

Jet Function

LO Jet: $J_i^{(0)}(m^2_{J_i}, p_{0,J_i}, R) = \delta(m^2_{J_i}).$

Quark Jet:

$$
\begin{aligned}
J_i^q(m_J^2, p_{0,J_i}, R) \;&=\; \frac{(2\pi)^3}{2\sqrt{2}\,(p_{0,J_i})^2}\frac{\xi_\mu}{N_c}\sum_{N_{J_i}} \mathrm{Tr}\left\{\gamma^\mu \langle 0|q(0)\Phi_\xi^{(\bar q)\dagger}(\infty,0)|N_{J_i}\rangle\langle N_{J_i}|\Phi_\xi^{(\bar q)}(\infty,0)\bar q(0)|0\rangle\right\} \\
&\times\delta\left(m_J^2 - \tilde m_J^2(N_{J_i},R)\right)\delta^{(2)}(\hat n - \tilde n(N_{J_i}))\delta(p_{0,J_i} - \omega(N_{J_c})), \qquad \text{(A.3)}
\end{aligned}
$$

Gluon Jet:

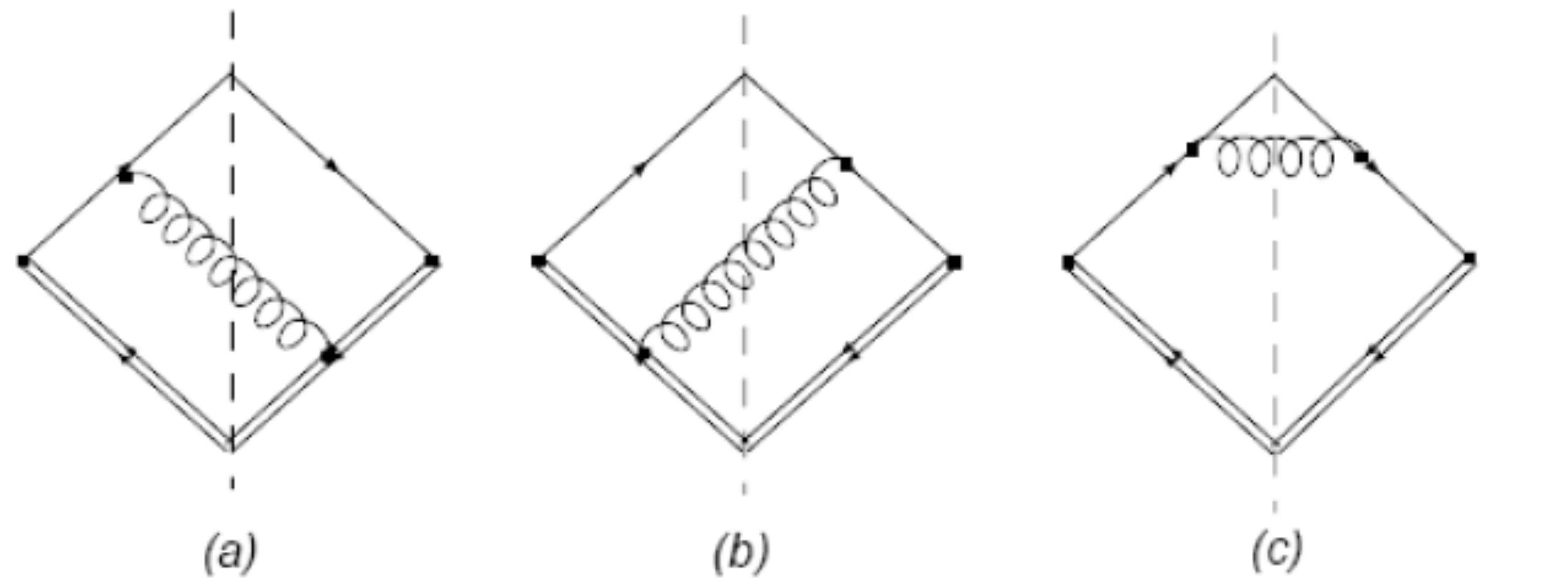
$$
\begin{aligned}
J_i^g(m_J^2, p_{0,J_i}, R) \;&=\; \frac{(2\pi)^3}{2(p_{0,J_i})^3}\sum_{N_{J_i}} \langle 0|\xi_\sigma F^{\sigma\nu}(0)\Phi_\xi^{(g)\dagger}(0,\infty)|N_{J_i}\rangle\langle N_{J_i}|\Phi_\xi^{(g)}(0,\infty)F_\nu{}^\rho(0)\xi_\rho|0\rangle \\
&\times\delta\left(m_J^2 - \tilde m_J^2(N_{J_i},R)\right)\delta^{(2)}(\hat n - \tilde n(N_{J_i}))\delta(p_{0,J_i} - \omega(N_{J_c})). \qquad \text{(A.4)}
\end{aligned}
$$

$$
\frac{d\sigma}{dP_T dM_J} = \sum_c 2M_J J^c(M_J, P_T, R)\frac{d\sigma^c}{dP_T}
$$

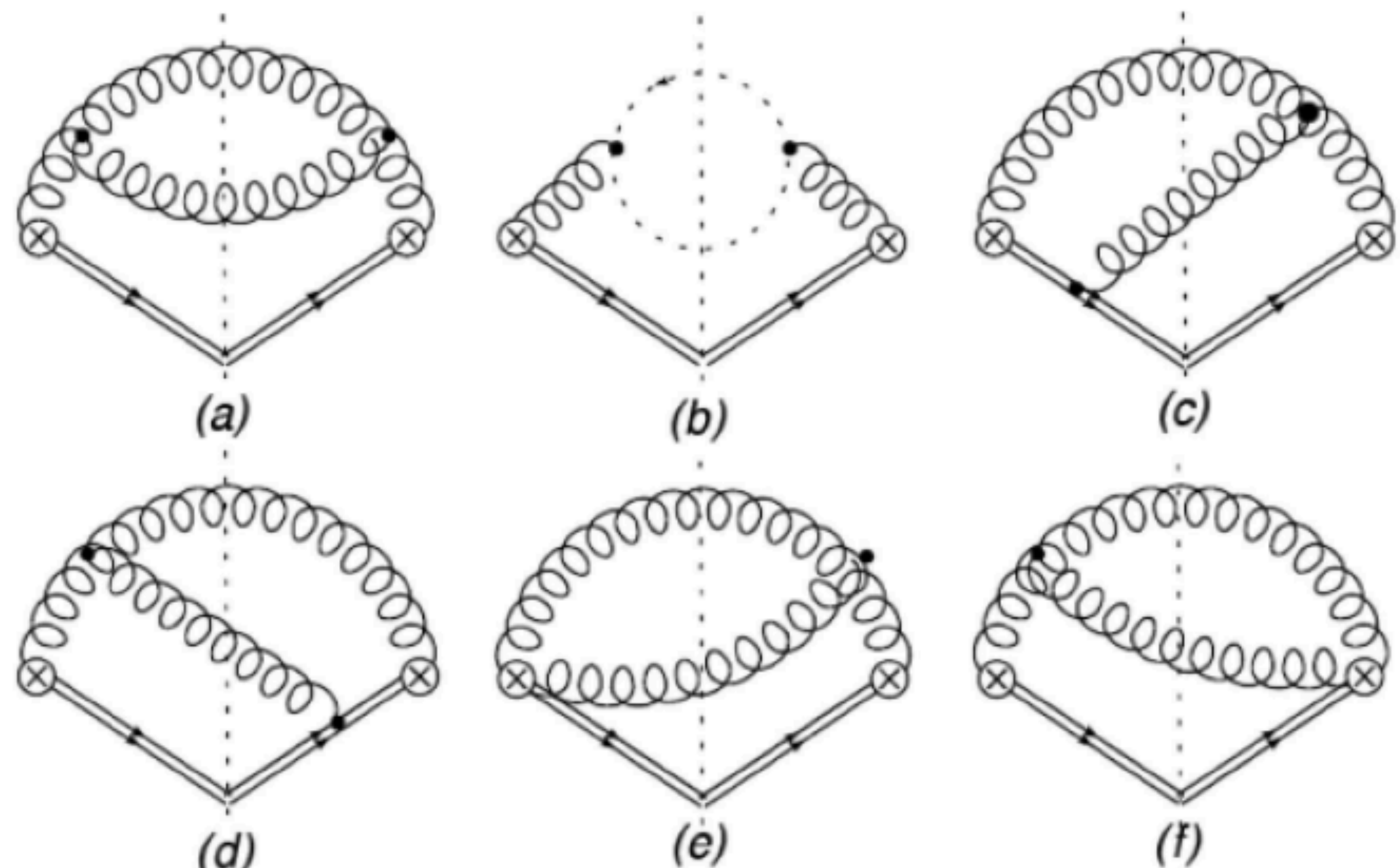
Almeida, arxiv:0810.0934

Diagrams for NLO calculations

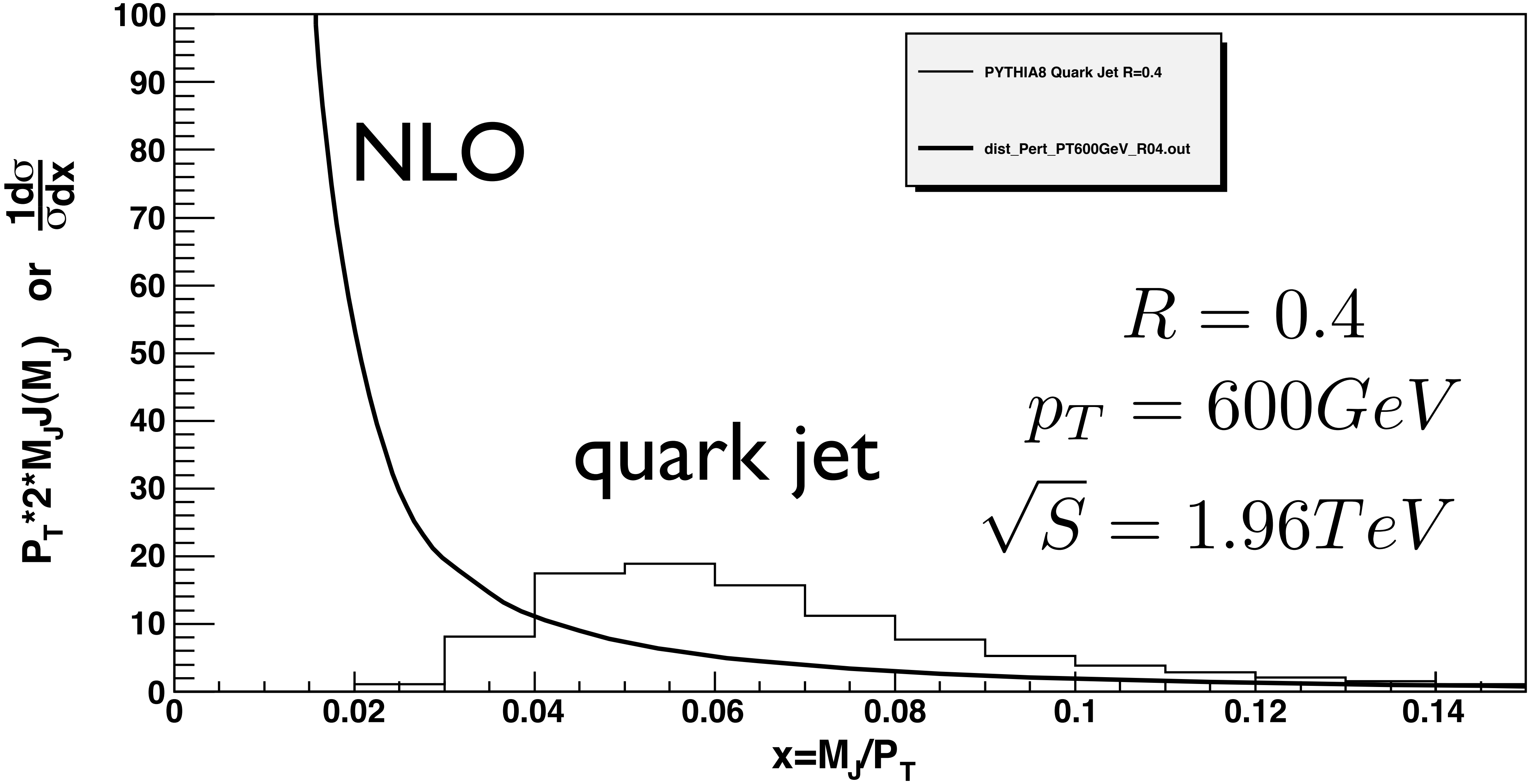
NLO Quark:



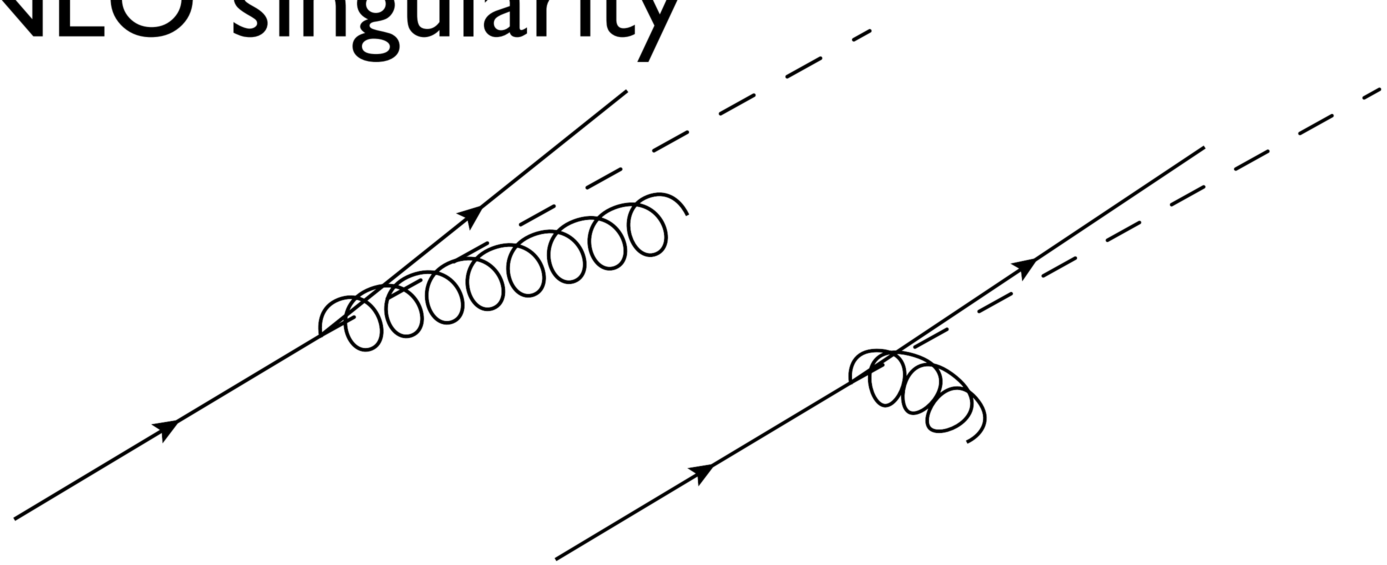
NLO Gluon:



Fixed order prediction has singularity at MJ->0



NLO singularity



anti-kT algorithm

$$D_i = P_{Ti}^{-2}$$

$$D_{ij} = min(P_{Ti}^{-2}, P_{Tj}^{-2})\frac{\Delta R_{ij}^2}{R^2}$$

$$d = min(\{D_{ij}, D_i\})$$

Resummation for Jet function (jet mass distribution)

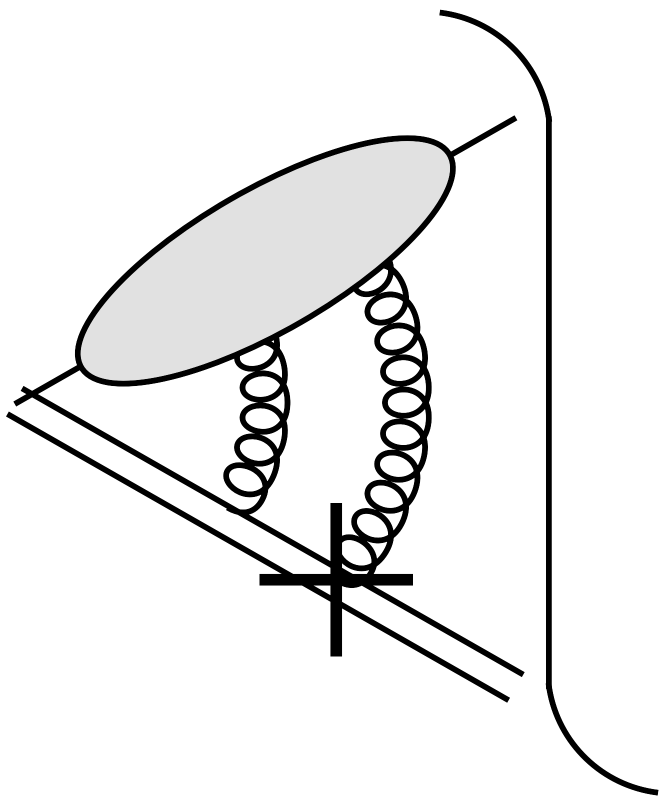
In fixed order calculations, there are large logarithmic terms of the ratio of jet pT to jet mass,

$$\ln\left(\frac{M_J}{RP_T}\right)$$

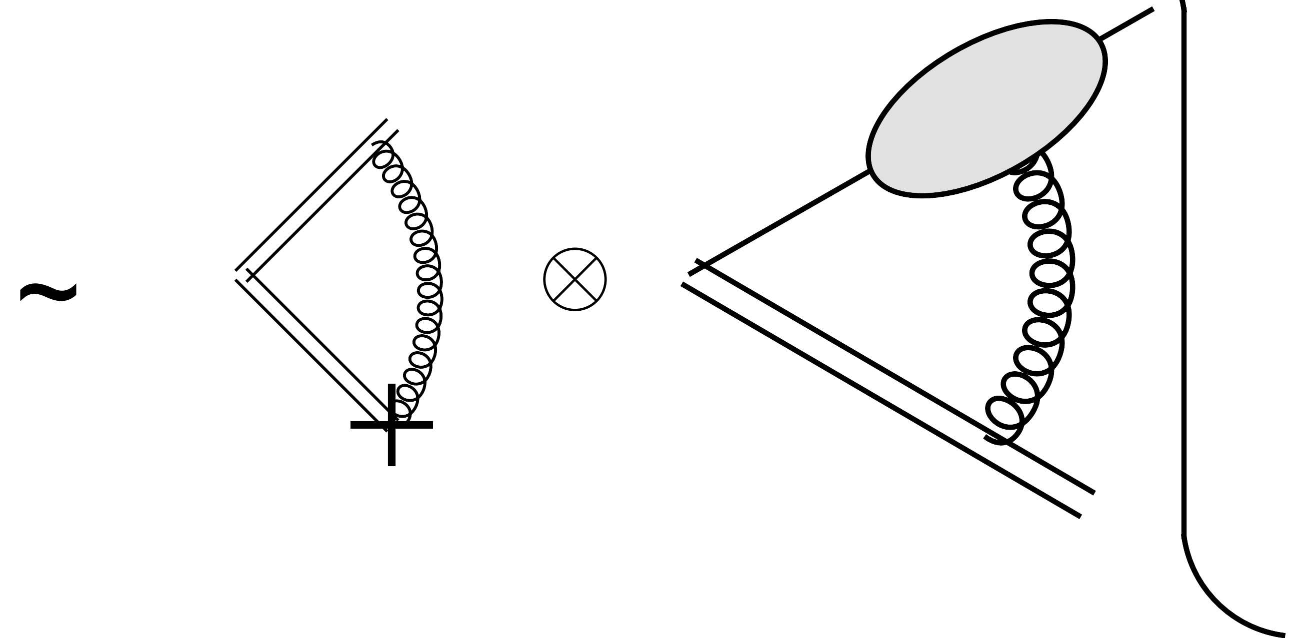
which can be resummed by applying renormalization group (RG) technique.

The key idea of resummation technique is to vary Wilson line direction to arbitrary gauge vector n, since collinear dynamics is independent of n.

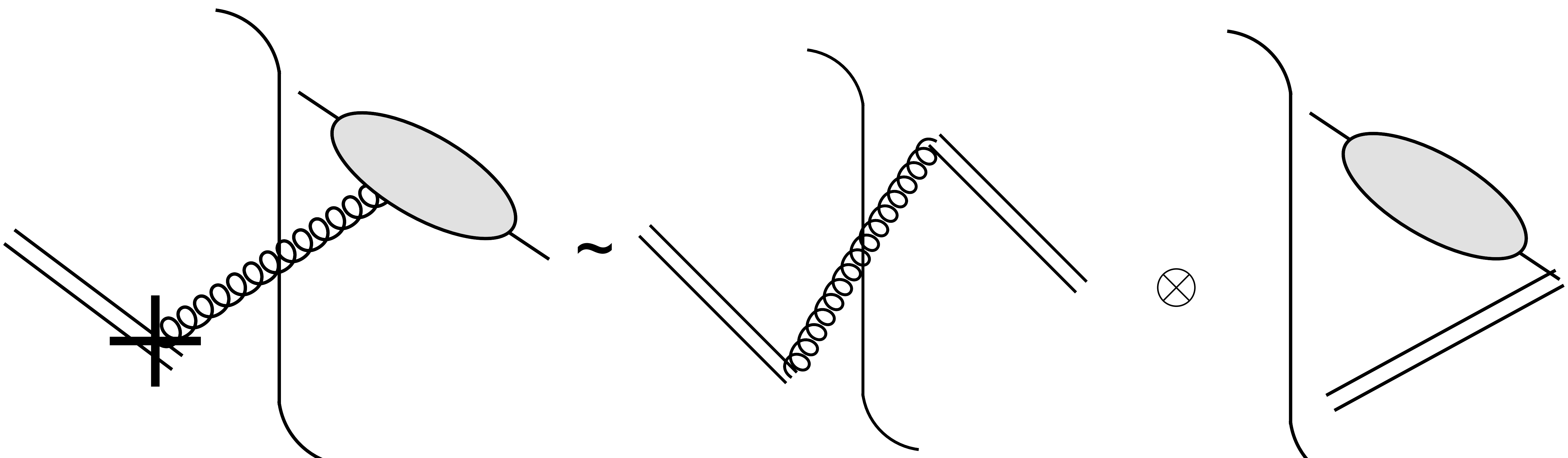
Soft Gluon Factorization



LO soft kernel $K_v^{(1)}$

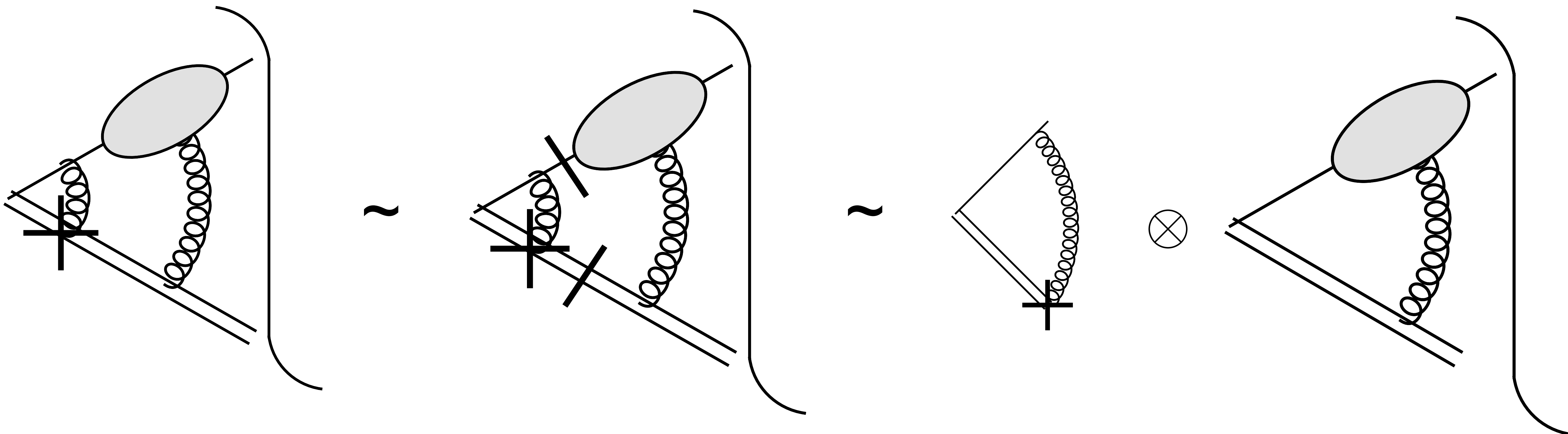


LO soft kernel $K_r^{(1)}$



RG Equation for Jet function

LO hard kernel $G^{(1)}$



Up to leading logarithms, RG equation is

$$-\frac{n^2}{P_J \cdot n} P_J^\alpha \frac{d}{dn^\alpha} J = [G^{(1)} + K_v^{(1)} + K_r^{(1)}] \otimes J$$

To include next-to-leading-log contribution, G and K are evaluated to two loops.

Mellin transform converts convolution to multiplication

$$\bar{J}_q(N, P_T, \nu^2, R, \mu^2) \equiv \int_0^1 dx (1-x)^{N-1} J_q(x, P_T, \nu^2, R, \mu^2),$$

$$x \equiv m_J^2/(RP_T)^2.$$

$$-\frac{n^2}{v \cdot n} v_\alpha \frac{d}{dn_\alpha} \bar{J}_q(N, P_T, \nu^2, R, \mu^2) = 2\nu^2 \frac{d}{d\nu^2} \bar{J}_q(N, P_T, \nu^2, R, \mu^2)$$

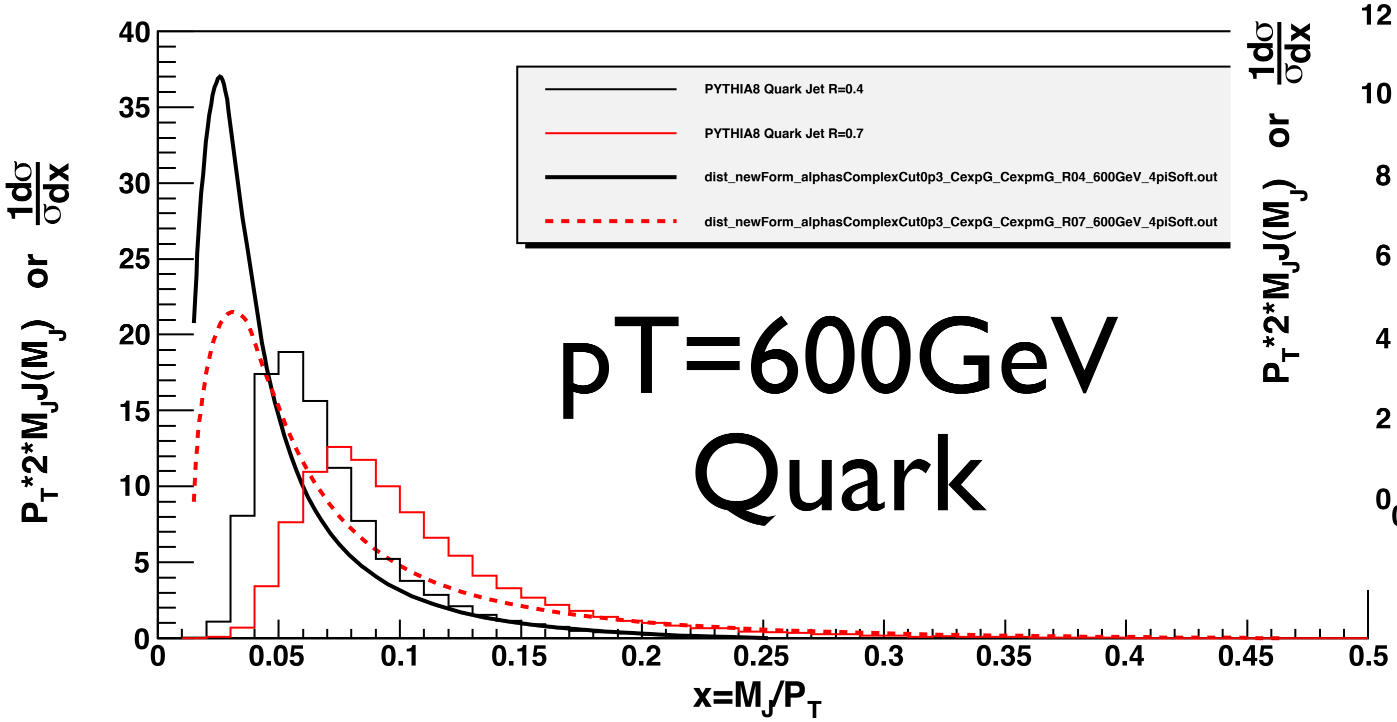
$$= 2 \left[K\left(\frac{RP_T C_1}{\bar{N}\mu}, \alpha_s(\mu^2) \right) + G\left(\frac{C_2 \nu^2 RP_T}{\mu}, \alpha_s(\mu^2) \right) \right] \bar{J}_q(N, P_T, \nu^2, R)$$

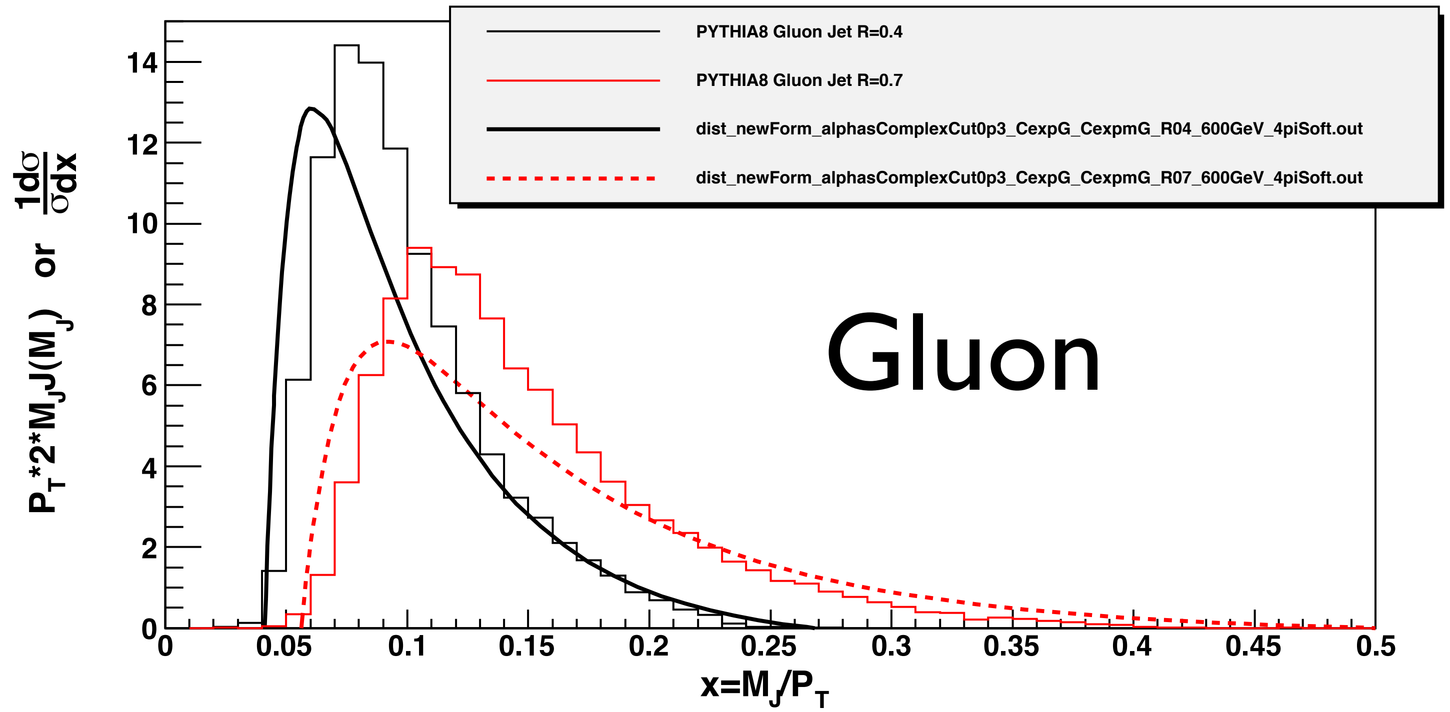
Jet function (in Mellin space), including resummation effect:

$$\bar{J}_q(N, P_T, \nu_{\rm fi}^2, R, \mu^2) \ = \ \bar{J}_q(N, P_T, \nu_{\rm in}^2, R, \mu^2) \exp[S(N)]$$

Jet mass distribution

PYTHIA8+SpartyJet





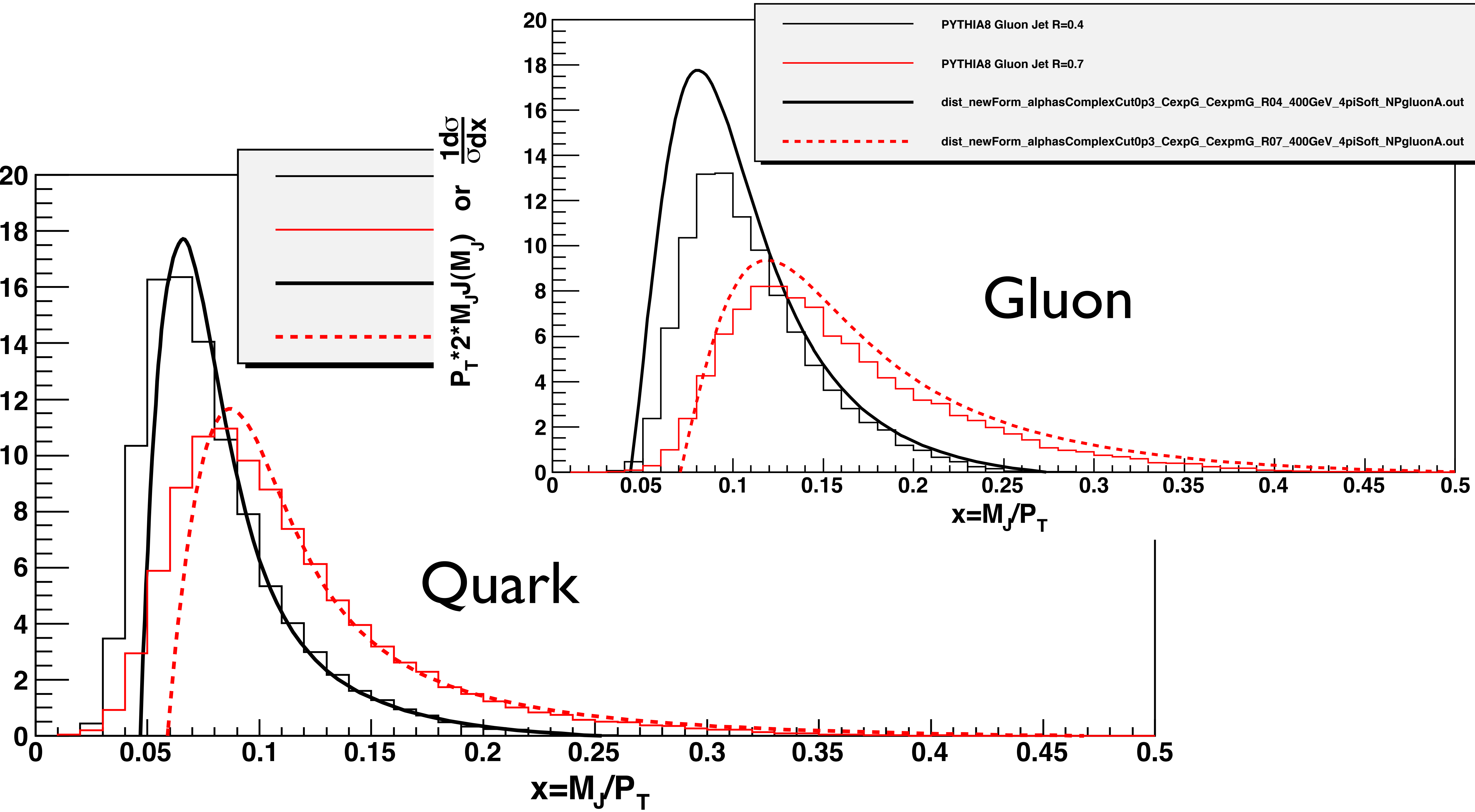
Non-perturbative contribution is needed for small MJ.

$$\bar{J}_q(N, P_T, \nu_{\rm fi}^2, R, \mu^2) = \bar{J}_q(N, P_T, \nu_{\rm in}^2, R, \mu^2)\exp[S(N) + S^{NP}(N)]$$

$$S^{NP}(N) = \frac{N^2 Q_0^2}{R^2 P_T^2}(C_c \alpha_0 \ln N + \alpha_1) + C_c \alpha_2 \frac{N Q_0}{R P_T}$$

Non-perturbative parameters were fit at pT=600GeV with R=0.7.

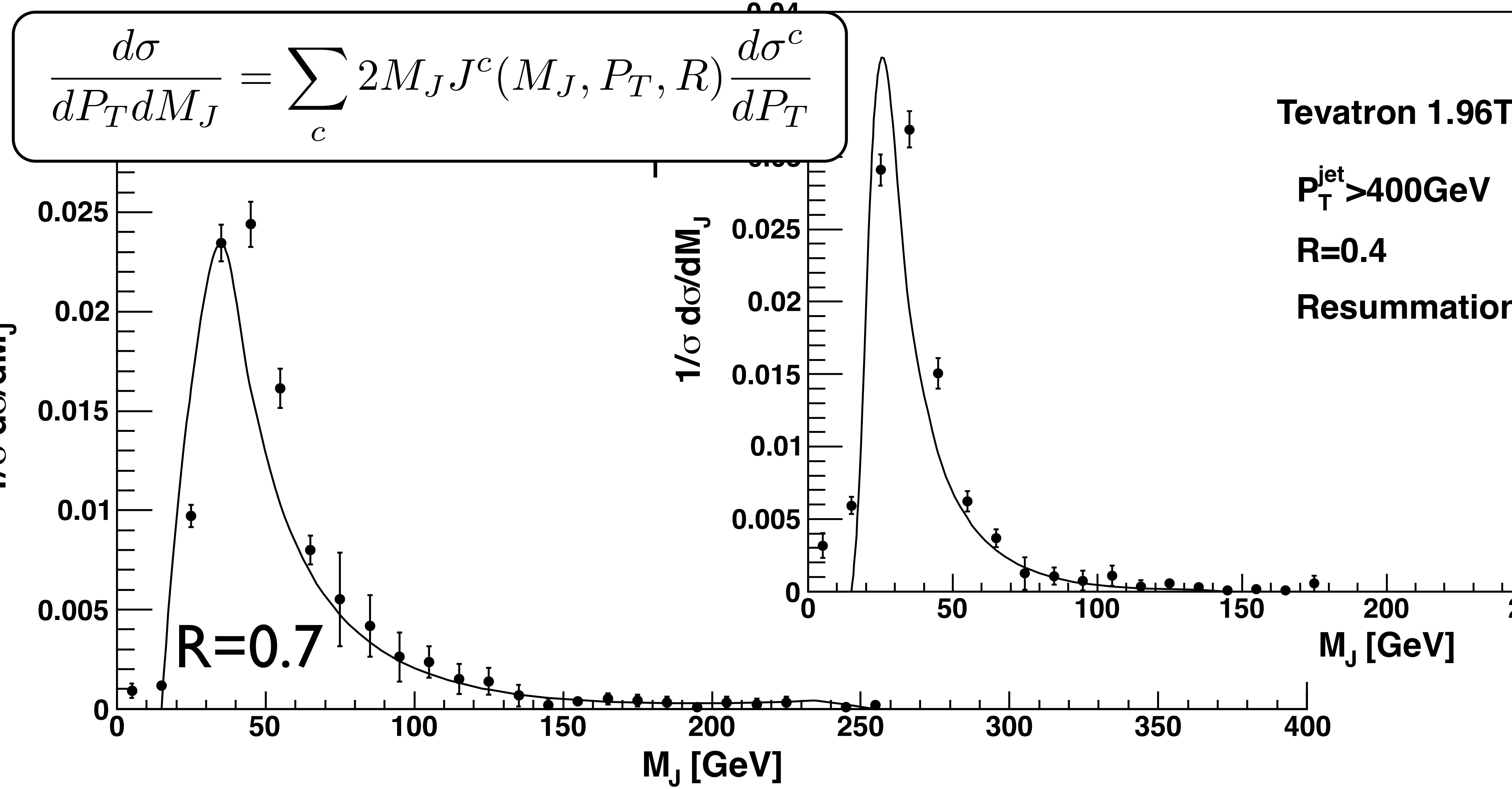
Quark and gluon jets at pT=400GeV



Agree with PYTHIA prediction for various pT and R.

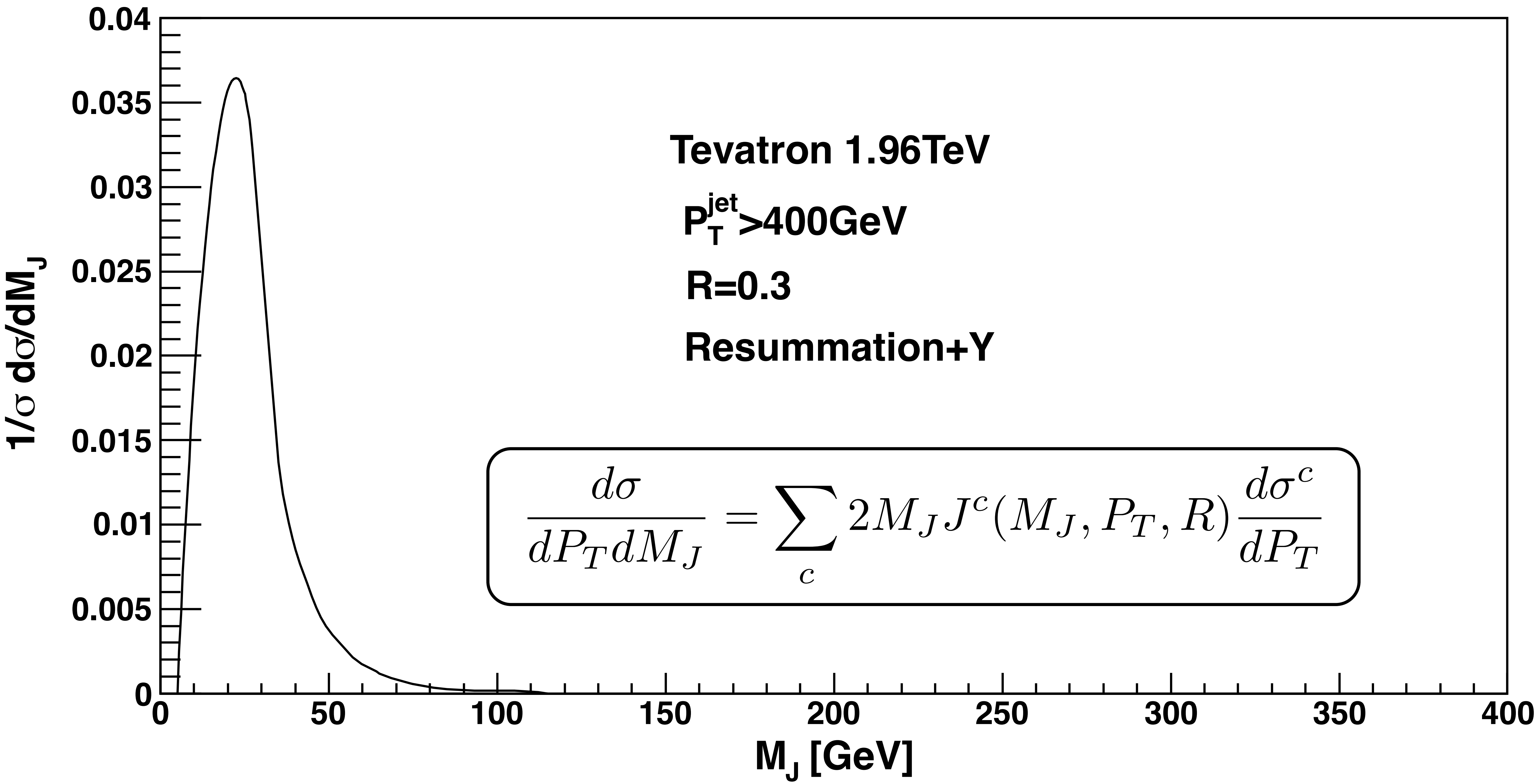
Compare with CDF data after convoluting with $d\sigma^c/dP_T$

$$\frac{d\sigma}{dP_T dM_J} = \sum_c 2M_J J^c(M_J, P_T, R) \frac{d\sigma^c}{dP_T}$$



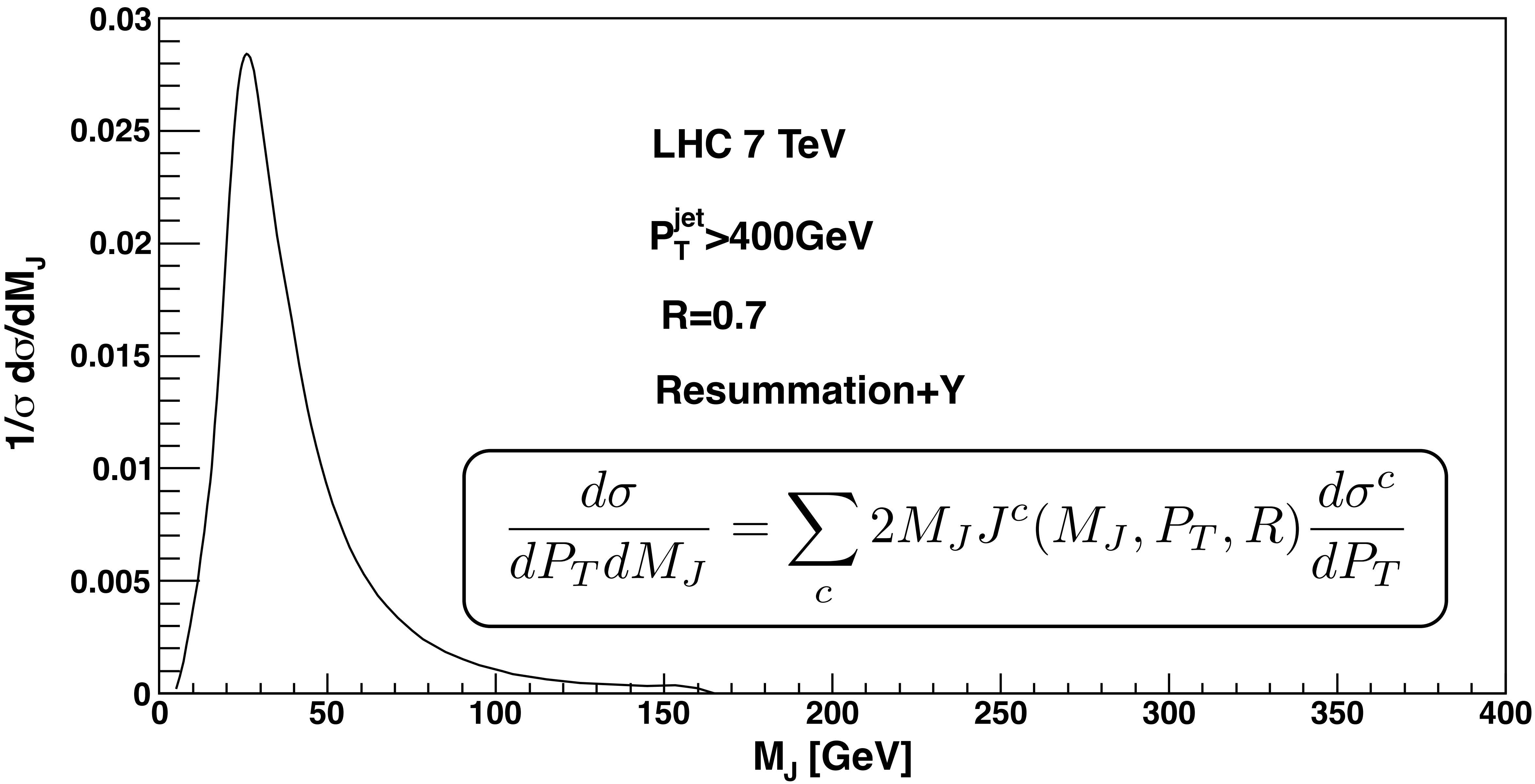
Resummation dramatically improve prediction in small to medium jet mass range

Resummation Prediction for R=0.3 @ Tevatron



Resummation prediction can be tested by CDF data.

Resummation Prediction for R=0.7 @ LHC



Resummation prediction to jet mass distribution can be tested in future by LHC data.

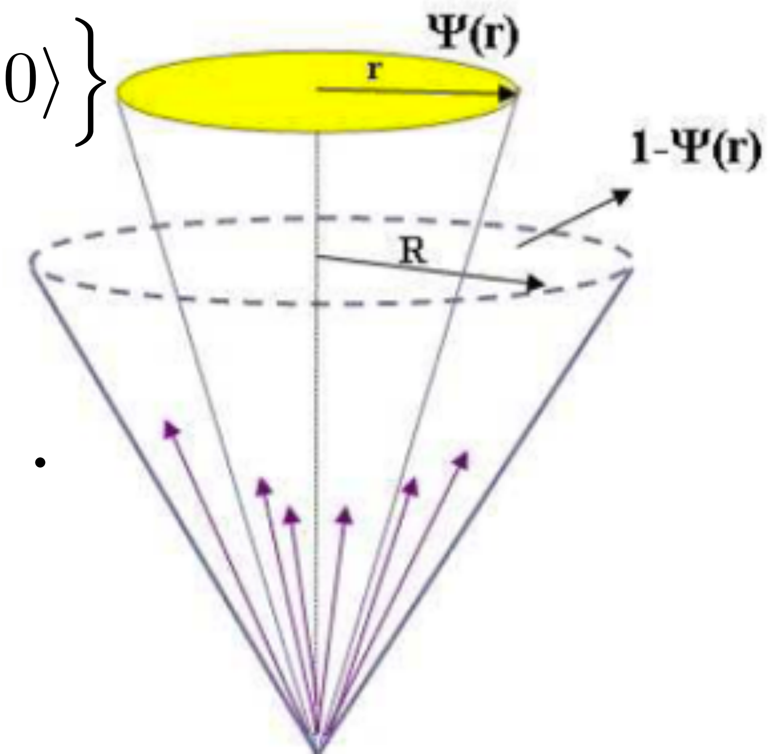
Jet energy profile

$$\Psi(r) = \frac{1}{\mathrm{N_{jet}}} \sum_{\mathrm{jets}} \frac{P_T(0,r)}{P_T(0,R)}, \qquad 0 \le r \le R$$

Jet energy profile J^E can be obtained by inserting the step function in the jet function:

$$J_q^{E(1)}(m_J^2, P_T, \nu^2, R, \mu^2) \; =$$

$$\frac{(2\pi)^3}{2\sqrt{2}(P_J^0)^2 N_c} \sum_{\sigma,\lambda} \int \frac{d^3p}{(2\pi)^3 2\omega_p} \frac{d^3k}{(2\pi)^3 2\omega_k} [p^0 \Theta(R-\theta_p) + k^0 \Theta(R-\theta_k)]$$
$$\times \mathrm{Tr}\left\{ \not{\xi} \langle 0|q(0) W_\xi^{(\bar{q})\dagger}(\infty,0)|p,\sigma;k,\lambda\rangle\langle k,\lambda;p,\sigma|W_\xi^{(\bar{q})}(\infty,0)\bar{q}(0)|0\rangle \right\}$$
$$\times \delta(m_J^2 - (p+k)^2)\delta(\hat{n} - \hat{n}_{\vec{p}+\vec{k}})\delta(P_J^0 - p^0 - k^0),$$

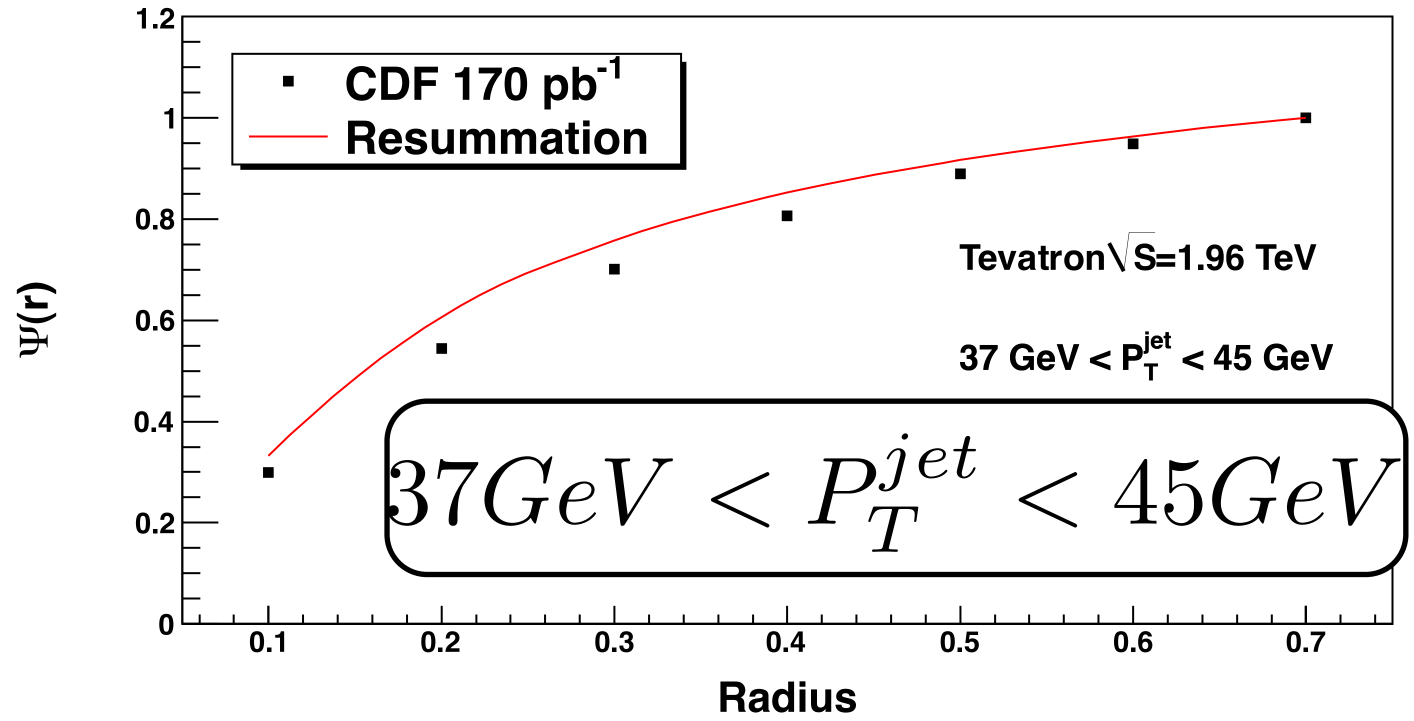


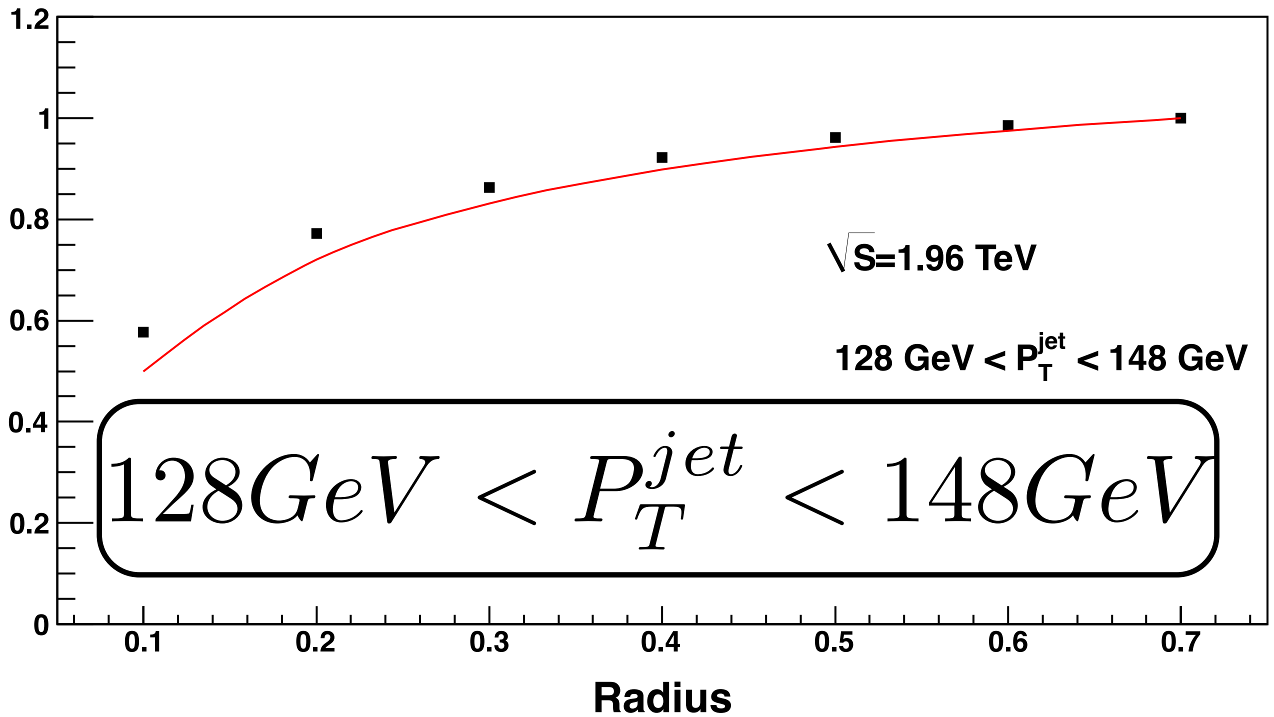
At NLO, $\bar{J}_E^q \approx \frac{\alpha_s C_F}{P_J^0 \pi} \left[-\frac{1}{4} \ln^2 \frac{R^2}{r^2} - \frac{3}{4} \ln \frac{R^2}{r^2} \right].$

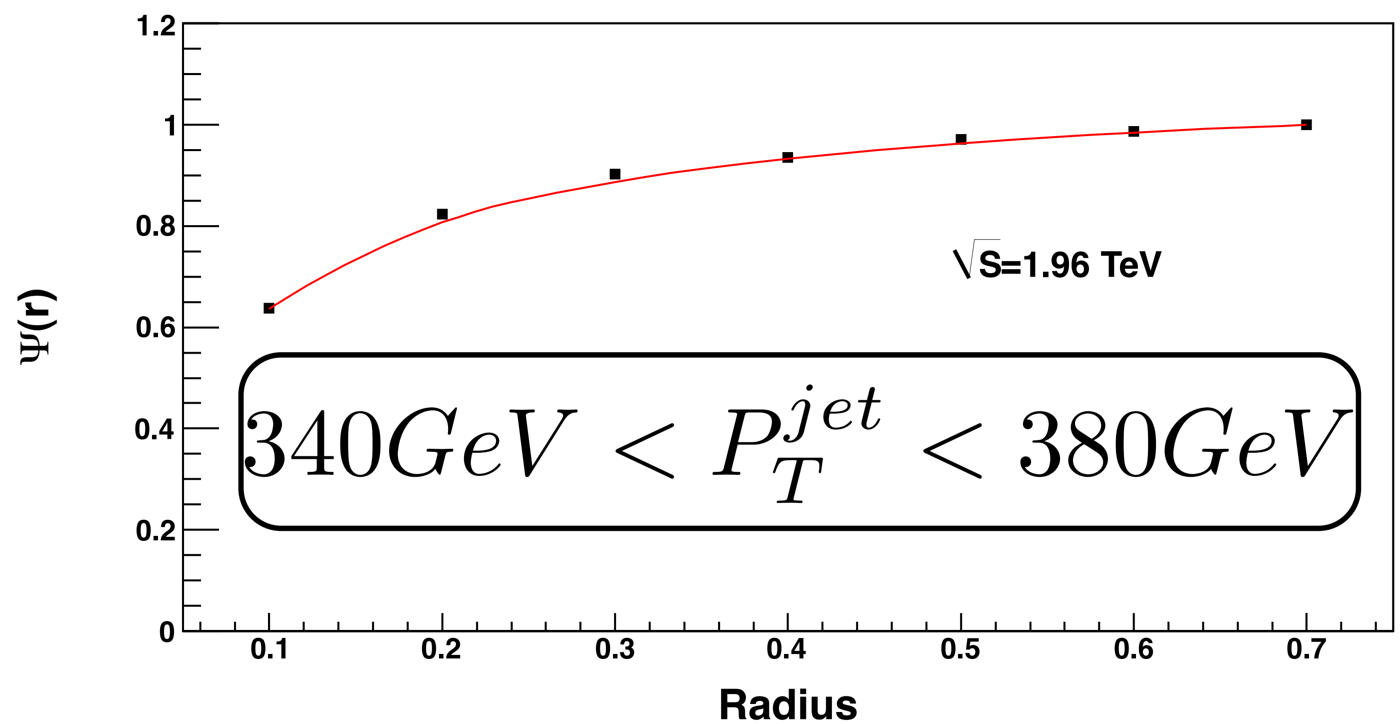
which is an integrable singularity.

Jet energy profile @ CDF

$$\Psi_q(r) \equiv \frac{\bar{J}_q^E(1, P_T, \nu_{\mathrm{fi}}^2, R, r)}{\bar{J}_q^E(1, P_T, \nu_{\mathrm{in}}^2, R, R)},$$



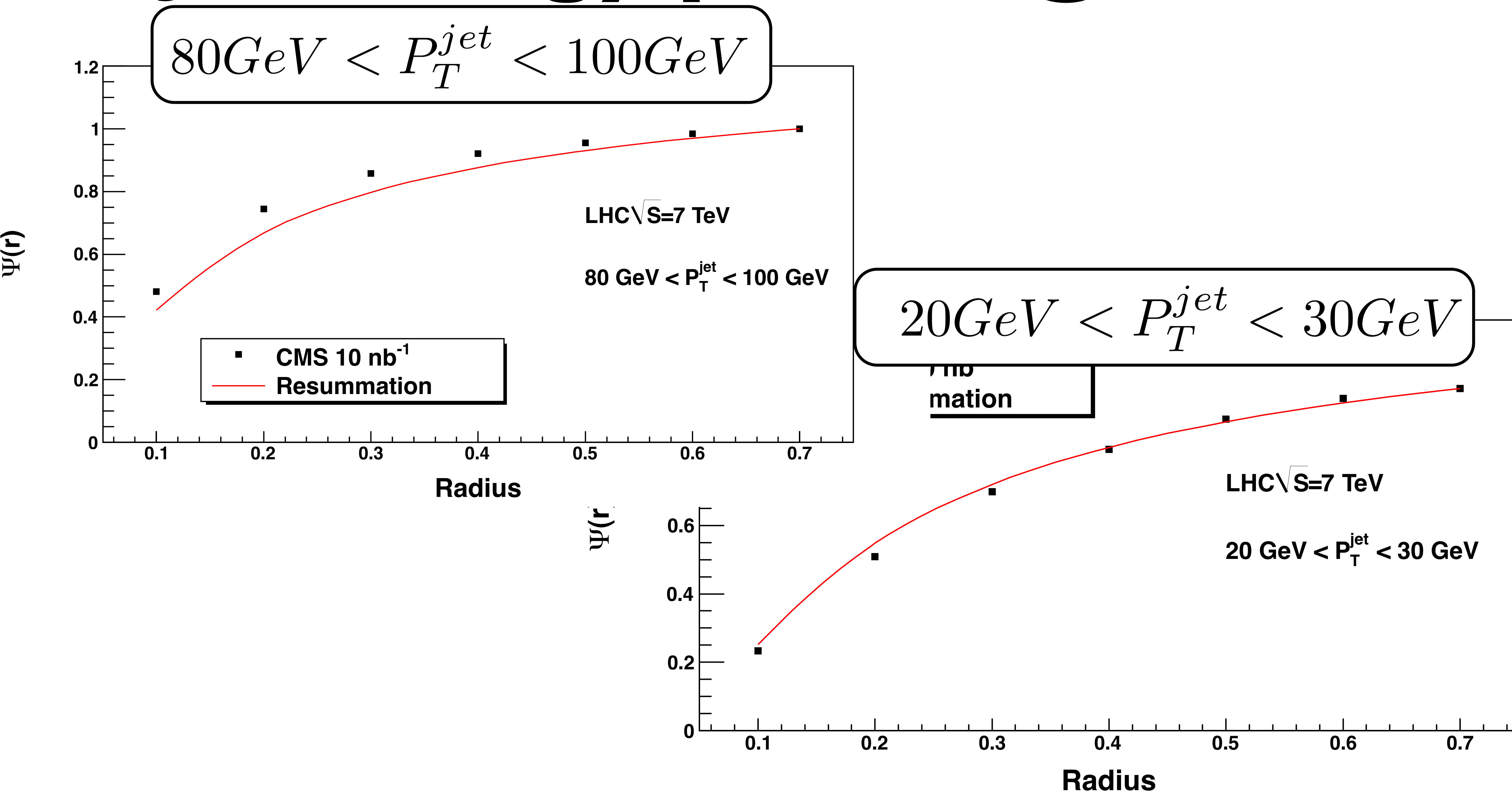




CDF data arxiv:hep-ex/0505013

Gluon jet dominates in low pT region.

Jet energy profile @ CMS



Predicted by perturbative resummation calculation
(no non-perturbative contribution is needed.

Summary

Studying jet substructure is useful for testing Standard Model and identifying New Physics.

- Fixed-order calculations in jet mass distribution and jet energy profile contain large logs, making predictions unreliable in small jet mass or small r region.
- QCD resummation provides reliable prediction and making independent check to full event generators.
- Resummed jet mass distribution including non-perturbative contribution agrees with PYTHIA8 for different jet pT and R, and Tevatron CDF data.
- Resummation predictions for jet energy profile agree with CDF and CMS data.
- Our formalism can be extended for heavy quark jet, e.g., a boosted top quark jet. (in progress)
- Same formalism can be used in jet study at HERA and RHIC.

Backup slides

Non-perturbative term

$$
\begin{aligned}
&\int_{1/\bar{N}}^{1} \frac{dy}{y} \int_{1/\bar{N}}^{y} \frac{d\omega}{\omega} \alpha_s(\omega R P_T) \\
&\approx \int_0^1 \frac{1-(1-y)^{N-1}}{y} dy \int_{1/\bar{N}}^{y} \frac{d\omega}{\omega} \alpha_s(\omega R P_T) \\
&= \int_0^1 \frac{d\omega}{\omega} \alpha_s(\omega R P_T) \int_\omega^1 \frac{1-(1-y)^{N-1}}{y} dy \\
&\to \int_0^\epsilon \frac{d\omega}{\omega} \alpha_s(\omega R P_T) \int_\omega^1 \frac{1-(1-y)^{N-1}}{y} dy \\
&\approx \int_0^\epsilon \frac{d\omega}{\omega} \alpha_s(\omega R P_T) \int_\omega^1 N dy \\
&= \frac{N}{R P_T} \int_0^\epsilon d\mu \alpha_s(\mu) \\
&\to \frac{N Q_0}{R P_T} \alpha_0
\end{aligned}
$$

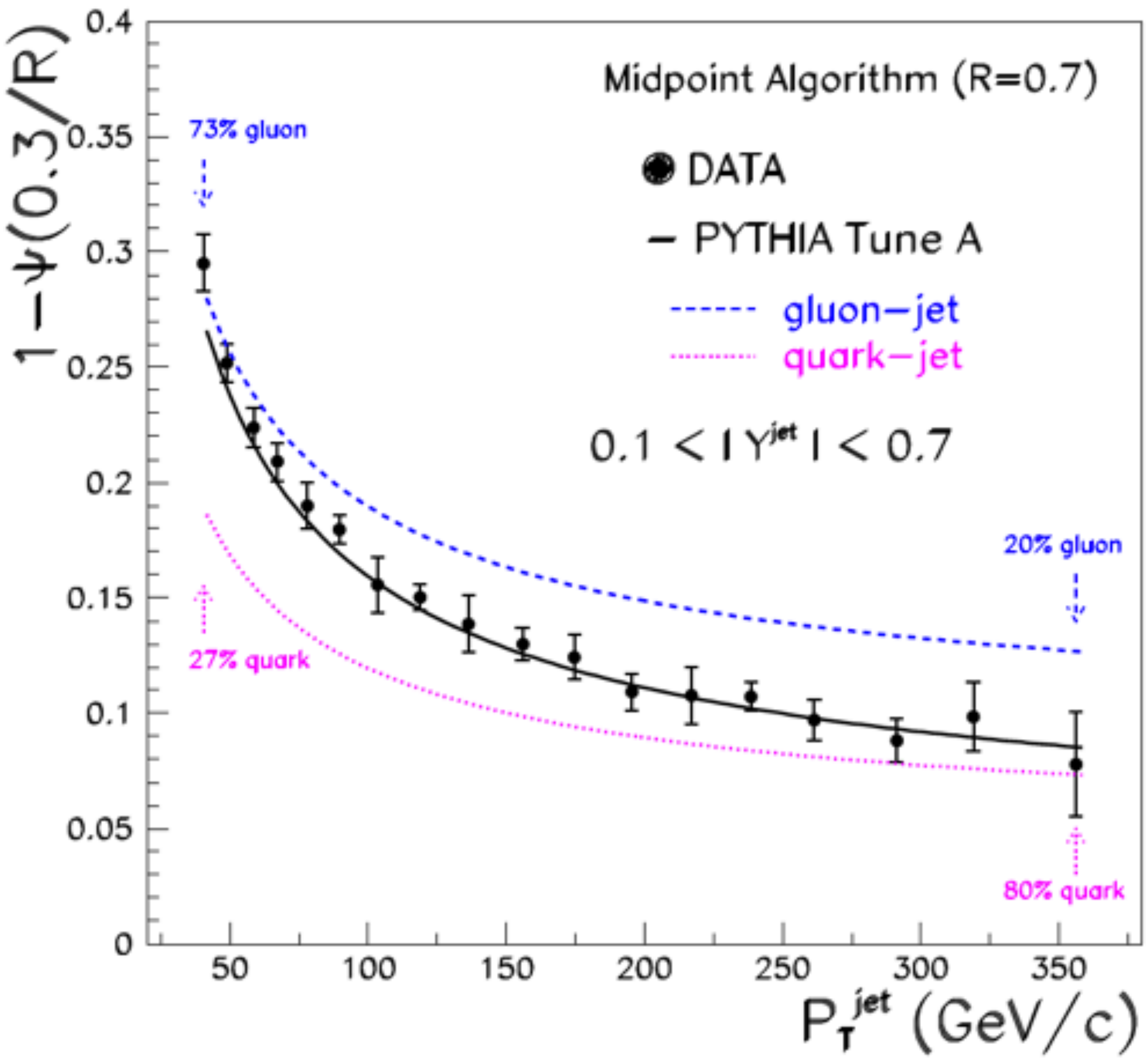
B.R. Webber et al.

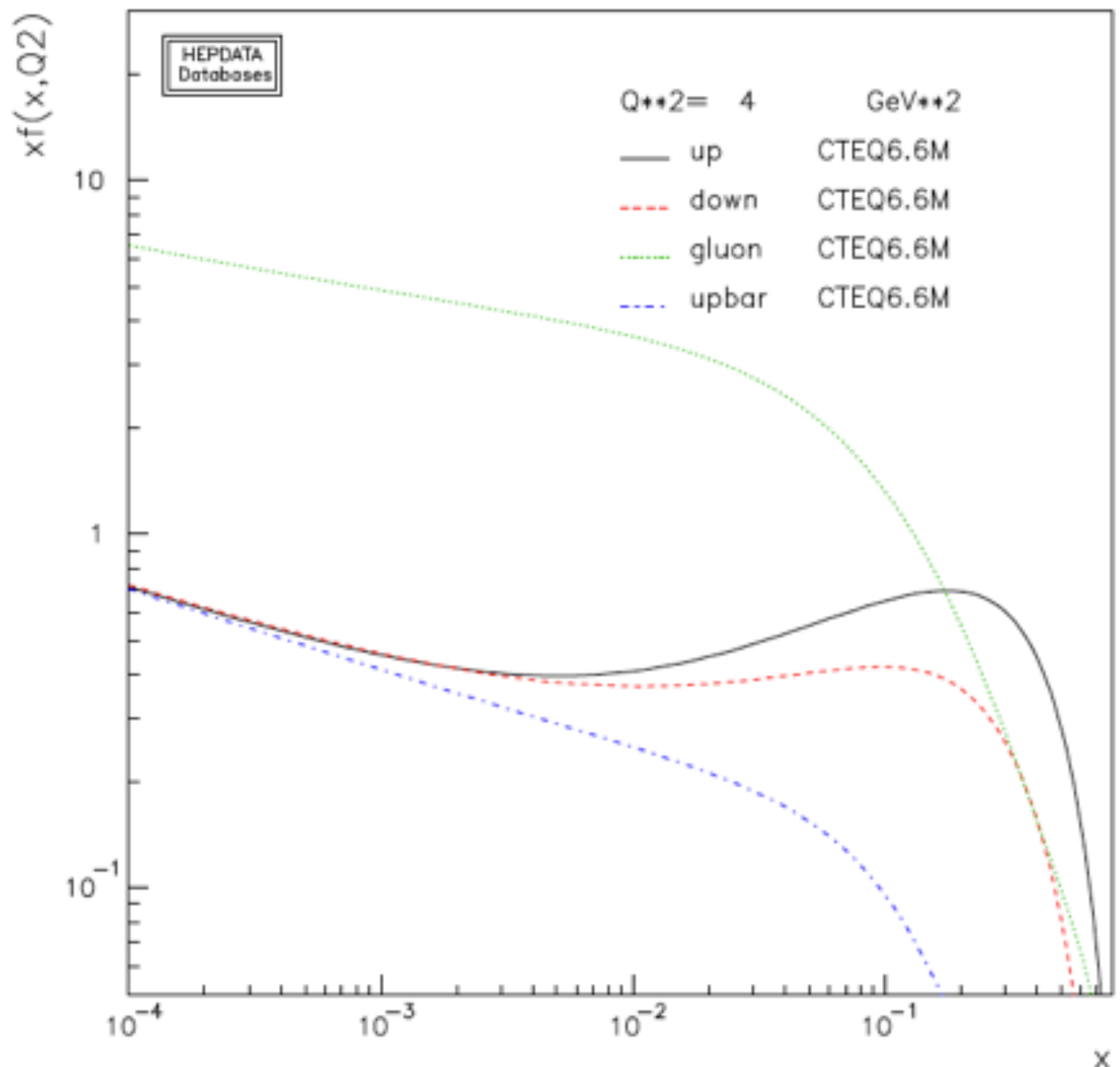
JHEP 9804:017,1998

$\frac{N^2 Q_0^2}{R^2 P_T^2} \ln N$ is implied by RG equation

$\frac{N^2 Q_0^2}{R^2 P_T^2}$ is a gaussian smearing

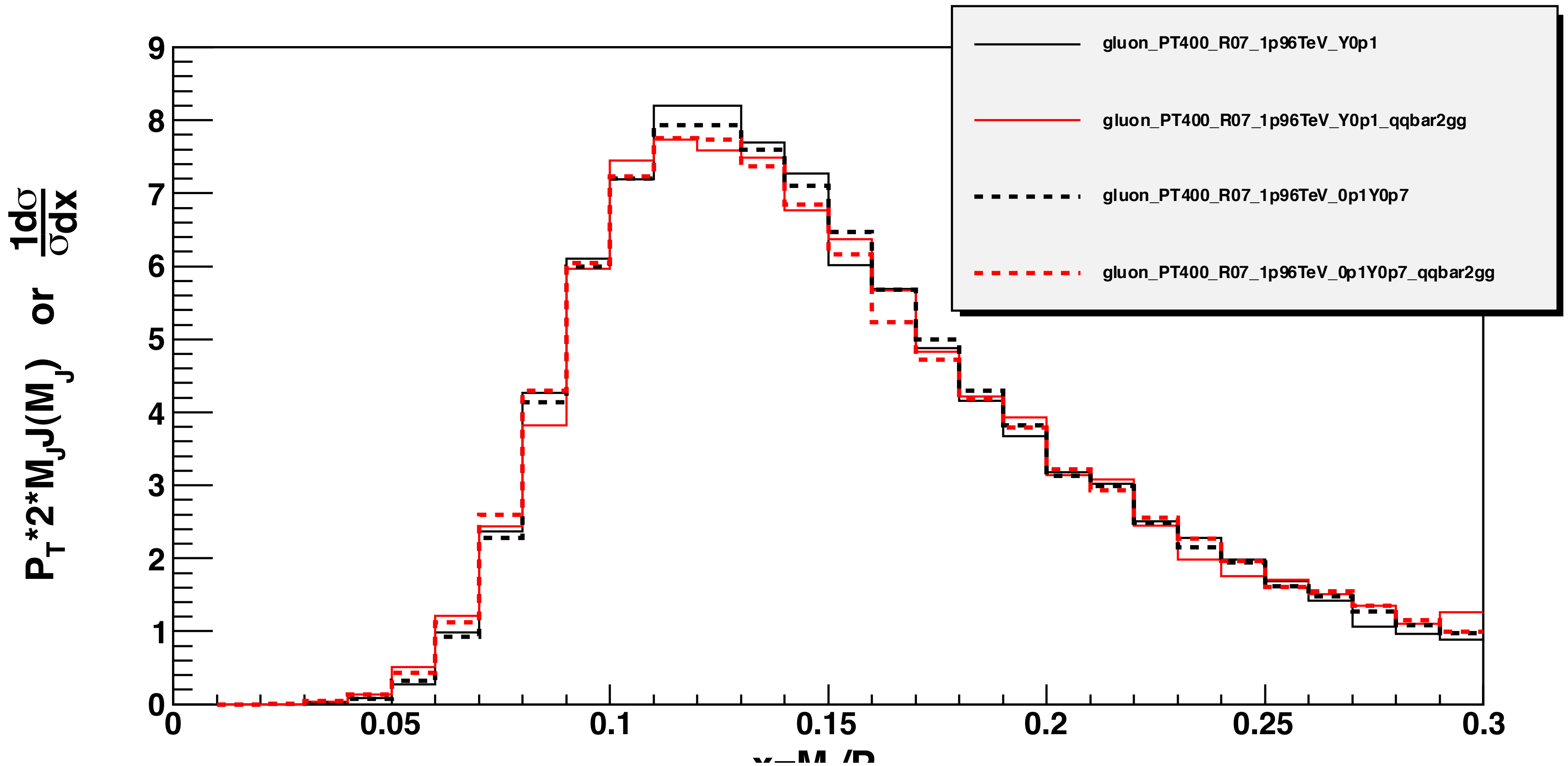
Dependence on pT@ CDF

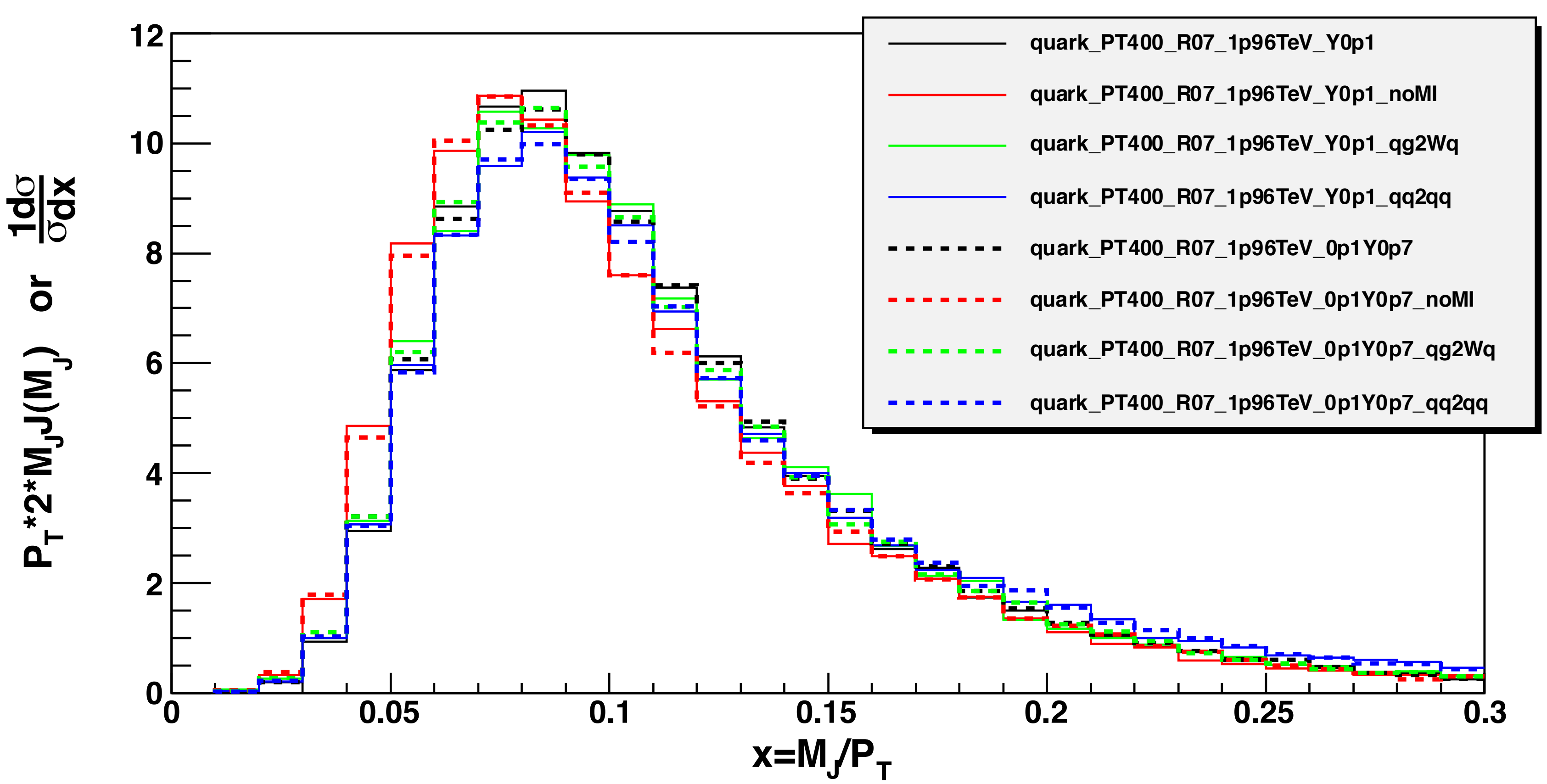




Gluon jet and quark jet dominates in low and high pT region, respectively, mainly caused by parton density (PDFs).

Dependence on different subprocesses





Dependence on collider energy