Generalized Parton Distribution and TMD modeling in terms of effective light-cone wave functions

D. Müller, LBNL

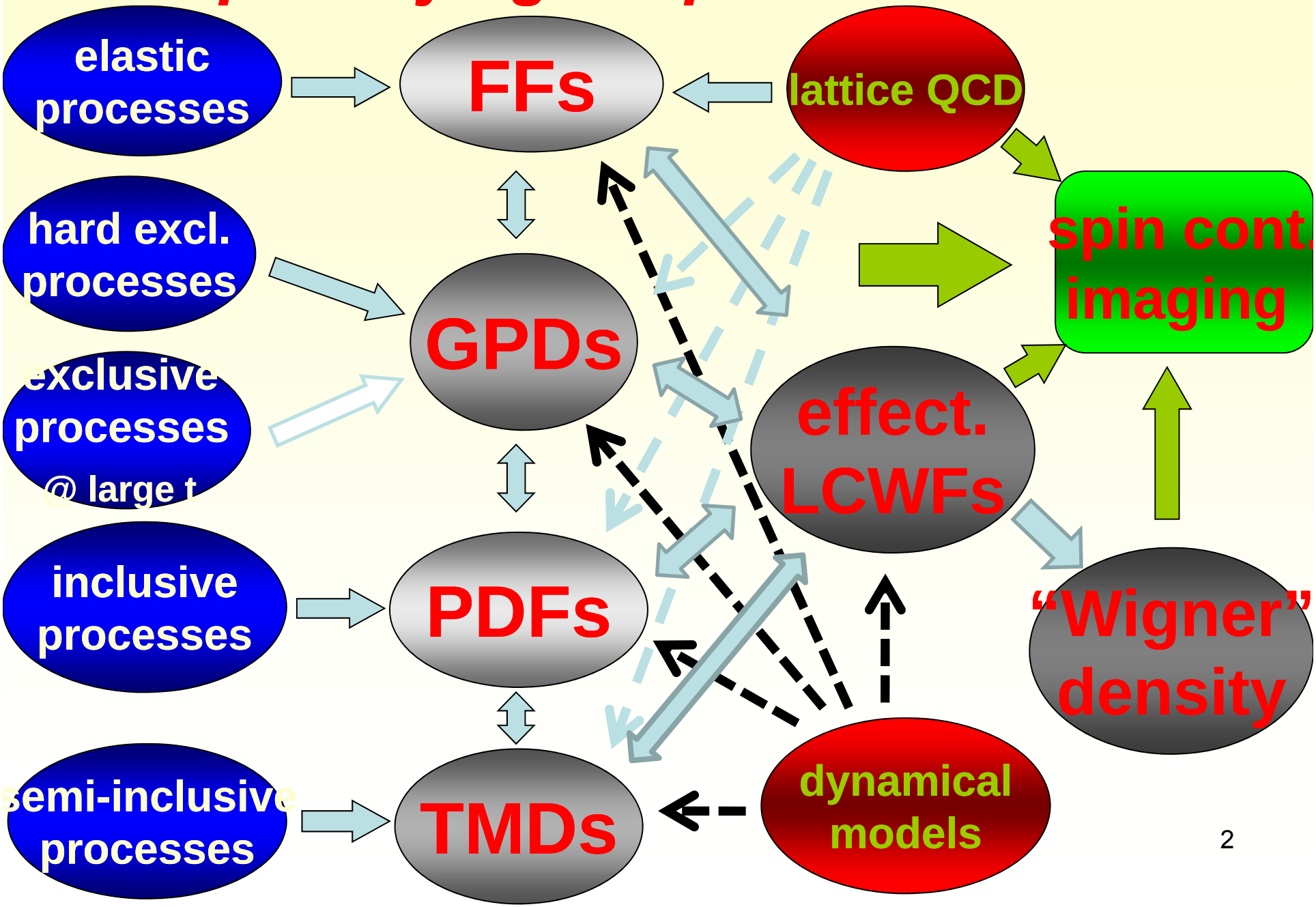
- ***Motivation/preliminaries***
- ***Problems with LCWF overlap representations***
- ***Simple models versus phenomenology***
- ***Conclusions***

in collaboration with D.S. Hwang (Sejong Univ.)

arXiv:0710.1567

http://gpd.gla.ac.uk/gpd2010/Talks/DieterMuellerGPD2010.pdf

quantifying the partonic content



The concept of an effective LCWF

- Is the LCWF approach well defined?
- certainly, all LCWFs are coupled by QCD dynamics
- a solution can not be expected in near future
- to stay with the LCWF concept, we introduce an effective one

$$|P,S\rangle = \sum_{h=-1/2}^{1/2} \int d\lambda^2 \int [dX\, d^2\mathbf{k}_\perp]$$
$$\times \sqrt{\rho(\lambda^2)}\Psi_h^S(X_i,\mathbf{k}_{\perp i}|\lambda^2) \prod_{j=1}^{2} \frac{1}{\sqrt{X_j}} |\lambda^2, X_i P^+, X_i \mathbf{P}_\perp + \mathbf{k}_{\perp i}, h\rangle$$

Ψ_h describes the dynamics of the struck quark with spin h

λ^2 denotes some collective degrees of freedom

$\rho(\lambda^2)$ is a density

Scalar di-quark model (Yukawa theory)

$$\mathcal{L} = \bar{\psi}\left(i\partial\!\!\!/ - m\right)\psi - \frac{1}{2}\phi\left(\partial^2 + \lambda^2\right)\phi + g\bar{\psi}\psi\phi$$

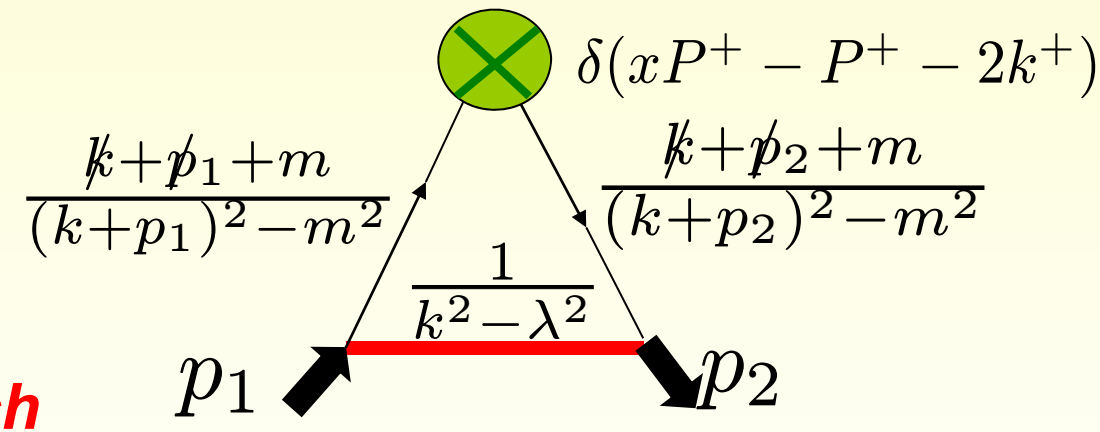
struck spin-*1/2* quark

collective scalar diquark spectator

coupling knows about spin

Diagrammatic approach:

via covariant time ordered perturbation theory



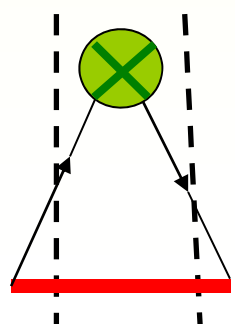
LC- Hamiltonian approach

$$k^\mu \to (k^+, k^-, \mathbf{k}_\perp),\ k^\pm = k^0 \pm k^3,\ \mathbf{k}_\perp = (k^1, k^2).$$

integrate out minus component to find LCWF

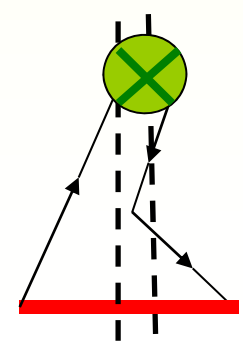
parton number conserved LCWF

(outer region)



parton number violating LCWF

(central region)



four parton number conserved LCWFs with angular momentum *L*

$$\psi^{\uparrow}_{+1/2}(X,\mathbf{k}_\perp) = \left(M+\frac{m}{X}\right)\varphi(x,\mathbf{k}_\perp)\,, \qquad L=0$$

$$\psi^{\downarrow}_{-1/2}(X,\mathbf{k}_\perp) = \left(M+\frac{m}{X}\right)\varphi(x,\mathbf{k}_\perp)\,, \qquad L=0$$

$$\psi^{\downarrow}_{+1/2}(X,\mathbf{k}_\perp) = \frac{k^1-ik^2}{X}\varphi(X,\mathbf{k}_\perp)\,, \qquad L=-1$$

$$\psi^{\uparrow}_{-1/2}(X,\mathbf{k}_\perp) = -\frac{k^1+ik^2}{X}\varphi(X,\mathbf{k}_\perp)\,, \qquad L=+1$$

in terms of one scalar LCWF (e.g., generalized Yukawa theory)

$$\varphi(X,\mathbf{k}_\perp) = \frac{gM^{2p}}{\sqrt{1-X}}X^{-p}\left(M^2-\frac{\mathbf{k}_\perp^2+m^2}{X}-\frac{\mathbf{k}_\perp^2+\lambda^2}{1-X}\right)^{-p-1}\,,$$

- Yukawa result follows with p=0
- p>0 serves as analytic regularization (model parameter)

axial-vector LCWFs have L= {0,+1,-1}

Unintegrated PDFs (diquark model)

- viewed as a phenomenological concept to describe data

? TMD counting, definition, factorization theorems, universality

✓ twist-2 and -3 PDFs are expressed by one LCWF overlap, e.g.

L=0 *L=1* *overlap (one should wonder)*

$$q(x,\mathbf{k}_\perp^2) = \frac{(m+xM)^2+\mathbf{k}_\perp^2}{M^2}\,\mathbf{\Phi}(x,\mathbf{k}_\perp^2)\,, \qquad \mathbf{\Phi}(x,\mathbf{k}_\perp^2) = \int d\lambda^2 \rho(\lambda^2)\,\frac{\left|\phi(x,\mathbf{k}_\perp|\lambda^2)\right|^2}{1-x}$$

$$\Delta q(x,\mathbf{k}_\perp^2) = \frac{(m+xM)^2-\mathbf{k}_\perp^2}{M^2}\,\mathbf{\Phi}(x,\mathbf{k}_\perp^2)\,, \qquad \Delta_\perp q(x,\mathbf{k}_\perp^2) = \frac{k_1(m+xM)}{M^2}\,\mathbf{\Phi}(x,\mathbf{k}_\perp^2)$$

$$\delta q(x,\mathbf{k}_\perp^2) = \frac{(m+xM)^2-k_1^2+k_2^2}{M^2}\,\mathbf{\Phi}(x,\mathbf{k}_\perp^2),\ \delta_{\rm L} q(x,\mathbf{k}_\perp^2) = \frac{-2k_1(m+xM)}{M^2}\,\mathbf{\Phi}(x,\mathbf{k}_\perp^2)$$

✓ elimination of LCWF overlap yields 12-1 constraints

✓ known results **[Jakob et al. (97)]** are recovered with

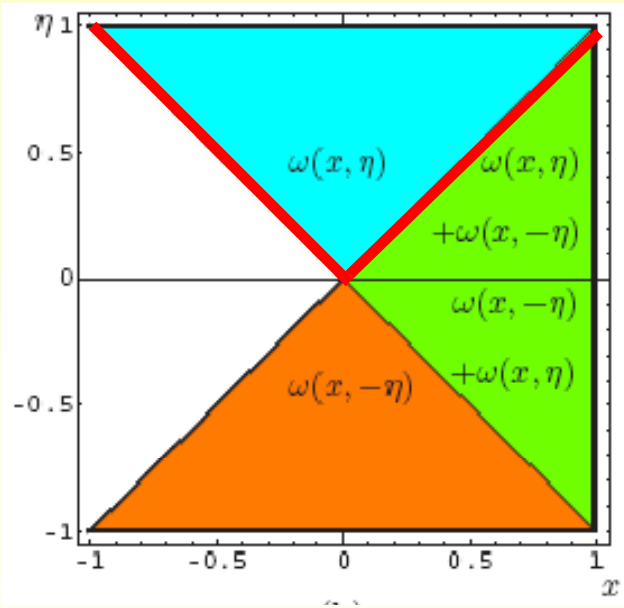
$$\mathbf{\Phi}(x,\mathbf{k}_\perp^2) = \frac{g^2}{M^2}\,\frac{(1-x)^{2p+1}}{\left[(1-x)\frac{m^2}{M^2}+x\frac{\lambda^2}{M^2}-x(1-x)+\frac{\mathbf{k}_\perp^2}{M^2}\right]^{2p+2}}$$

A partonic GPD duality interpretation

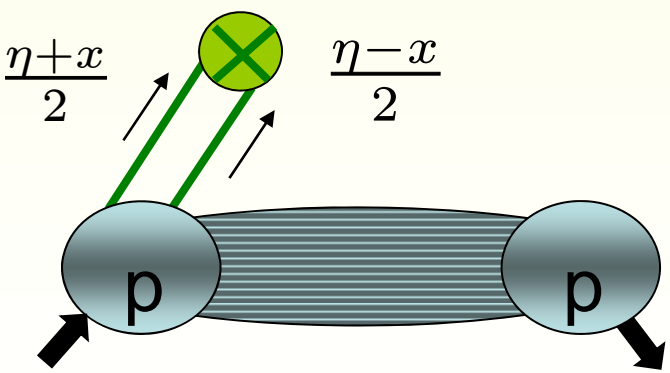
quark GPD (anti-quark x → -x):

$$F(x,\eta,t) \quad = \quad \theta(-\eta \le x \le 1)\,\omega(x,\eta,t) + \theta(\eta \le x \le 1)\,\omega(x,-\eta,t)$$

$$\omega\,(x,\eta,t) = \frac{1}{\eta}\int_0^{\frac{x+\eta}{1+\eta}} dy\, x^p f(y,(x-y)/\eta,t)$$

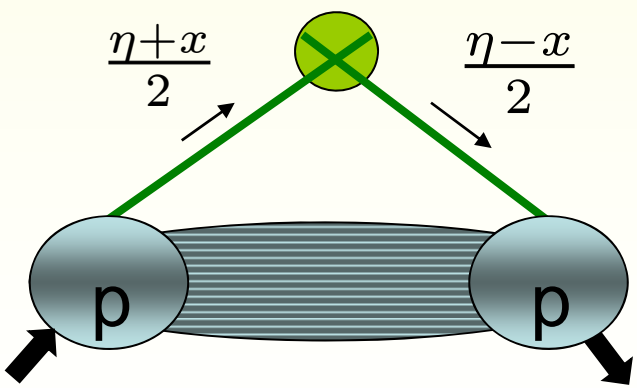


dual interpretation on partonic level:



support extension
is unique **[DM et al. 92]**

ambiguous (D-term)
[DM, A. Schäfer (05), KMP-K (07)]



central region - η < x < η

mesonic exchange in t-channel

outer region η < x

partonic exchange in s-channel

Restoring full GPD: in the outer region GPDs are given as overlap of parton number conserved LCWFs, e.g., **Diehl et al. (00)**

$$\frac{\Delta^1 - i\Delta^2}{2M} E(x \geq \eta, \eta, t))$$

early work on FFs & PDFs:
Drell, Yan (69)
Drell, Brodsky

$$= \sqrt{1-\zeta} \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} \left[\psi^{\uparrow *}_{+1/2}(X', \mathbf{k}'_\perp) \psi^{\downarrow}_{+1/2}(X, \mathbf{k}_\perp) + \psi^{\uparrow *}_{-1/2}(X', \mathbf{k}'_\perp) \psi^{\downarrow}_{-1/2}(X, \mathbf{k}_\perp) \right]$$

double distribution representation' for the outer region can be read off

$$E(x \geq \eta, \eta, t) = (1-x) \int_{\frac{x-\eta}{1-\eta}}^{\frac{x+\eta}{1+\eta}} \frac{dy}{\eta} \, e\left(y, \frac{x-y}{\eta}, t\right)$$

Hwang, DM (07)
Tiburzi, Miller (01,03)
Radyushkin et al. (03)
Tiburzi et al. (04)

with double distribution

$$e(y,z,t) = N \frac{\left(\frac{m}{M} + y\right) \left((1-y)^2 - z^2\right)^p}{\left[(1-y)\frac{m^2}{M^2} + y\frac{\lambda^2}{M^2} - y(1-y) - \left((1-y)^2 - z^2\right)\frac{t}{4M^2}\right]^{2p+1}}$$

- central region follows by restoring the full support
- polynomiality is restored
- ambiguous modification of LCWFs is not allowed

Exploring the LCWF overlap

- ✓ duality between *s*- and *t*-channel allows to find the central region
- ➢ folklore that central region always restores polynomiality
  rather LCWFs have to respect hidden Lorentz covariance
- ✓ positivity constraints should be automatically satisfied
- ✓ all twist-two and twist-three related quantities can be evaluated
- ✓ N, m, λ, and p can be used as model parameters
- ✓ concept: non-perturbative quantities in terms of LCWFs models

missing ingredients and open questions:

- Regge behavior as a collective phenomena is missing
- hard to understand from the naive point of view:

  t and $k_\perp$ are tied to each other
  μ^2 evolution arises from large $k_\perp$ behavior

? contradiction with power counting rules

Converting $\Delta_\perp$ into t dependence

unintegrated scalar LCWF overlap:

$$\mathbf{\Phi}(x \geq \eta, \eta, \mathbf{\Delta}_\perp, \mathbf{k}_\perp) = \frac{2-\zeta}{2\sqrt{1-\zeta}\sqrt{1-X'}\sqrt{1-X}} \phi^*\left(X', (\mathbf{k}'_\perp - \mathbf{\Delta}'_\perp)^2\right) \phi\left(X, \mathbf{k}_\perp^2\right)$$

$$\frac{1-X}{1-\zeta}\mathbf{\Delta}_\perp$$

has to possesses a scalar DD representation

$$\mathbf{\Phi}(x, \eta, t) \;=\; \int_0^1 dy \int_{-1+y}^{1-y} dz\; \delta(x - y - z\eta)\, \hat{\mathbf{\Phi}}(y, z, t)$$

Mandelstam variable

in terms of an ***unintegrated double distribution***

$$\hat{\mathbf{\Phi}}(y, z, t) = \iint d\overline{\mathbf{k}}_\perp^2\; \hat{\mathbf{\Phi}}\left(y, z, t, \overline{\mathbf{k}}_\perp^2\right)\;, \quad \overline{\mathbf{k}}_\perp = \mathbf{k}_\perp - (1 - y + z)\mathbf{\Delta}_\perp/2$$

a general ansatz (Laplace transform) solves the problem

$$\boxed{\phi(X, \mathbf{k}_\perp) = \int_0^\infty d\alpha\; \varphi(X, \alpha) \exp\left\{-\alpha \frac{\mathbf{k}_\perp^2 - X(1-X)M^2}{(1-X)M^2}\right\}}$$

Constrain for Laplace transformed LCWF

to restore polynomiality a (sufficient) constraint must be satisfied:

$$\frac{d}{d\eta}\left[\varphi^*\left(\frac{y-\eta(1-z)}{1-\eta}, A\frac{1-y+z}{2}\right)\varphi\left(\frac{y+\eta(1+z)}{1+\eta}, A\frac{1-y-z}{2}\right)\right]=0$$

a simple, however, non-trivial Laplace transformed LCWF:

$$\varphi(X,\alpha)=\varphi(\alpha)\exp\left\{-\alpha\frac{m^2}{M^2}-\alpha\frac{X}{1-X}\frac{\lambda^2}{M^2}\right\}$$

reduced LCWF is not fixed

yields the desired result:

$$\varphi^*(X',\alpha_2)\varphi(X,\alpha_1)$$

$$\to\varphi^*\left(A\frac{1-y+z}{2}\right)\varphi\left(A\frac{1-y-z}{2}\right)\exp\left\{-A(1-y)\frac{m^2}{M^2}-Ay\frac{\lambda^2}{M^2}\right\}$$

NOTE:

$\varphi\propto\delta(\alpha-\alpha_0)$ ➡ exponential *t*-dependence **(disfavored @ large -*t*)**

$\varphi\propto\alpha^p$ ➡ power-like *t*-dependence

Spin structure of chiral even GPDs

specified spin coupling yields various GPD representations:

$$\left\{\begin{array}{c} H \\ E \\ \widetilde{H} \\ \widetilde{E} \end{array}\right\}(x,\eta,t) = \int_0^1 dy \int_{-1+y}^{1-y} dz\, \delta(x-y-\eta z) \left\{\begin{array}{c} h+(x-y)e \\ (1-x)e \\ \widetilde{h} \\ \widetilde{e}+(1-y-z/\eta)\, e \end{array}\right\}(y,z,t)$$

in terms of DDs:

$$\begin{aligned}
h &= \frac{(m+yM)^2}{M^2}\hat{\mathbf{\Phi}}(y,z,t) + \left[(1-y)^2 - z^2\right]\left[\frac{t}{4M^2}\hat{\mathbf{\Phi}}(y,z,t) + \int_{-\infty}^{t}\frac{dt'}{M^2}\hat{\mathbf{\Phi}}(y,z,t')\right] \\
e &= 2\left(\frac{m}{M}+y\right)\hat{\mathbf{\Phi}}(y,z,t) \\
\widetilde{h} &= \frac{(m+yM)^2}{M^2}\hat{\mathbf{\Phi}}(y,z,t) - \left[(1-y)^2 - z^2\right]\left[\frac{t}{4M^2}\hat{\mathbf{\Phi}}(y,z,t) + \int_{-\infty}^{t}\frac{dt'}{M^2}\hat{\mathbf{\Phi}}(y,z,t')\right] \\
\widetilde{e} &= 2\left((1-y)^2 - z^2\right)\hat{\mathbf{\Phi}}(y,z,t)
\end{aligned}$$

NOTE: $k_\perp$-integration can be replaced by t-integration

three constraints among four GPDs

Spin structure of chiral odd GPDs

chiral odd GPDs arise from the interference of *L=0* & *L=1* LCWFs

$$F_{\rm T}(x,\eta,t) = \int_0^1 dy \int_{-1+y}^{1-y} dz\, f_{\rm T}(y,z,t) \text{ for } F_{\rm T} = \left\{ H_{\rm T}, E_{\rm T}, \widetilde{H}_{\rm T}, \widetilde{E}_{\rm T} \right\}$$

$$\begin{aligned}
h_{\rm T} &= \left[\left(\frac{m}{M} + y \right)^2 + \left((1-y)^2 - z^2 \right) \frac{t}{4M^2} \right] \hat{\mathbf{\Phi}}(y,z,t) \\
e_{\rm T} &= 2 \left[\left(\frac{m}{M} + y \right) (1-y) + (1-y)^2 - z^2 \right] \hat{\mathbf{\Phi}}(y,z,t) \\
\widetilde{h}_{\rm T} &= -\left[(1-y)^2 - z^2 \right] \hat{\mathbf{\Phi}}(y,z,t) \\
\widetilde{e}_{\rm T} &= 2 \left(y + \frac{m}{M} \right) z\, \hat{\mathbf{\Phi}}(y,z,t)
\end{aligned}$$

(common DD representation)

obviously, another set of constraints, e.g.,

$$\begin{aligned}
h_{\rm T}(y,z,t) &= \frac{1}{2} \left[h + \widetilde{h} \right] (y,z,t) + \frac{t}{8M^2} \widetilde{e}(y,z,t), \quad \widetilde{h}_{\rm T}(y,z,t) = -\frac{1}{2} \widetilde{e}(y,z,t) \\
e_{\rm T}(y,z,t) &= (1-y) e(y,z,t) + \widetilde{e}(y,z,t)\,, \quad \widetilde{e}_{\rm T}(y,z,t) = z\, e(y,z,t)
\end{aligned}$$

Regge improved PDFs and GPDs

collective degrees of freedom:

- *g* an effective coupling (normalization fixed by parton number)
- *m* struck quark mass (containing a bunch of partons)
- *λ* spectator diquark mass

[Landshoff, Polkinghorne (71); Pobylitsa (03)]

naively, any transformation within these parameters is allowed

$$\rho_\alpha(\lambda,\lambda_c) = \theta(\lambda-\lambda_c)\frac{2^{-\alpha}\Gamma(2+2p)}{\Gamma(2+2p-\alpha)\Gamma(\alpha)}M^{-2\alpha}(\lambda^2-\lambda_c^2)^{\alpha-1}$$

this simple Regge ansatz with intercept *α yields PDFs and DDs:*

$$\mathbf{\Phi}(x,\mathbf{k}_\perp^2) = \frac{g^2}{M^2}\frac{x^{-\alpha}(1-x)^{2p+1+\alpha}}{\left[(1-x)\frac{m^2}{M^2}+x\frac{\lambda^2}{M^2}-x(1-x)+\frac{\mathbf{k}_\perp^2}{M^2}\right]^{2p+2}}$$

$$\hat{\mathbf{\Phi}}(y,z,t) = N\frac{y^{-\alpha}\left((1-y)^2-z^2\right)^{p+\alpha/2}}{\left[(1-y)\frac{m^2}{M^2}+y\frac{\lambda^2}{M^2}-y(1-y)-\left((1-y)^2-z^2\right)\frac{t}{4M^2}\right]^{2p+1}}$$

NOTES:

- a *t*-dependent *α(t)* might violate positivity constraints
- linear Regge trajectory has not been established from field theory
- *t*-dependence requires a more intricate integral transformation

- there is a mismatch of dimensional counting (P) and UV behavior
- Drell-Yan (perturbative behavior) and West (IR behavior) derived the same exclusive-inclusive (form factor-PDF) relation

- collinear gluon degrees of freedom are effectively absorbed
- twist-3 rel. unintegrated PDFs are affected by transverse gluons

- matching scale is ambiguous (evolution below $<< 1\ GeV^2$???)
  ? not working GR(77) ➡ GRV(98) (has steep gluons)

Models versus phenomenology

only for the scalar content of the proton, related to a ud-diquark SU(6) symmetry states

$$|p\rangle = \frac{1}{\sqrt{2}}|u,(ud,0)^+\rangle + \frac{1}{\sqrt{6}}|u,(ud,0)^-\rangle - \frac{1}{\sqrt{3}}|d,(uu,1)^-\rangle$$

unintegrated PDFs (as below + Regge improved LCWF overlap)

GPDs:

$$H(x,\eta,t) = \frac{1}{3}\left[2H_{u_{\rm val}}(x,\eta,t) - H_{d_{\rm val}}(x,\eta,t)\right],$$

$$E(x,\eta,t) = \frac{1}{3}\left[2E_{u_{\rm val}}(x,\eta,t) - E_{d_{\rm val}}(x,\eta,t)\right],$$

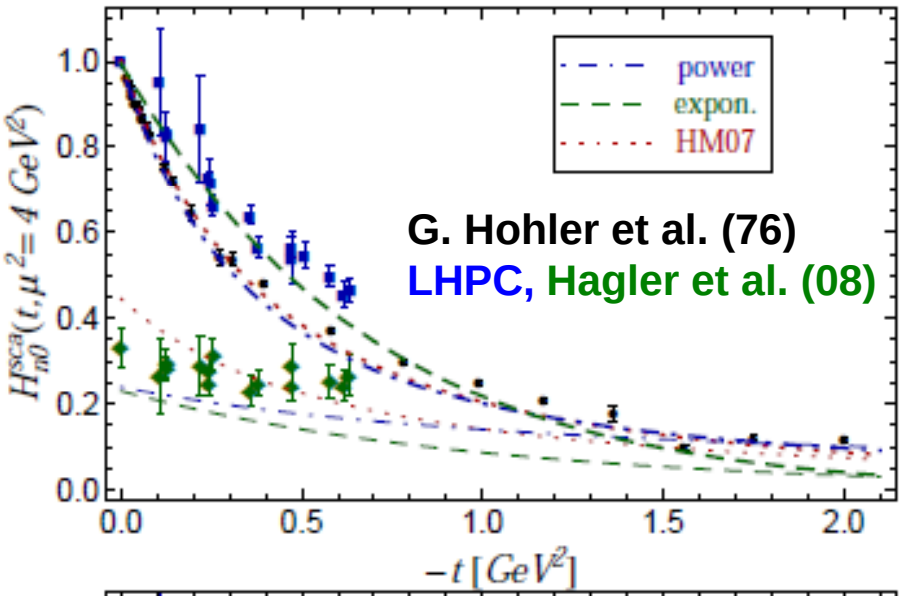
(generalized) form factors

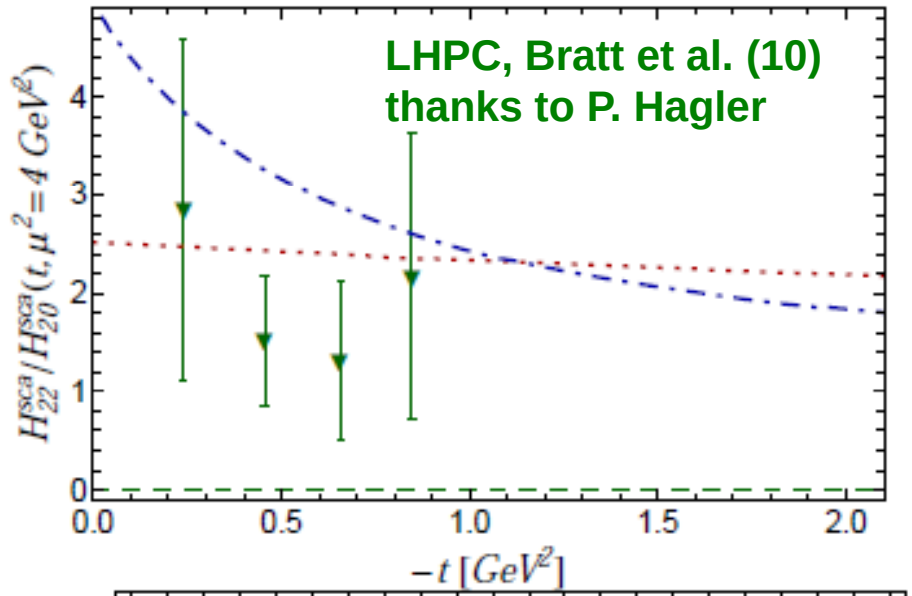
$$F_1(t) = \int_{-\eta}^{1} dx H(x,\eta,t) \quad F_2(t) = \int_{-\eta}^{1} dx E(x,\eta,t)$$

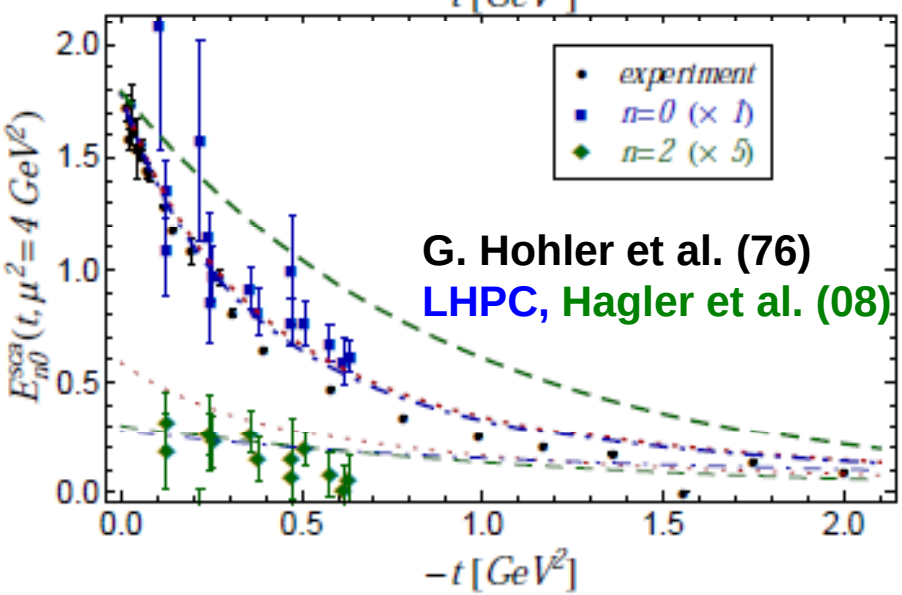
parton distribution functions

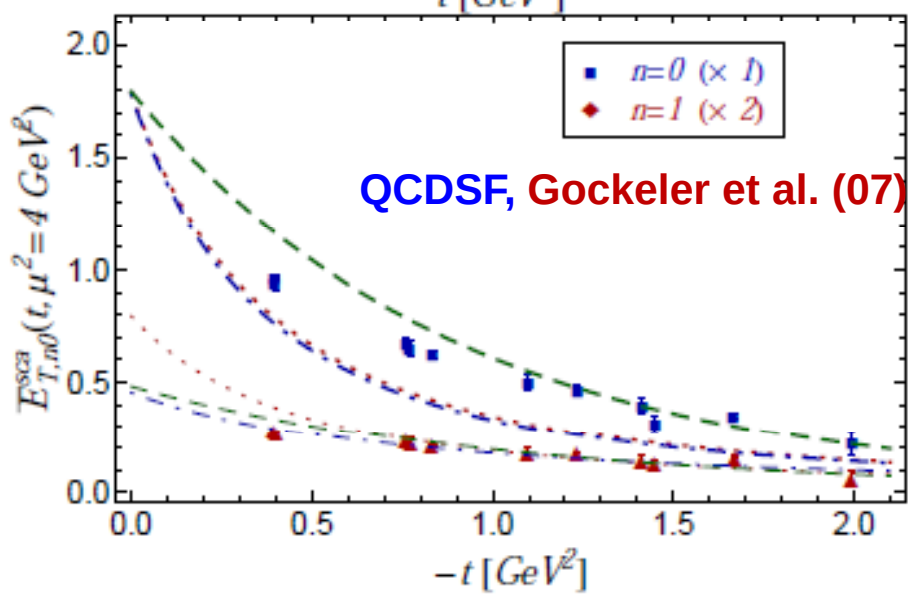
$$q(x) = H(x,\eta=0,t=0)$$

Comparison with Lattice data

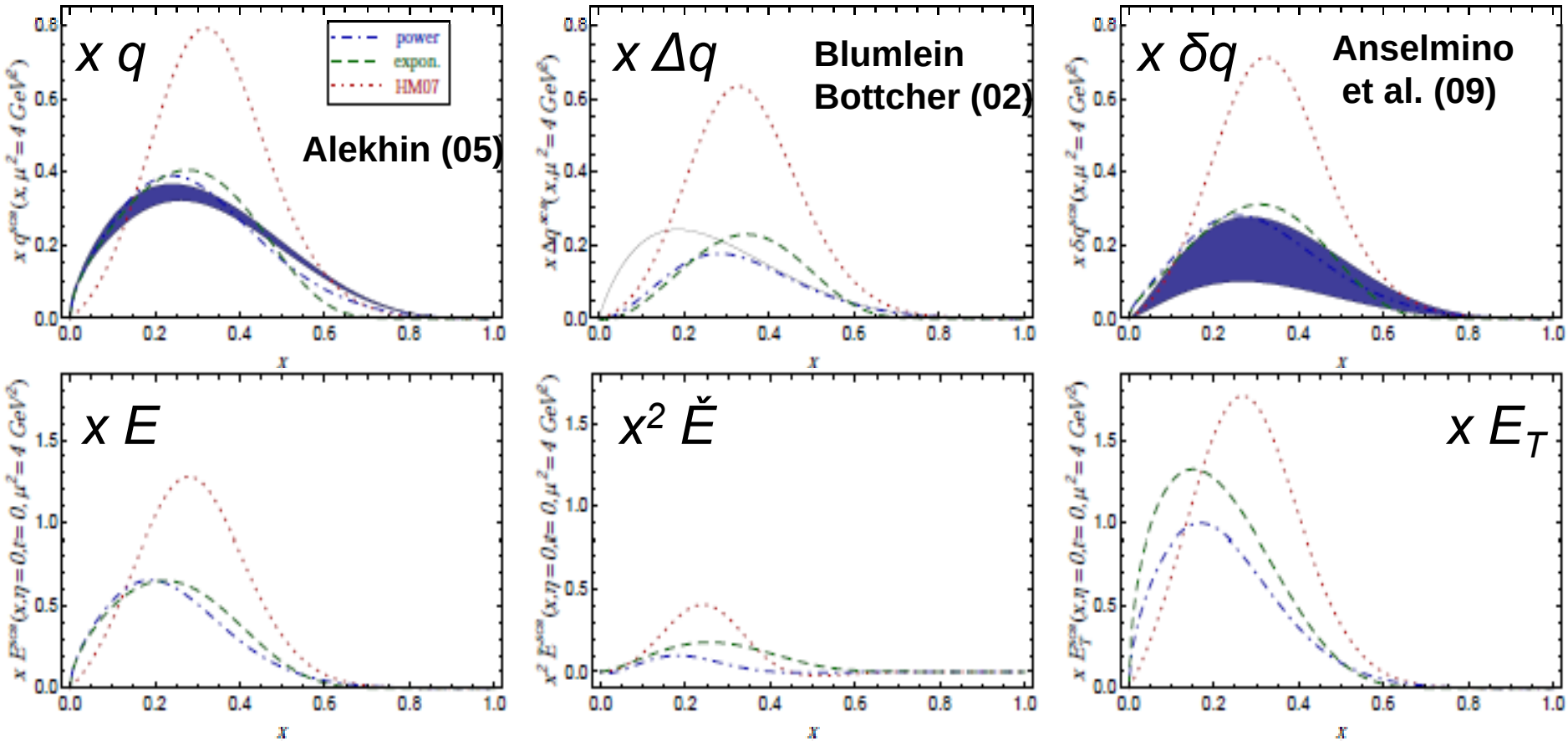






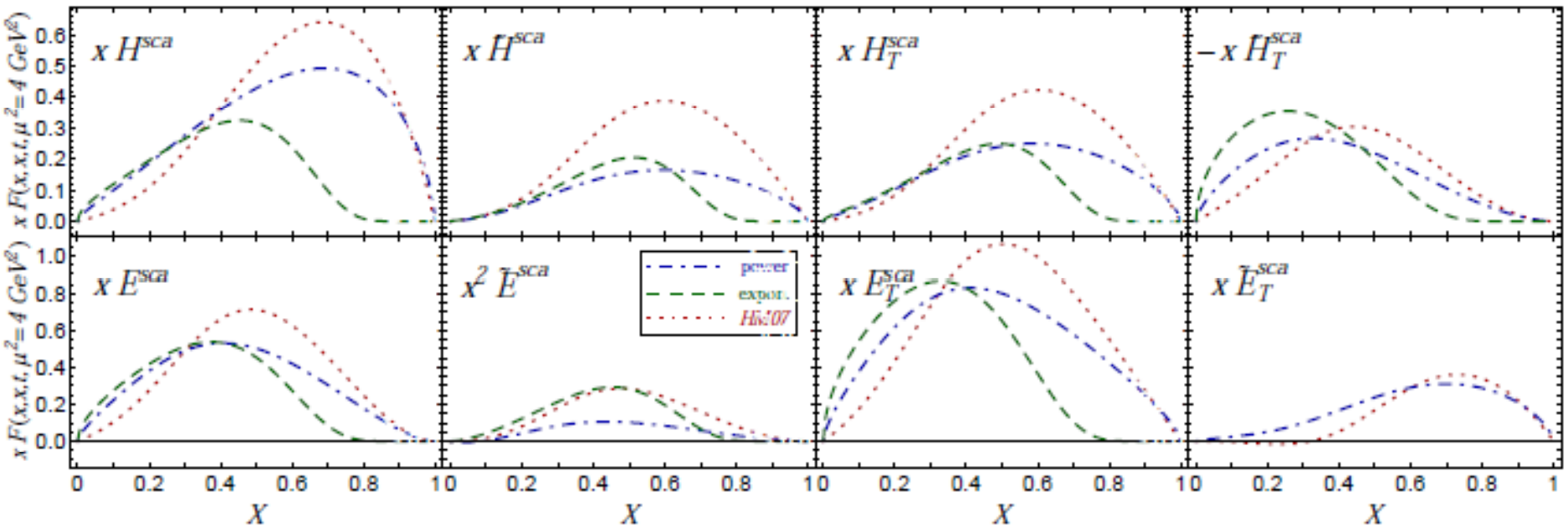


Comparison with PDFs and predictions for zero skewness GPDs



GPD predictions for η=x

GPD at $\eta=x$ is accessible @ LO and fixed Q^2 + subtraction const.



- ✓ *1-x* term "outside" of DD *e* completes polynomiality
- ✓ duality is respected, no ambiguous *D*-term
- ✓ projecting on the *D*-term $D(x,t) = -\lim_{\eta\to\infty} E(x\eta,\eta,t)$
- ✓ "pomeron" exchange $\alpha \gtrsim 1$ we find a large negative *D*-term

[Polyakov et al (01)]

Conclusions

unintegrated PDFs and GPDs are given as LCWF overlap

- scalar diquark spectator model is well understood
- simple for (unintegrated) PDFs and intricate for GPDs
- Regge improvement from s-channel is to reconsider
- a simple model describes phenomenological findings

quark orbital angular momentum is an appropriate label:

- our axial-vector part is so far restricted to forward case
- $|L| \leqq 1$ for scalar and axial-vector exchanges
- $|L| \geqq 2$ requires tensor (rank $\geqq 2$) exchanges
- angular momentum appears everywhere, e.g., in PDFs
- spin puzzle can be simply quantified in terms of LCWFs