Generalized TMDs of the Proton

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Pavia U. & INFN, Pavia

in collaboration with

Cédric Lorcé

Outline

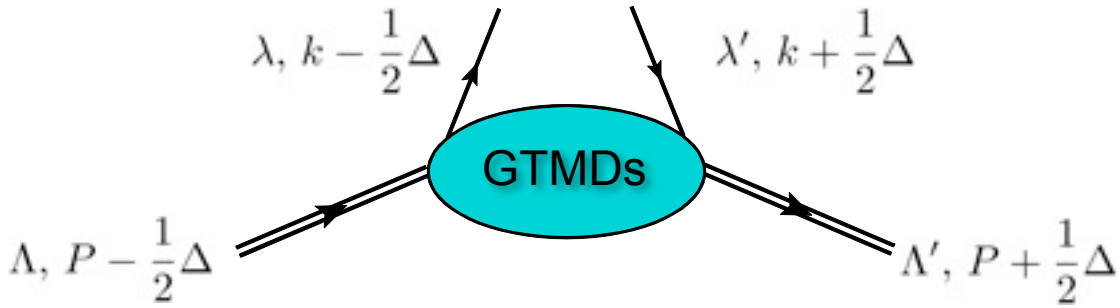
Generalized Transverse Momentum Dependent Parton Distributions (GTMDs)

$$\updownarrow \quad \text{FT } \bar{\Delta}_\perp \leftrightarrow \bar{b}_\perp$$

Wigner Distributions

- Results in light-cone quark models
  - unpolarized quarks in unpolarized nucleon (generalized transverse charge density)

Generalized TMDs



$$W^{[\Gamma]}_{\Lambda'\Lambda} = \frac{1}{2}\int \frac{\mathrm{d}^4 z}{(2\pi)^4} e^{ik\cdot z} \langle p', \Lambda' | \bar{\psi}(-\frac{1}{2}z)\, \Gamma\, \mathcal{W}(-\frac{1}{2}z, \frac{1}{2}z|n)\psi(\frac{1}{2}z)|p, \Lambda\rangle$$

❖ Complete parametrization : 16 GTMDs at twist-2

$$X(x, \xi, \vec{k}_\perp^2, \vec{\Delta}_\perp^2, \vec{k}_\perp \cdot \vec{\Delta}_\perp; \eta) \in \mathcal{C}$$

[Meißner, Metz, Schlegel (2009)]

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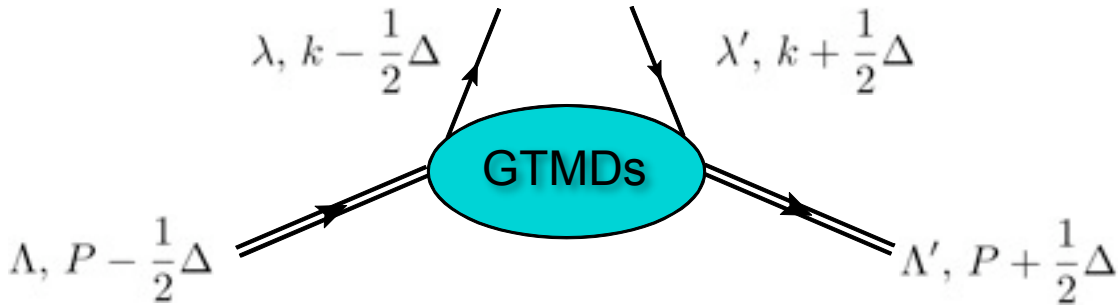

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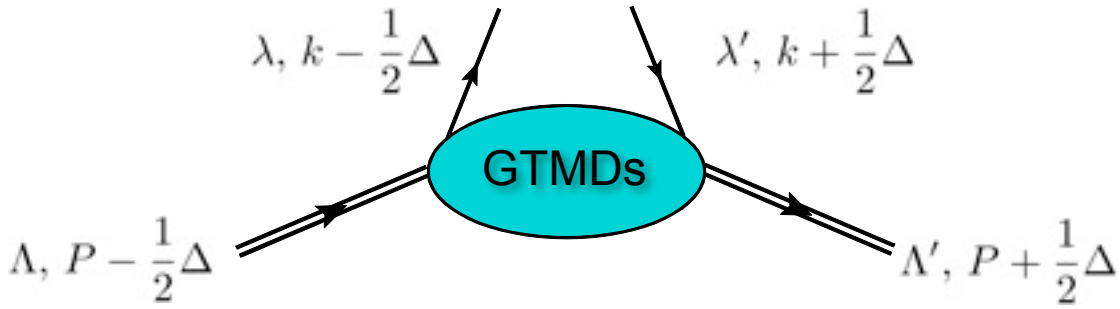

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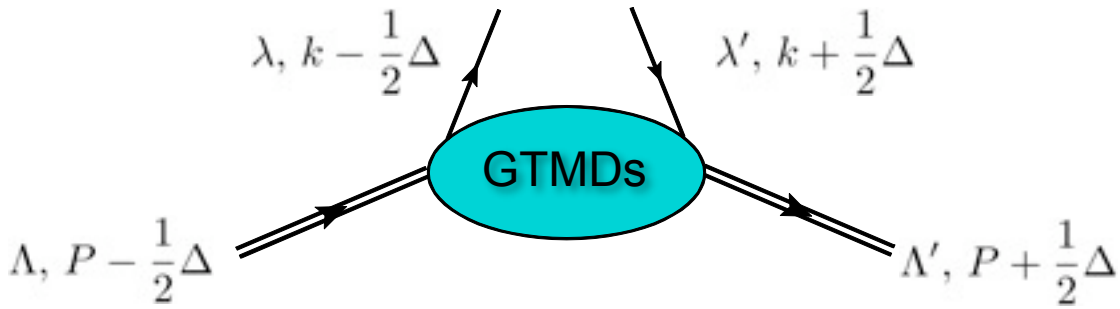

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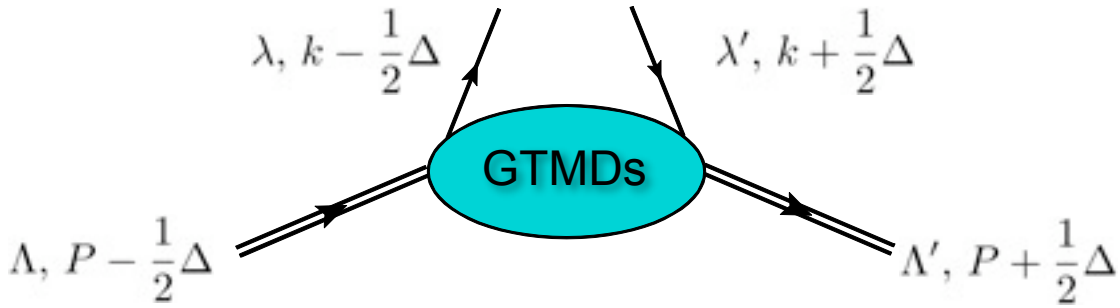

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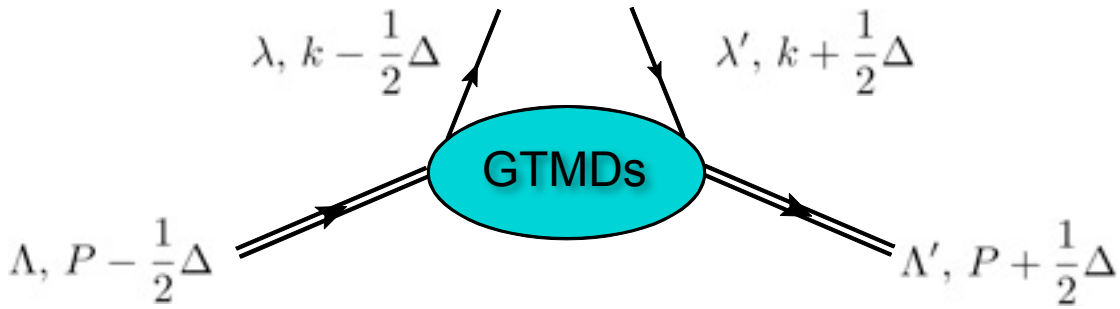

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❖ Fourier Transform $\vec{\Delta}_\perp \leftrightarrow \vec{b}_\perp$: 16 Wigner distributions

$$X(x, \xi, \vec{k}_\perp^2, \vec{b}_\perp^2, \vec{k}_\perp \cdot \vec{b}_\perp; \eta) \in \mathcal{C}$$

[Belitsky, Ji, Yuan (2004)]

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GTMDs

$$x, \xi, k_\perp^2, \vec{k}_\perp \cdot \vec{\Delta}_\perp, \Delta_\perp^2$$

[Lorce, BP, Vanderhaeghen, arXiv:1102.4704]

GTMDs

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$\Delta_\perp \leftrightarrow b_\perp$

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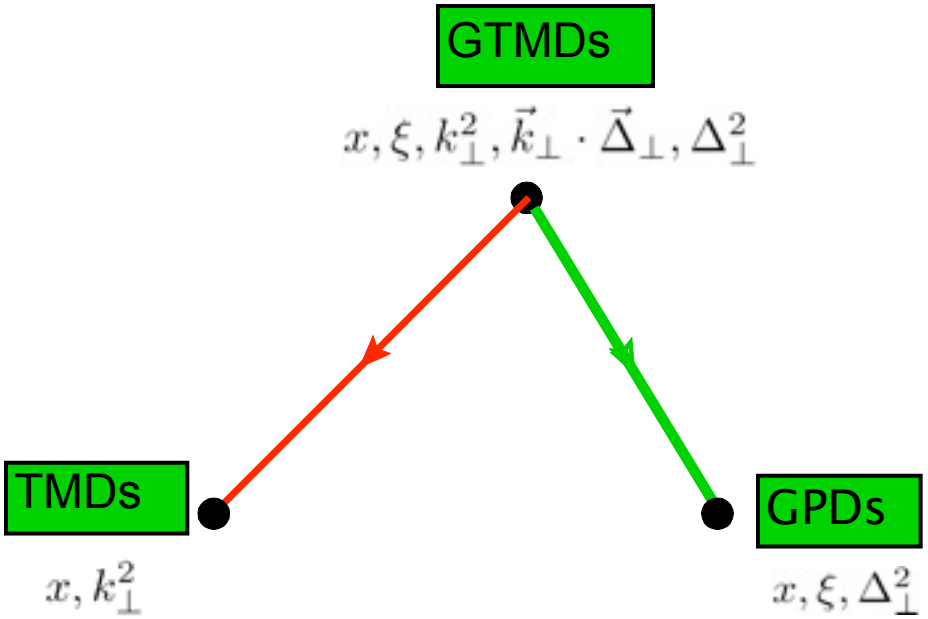

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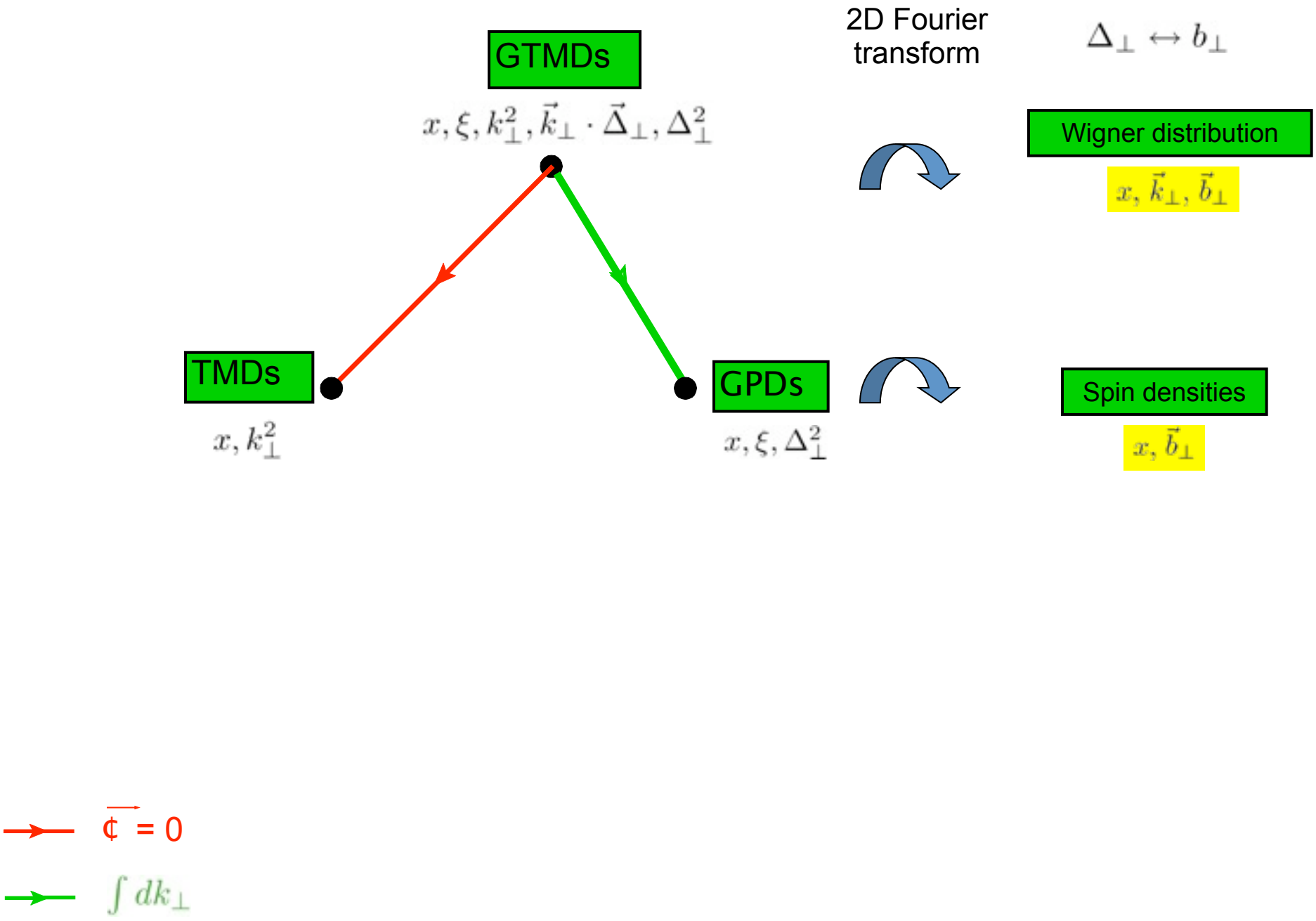

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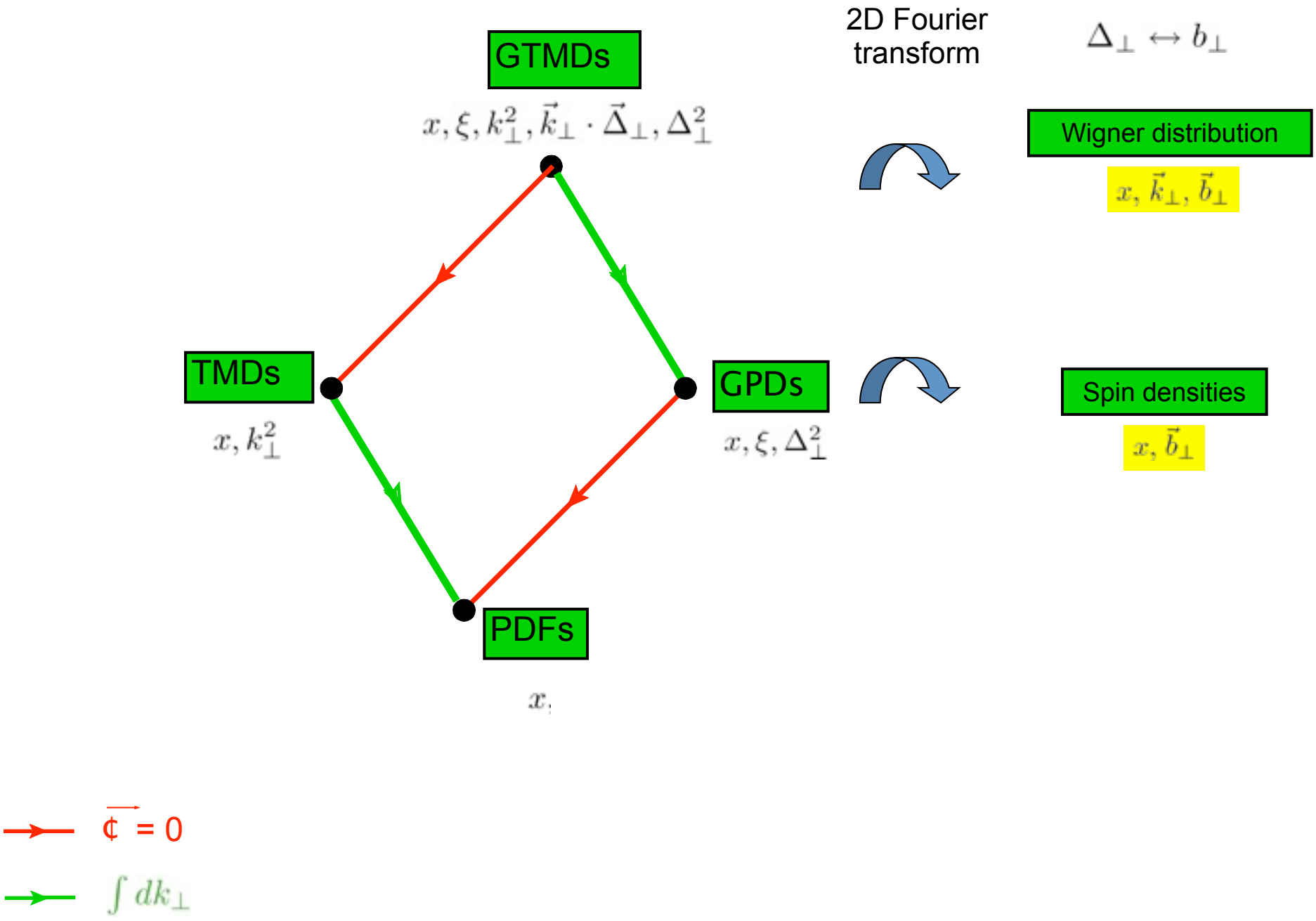

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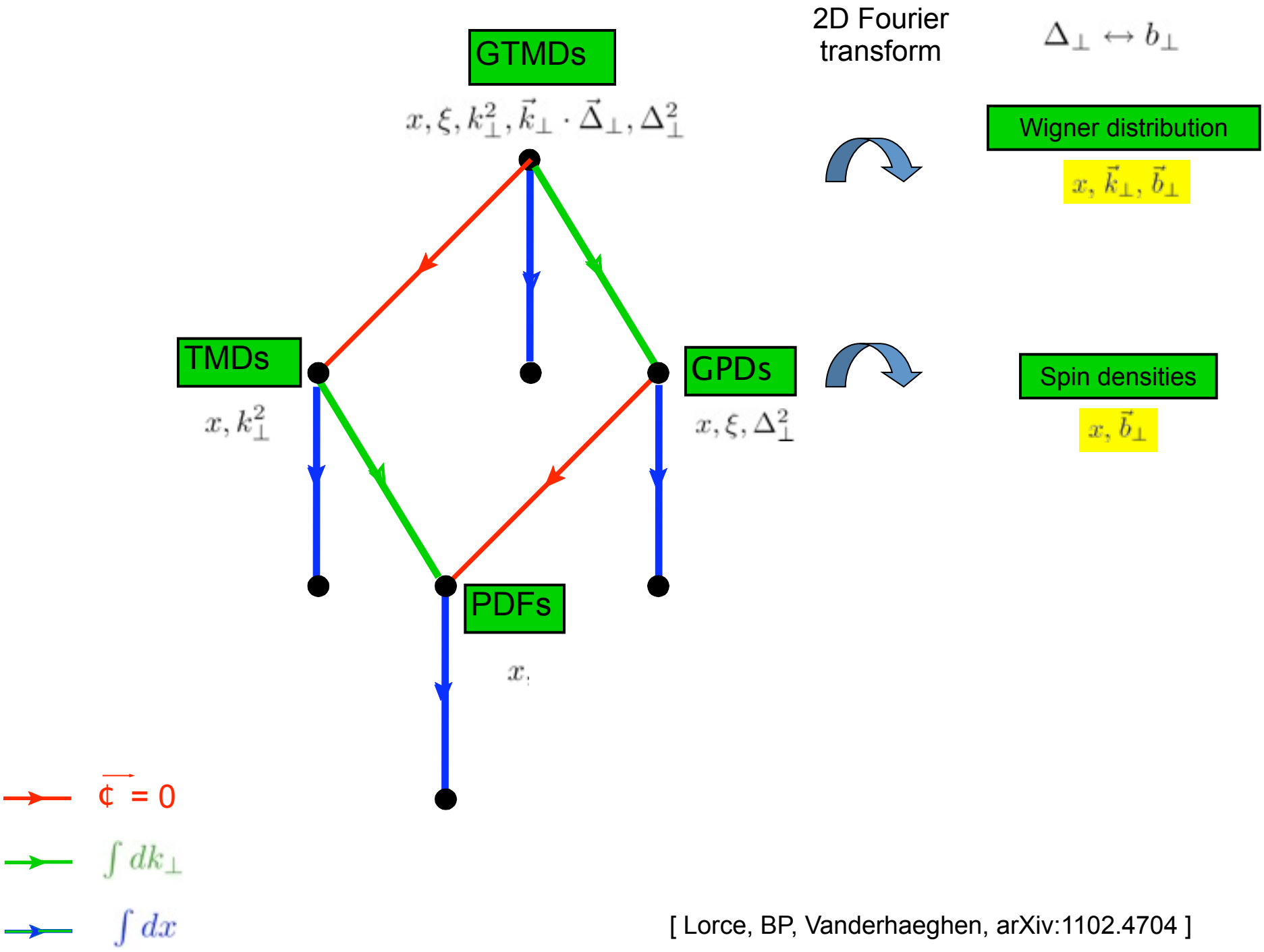

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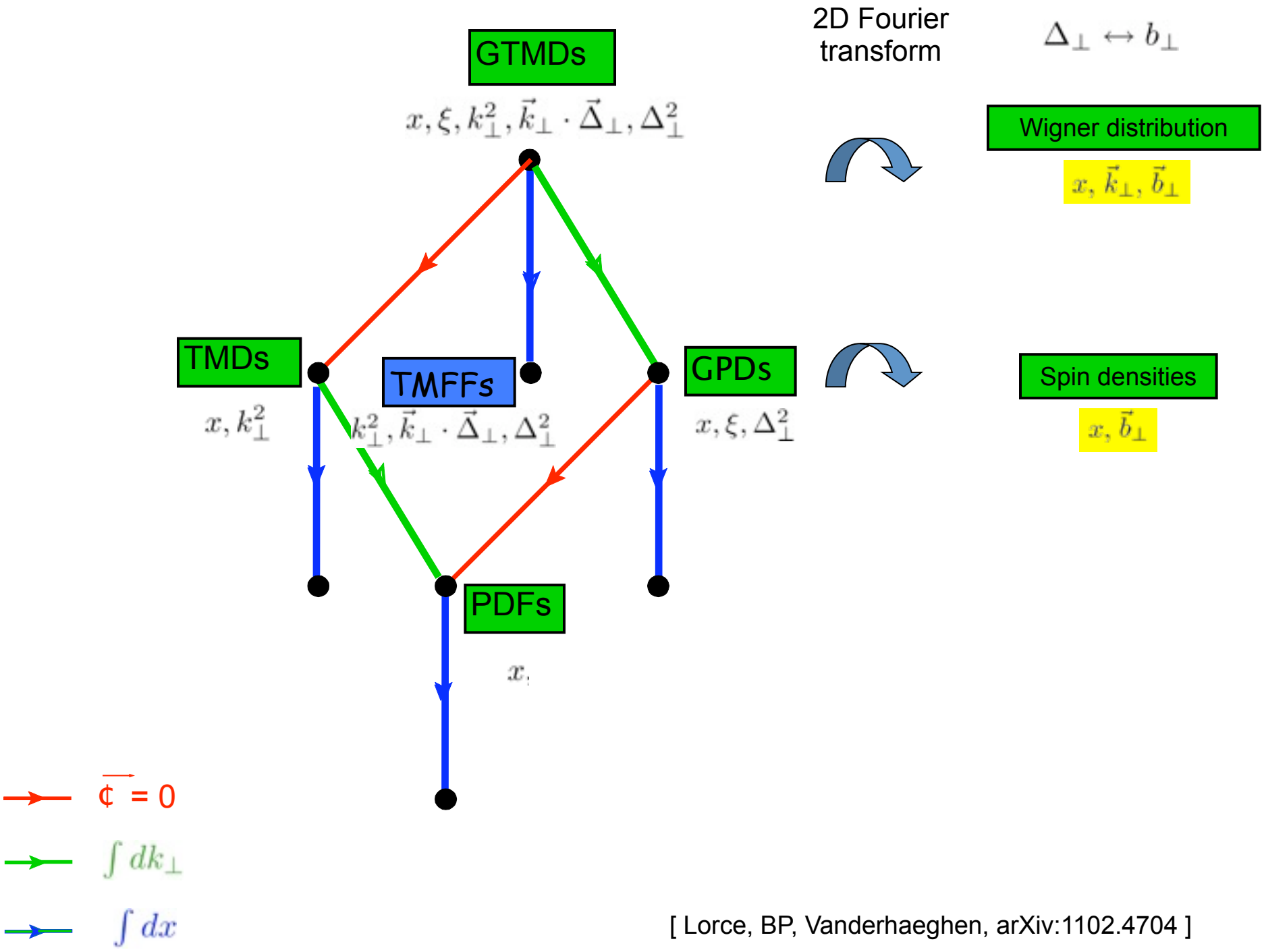

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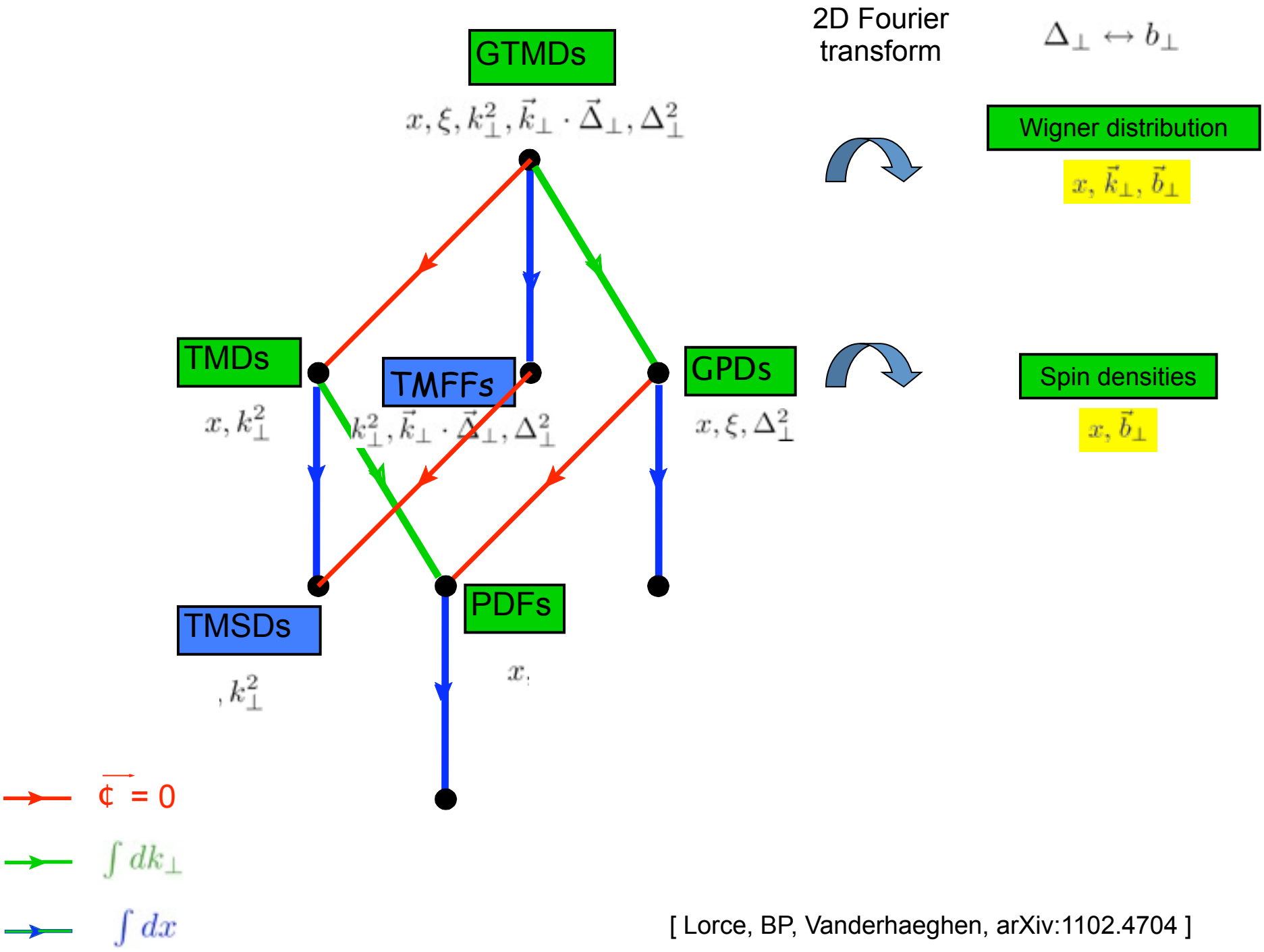

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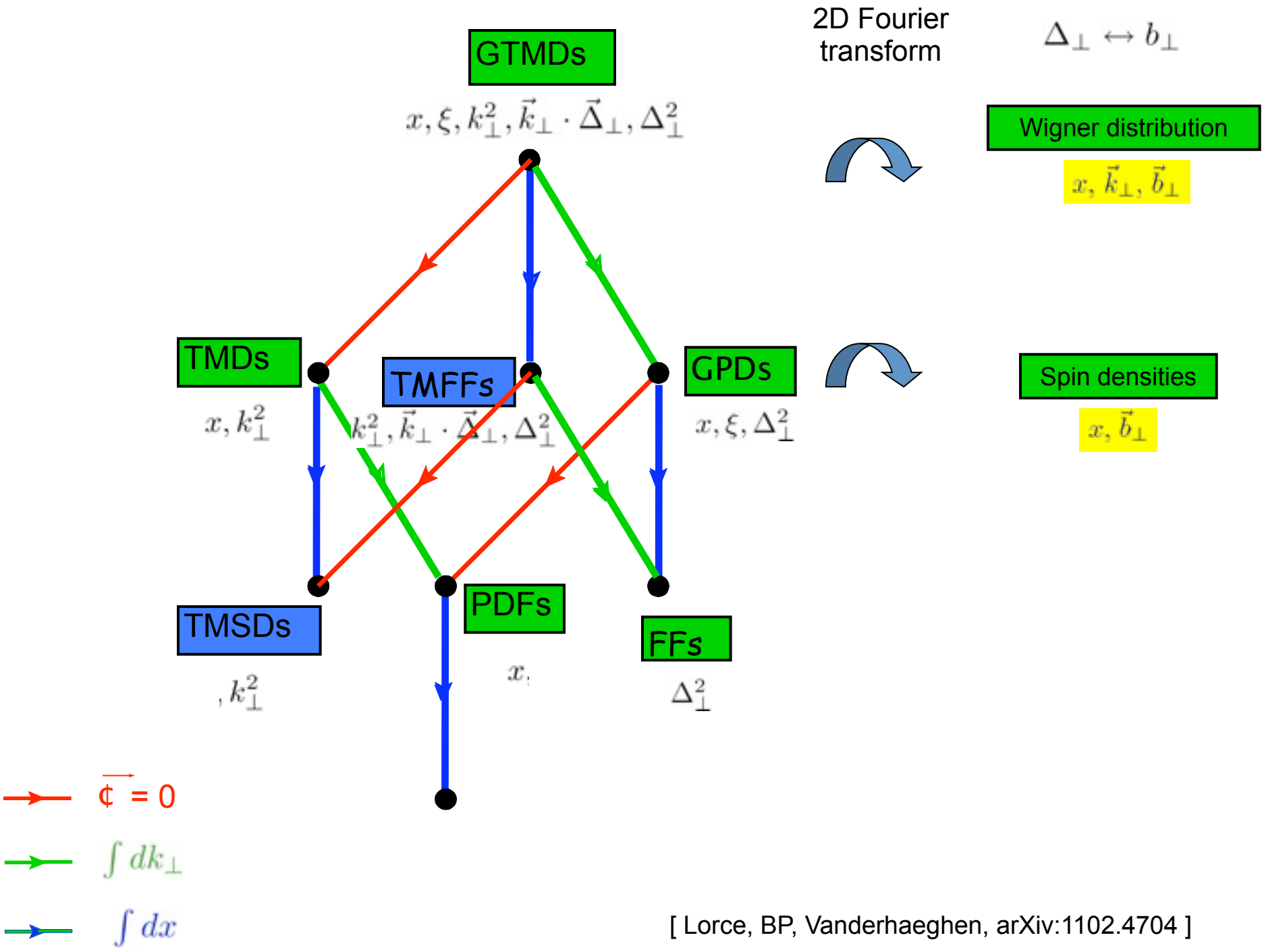

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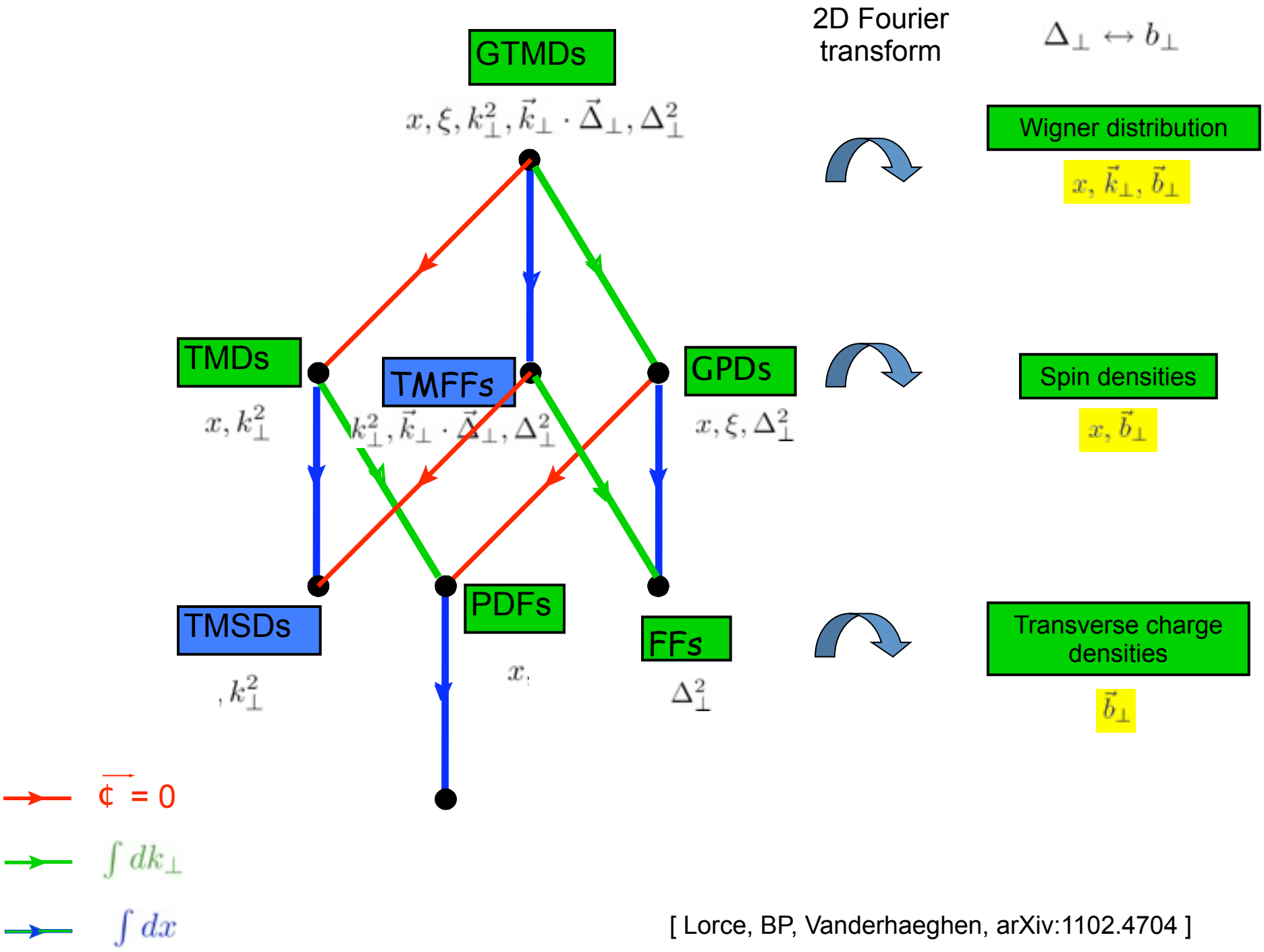

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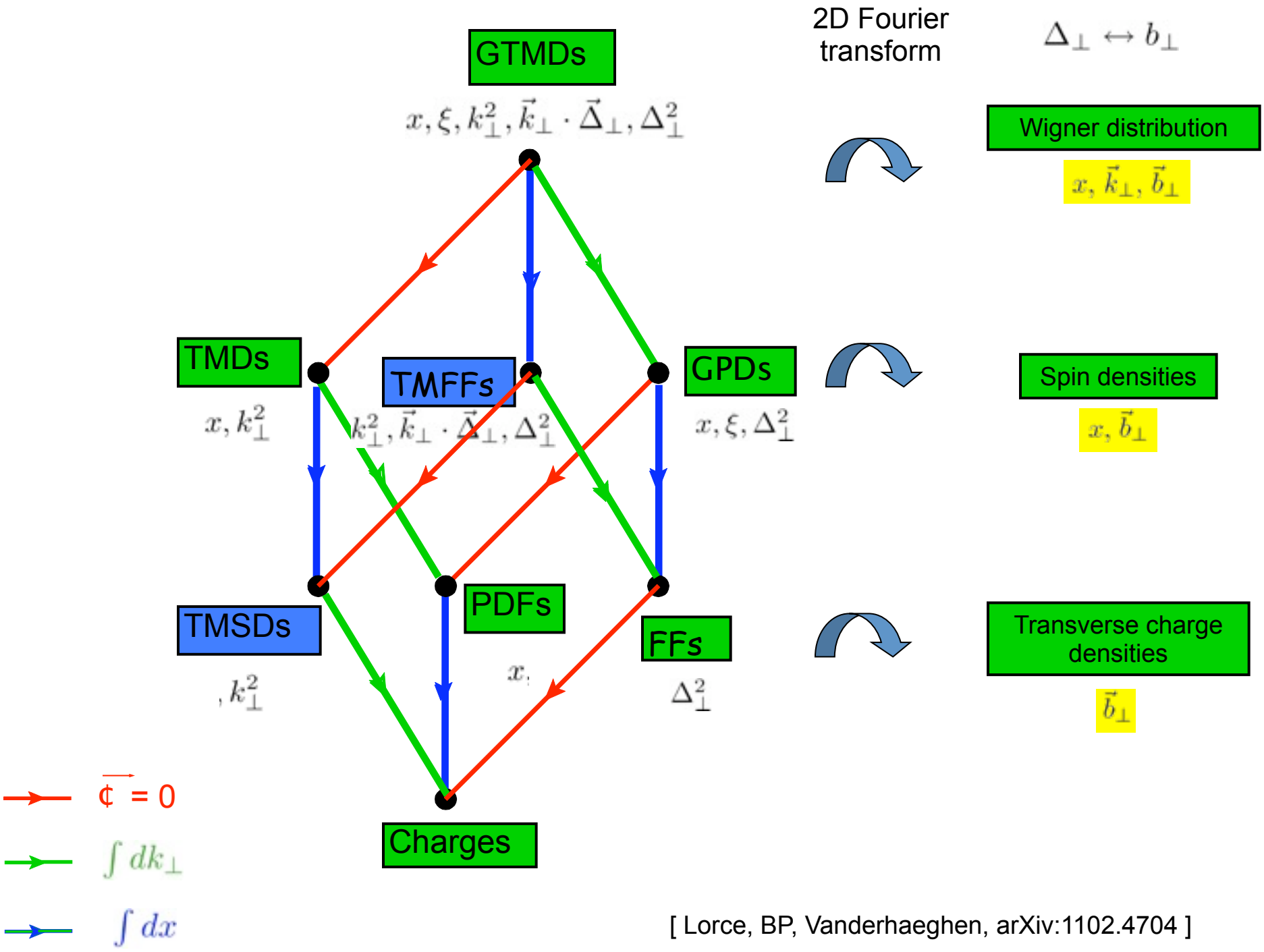

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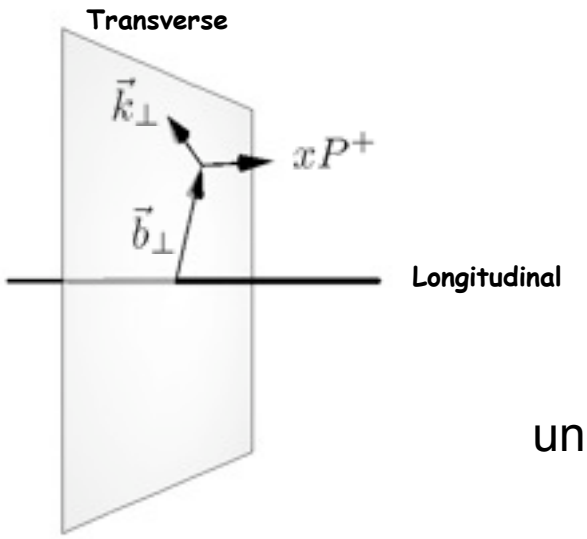
Wigner Distributions



[Wigner (1932)] QM
[Belitsky, Ji, Yuan (04)] QFT (Breit frame)
[Lorce', BP: arXiv:1012.0169] QFT (light cone)

Heisenberg's uncertainty relations → Quasi-probabilistic

GPDs → $\rho(x, b_x, b_y)$

TMDs → $\rho(x, k_x, k_y)$

GTMDs → $\rho(x, b_x, k_y)$ Third 3D picture !

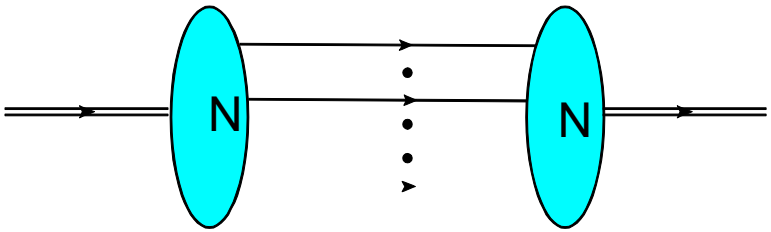
No restrictions from Heisenberg's uncertainty relations

Wigner Distributions

Wigner Distributions

❖ Wigner distributions in QCD:

at »=0 ! diagonal in the Fock-space



N=3 ! overlap of quark light-cone wave-functions

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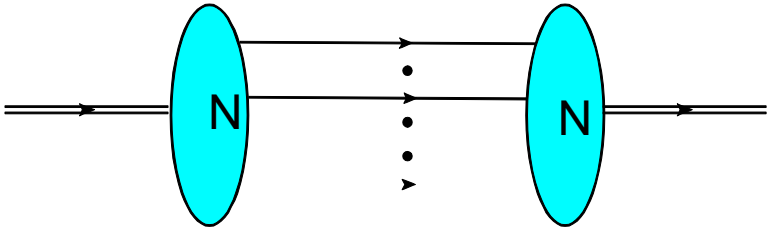

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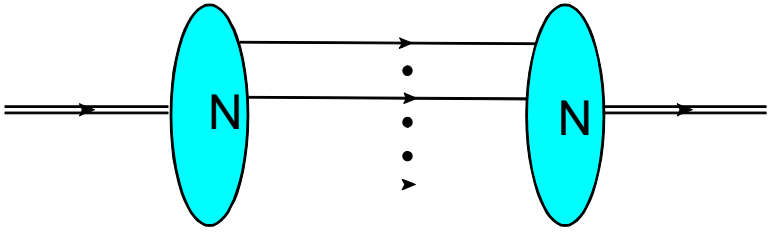

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➡ correlations of quark momentum and position in the transverse plane as function of quark and nucleon polarizations

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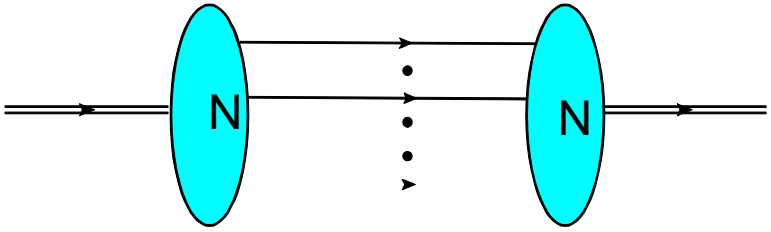

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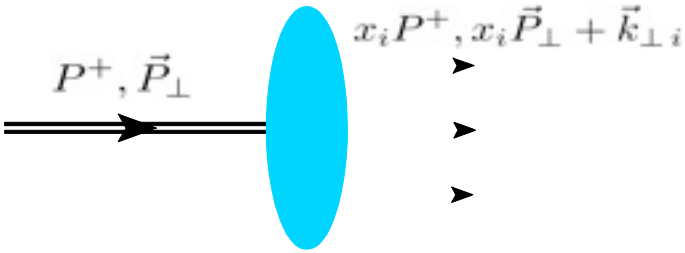
❖ no known experiments can directly measure them ! needs phenomenological models

Light-Cone Quark Models



LCWF: $\Psi^f_{\Lambda,\beta}(x_i, \vec{k}_{\perp,i})$

invariant under boost, independent of P^μ

internal variables: $\sum_{i=1}^{3} x_i = 1, \sum_{i=1}^{3} \vec{k}_{\perp i} = \vec{0}_\perp$

[Brodsky, Pauli, Pinsky, '98]

$$\Psi^f_{\Lambda,\beta}(x_i, \vec{k}_{\perp,i}) = \psi(x_i, \vec{k}_{\perp,i}) \sum_{s_i} \Phi_\Lambda^{s_1 s_2 s_3} \prod_i D^{1/2*}_{s_i \lambda_i}(R_{cf})$$

momentum wf — spin-flavor wf — rotation from canonical spin to light-cone spin

Bag Model, ÂQSM, LCQM, Quark-Diquark and Covariant Parton Models

<u>Common assumptions :</u>

- No gluons
- Independent quarks

[Lorce, BP, Vanderhaeghen, arXiv:1102.4704]

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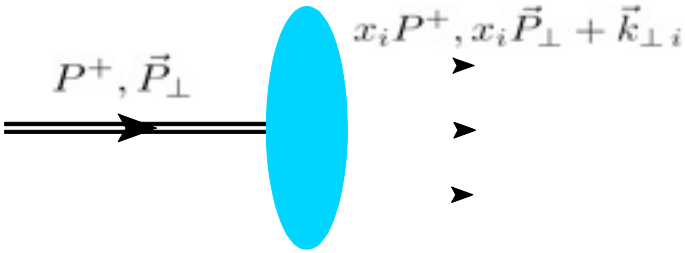

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Light-Cone Constituent Quark Model

- momentum-space wf $\Psi(k_i) = \frac{N}{(M_0^2 + \beta^2)^\gamma}$ $M_0 = \sum_i^3 \sqrt{m_i^2 + \vec{k}_i^2}$

  [Schlumpf, Ph.D. Thesis, hep-ph/9211255]

  β, γ parameters fitted to anomalous magnetic moments of the nucleon

  N: normalization constant

- spin-structure: $q_\lambda^{LC}(k) = D_{\lambda s}^{(1/2)*} q_s^C(k)$ $D_{\lambda s}^{(1/2)*}(k) = \frac{1}{|\vec{K}|} \begin{pmatrix} K_z & K_L \\ -K_R & K_z \end{pmatrix}$

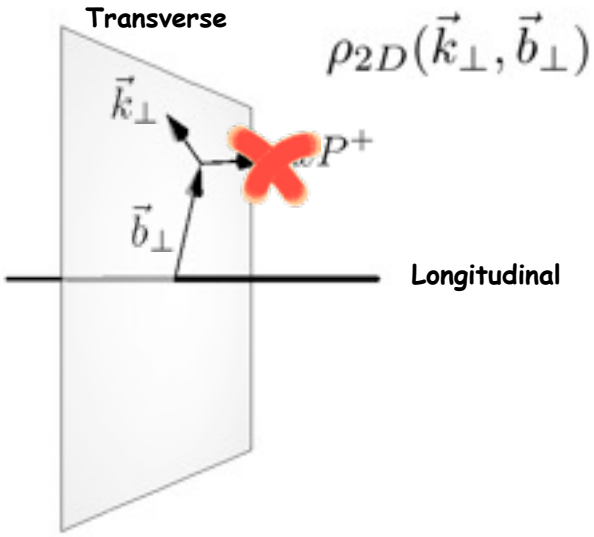
  free quarks → $K_z = m + x\mathcal{M}_0$ $\vec{K}_\perp = \vec{k}_\perp$ (Melosh rotation)

- SU(6) symmetry

Applications of the model to:

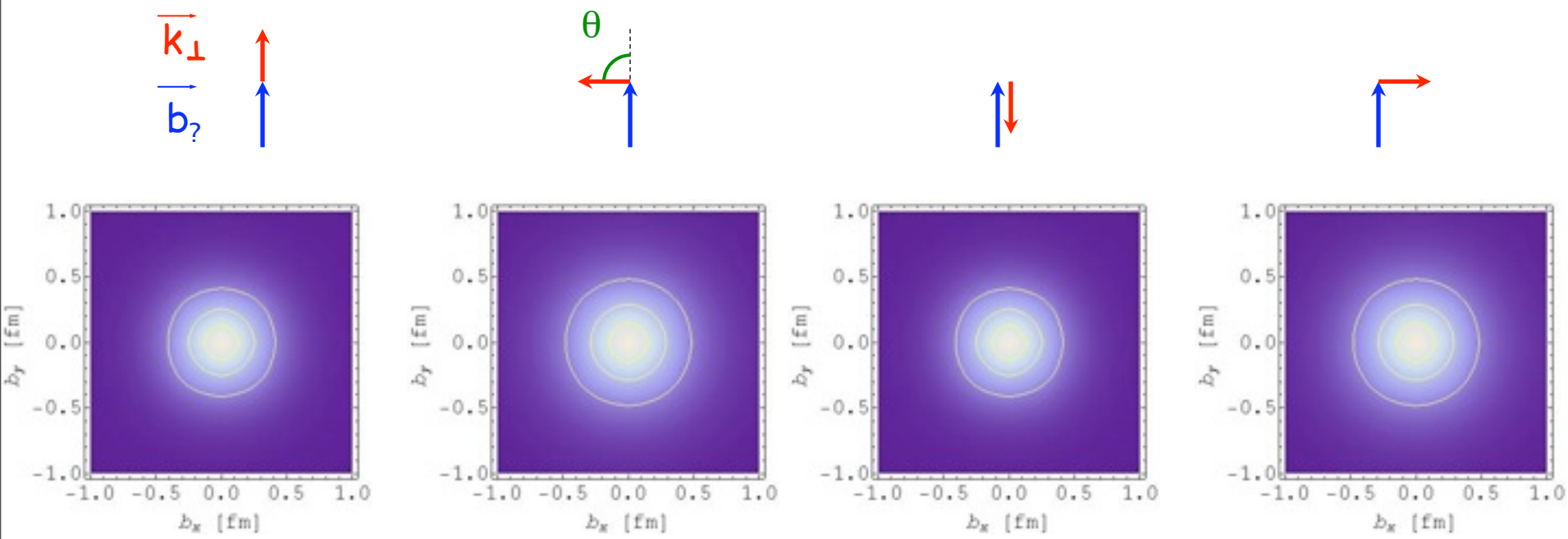
GPDs and form factors: BP, Boffi, Traini (2003)-(2005);
TMDs: BP, Cazzaniga, Boffi (2008); BP, Yuan (2010);
Azimuthal asymmetries: Schweitzer, BP, Boffi, Efremov (2009)

Example: Unpol. up Quark in Unpol. Proton



Generalized Transverse Charge Density

fixed angle between $\vec{k}_?$ and $\vec{b}_?$ and fixed value of $|\vec{k}_?|$



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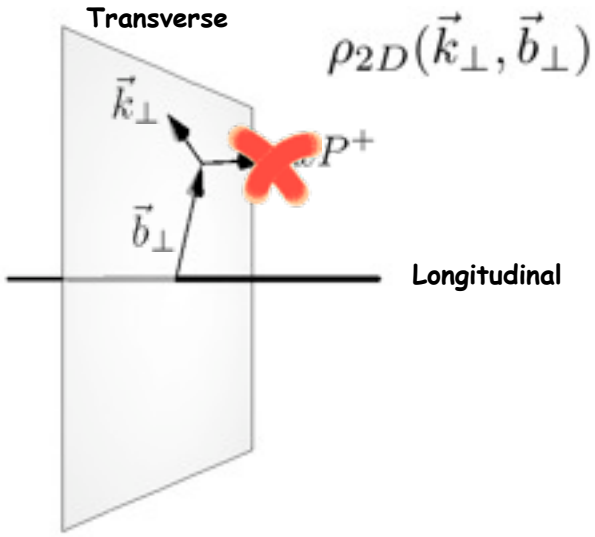

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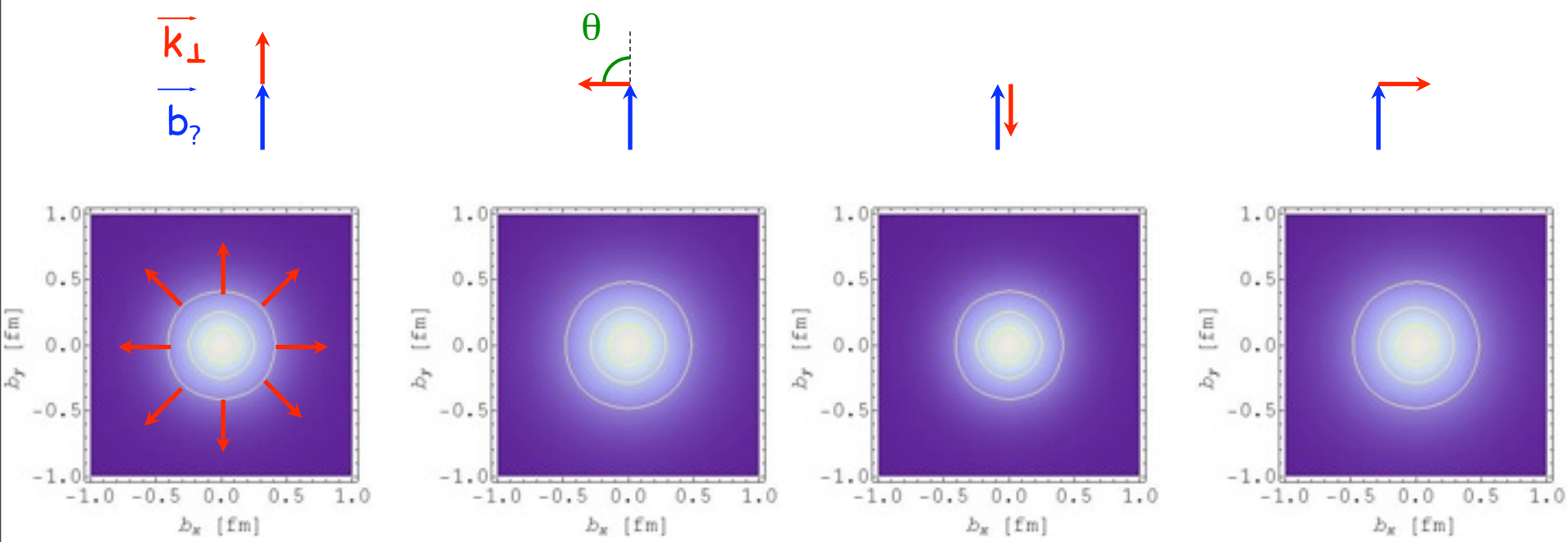

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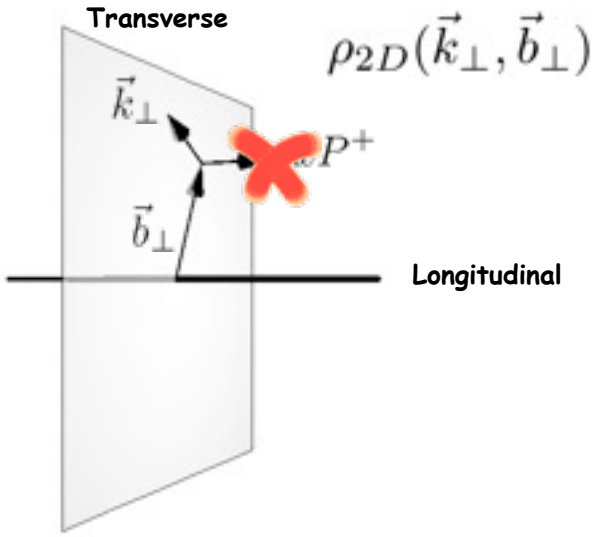

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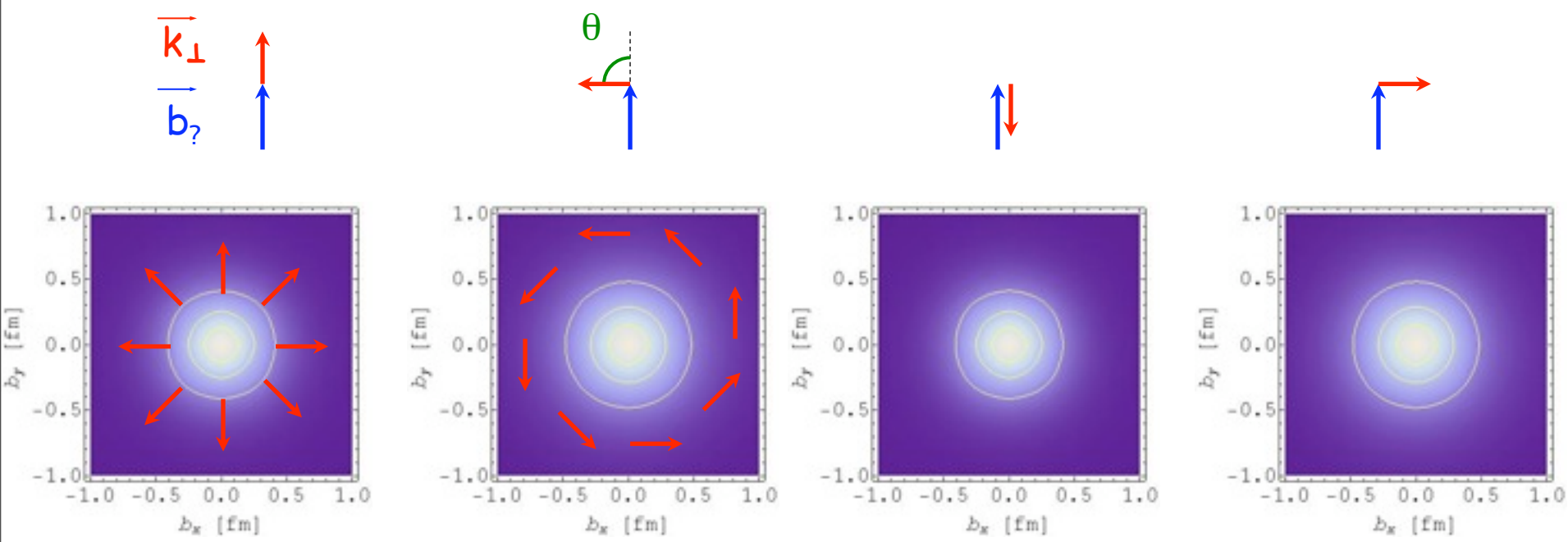

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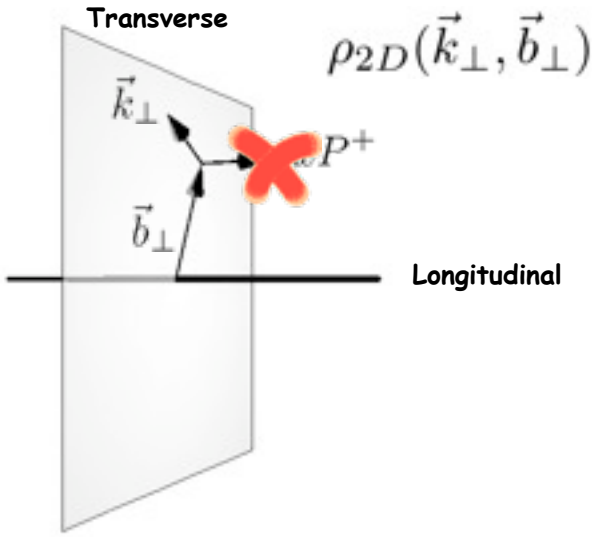

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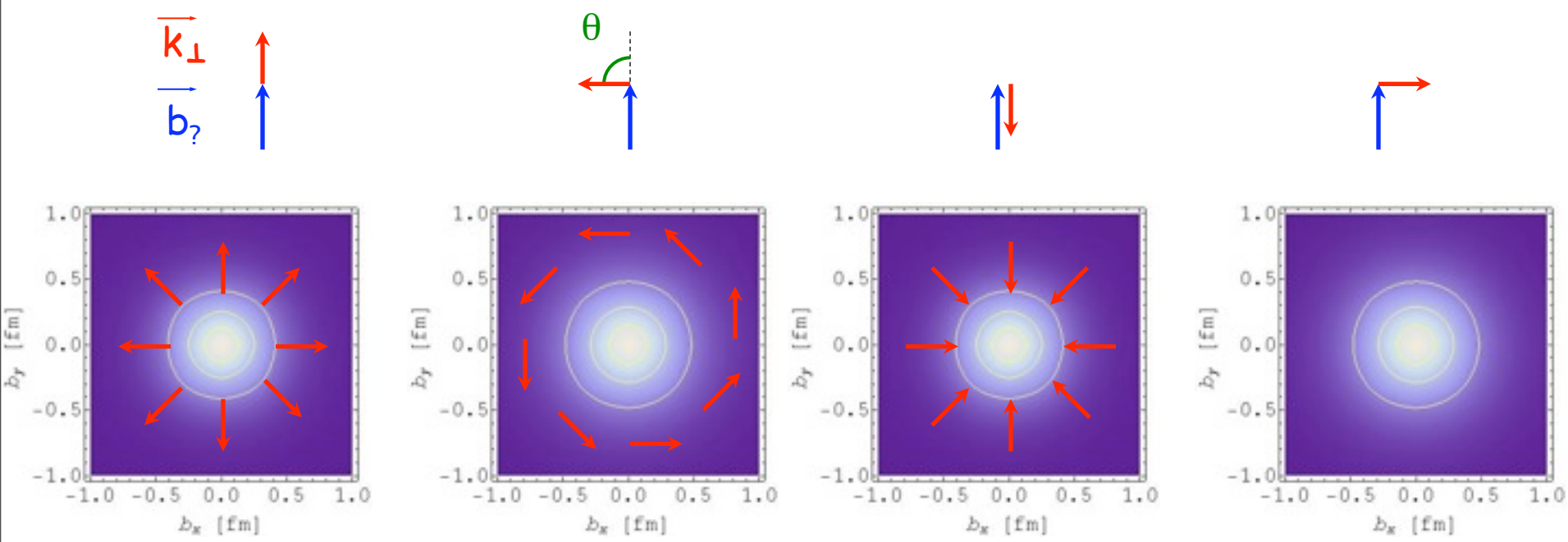

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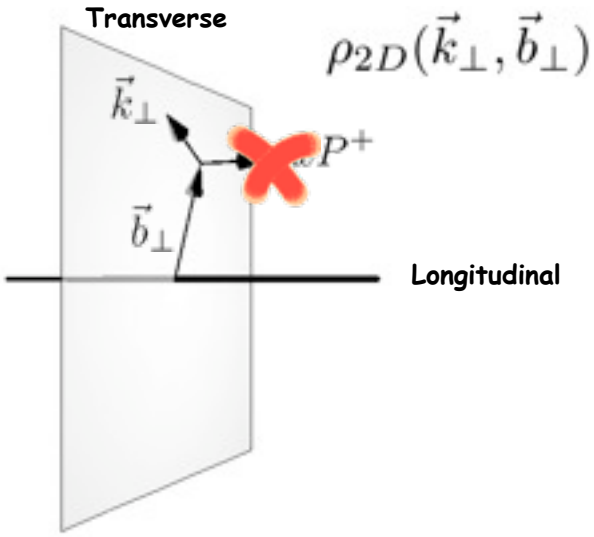

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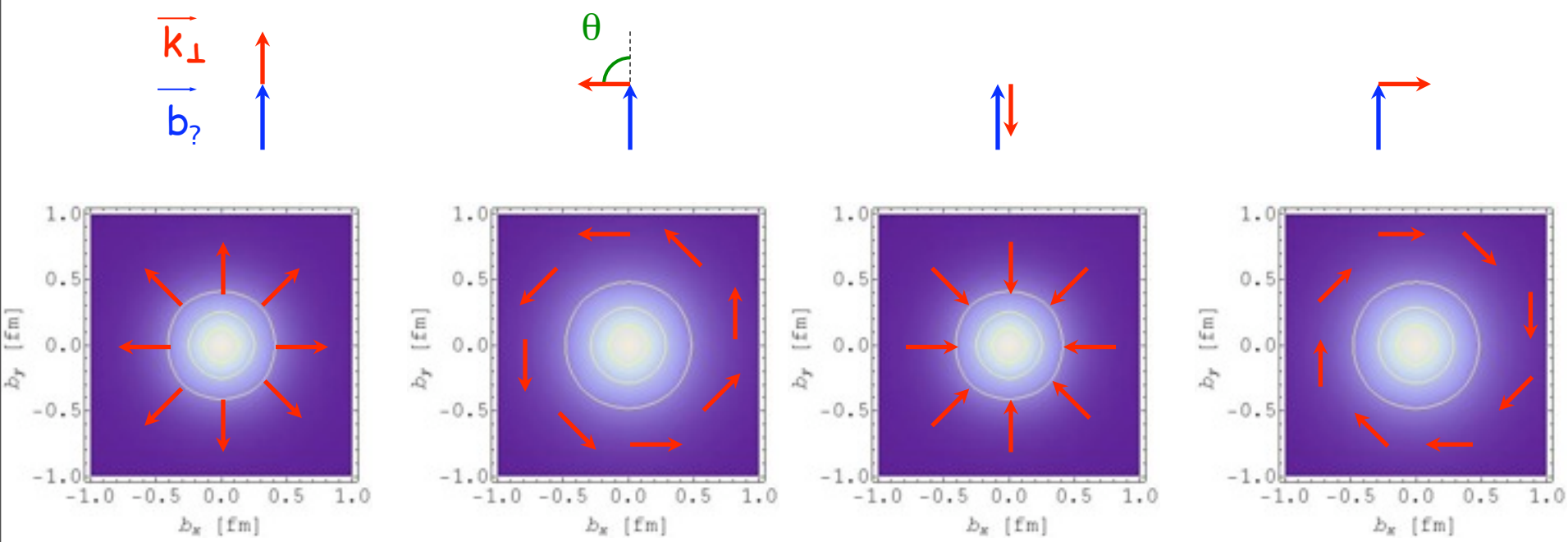

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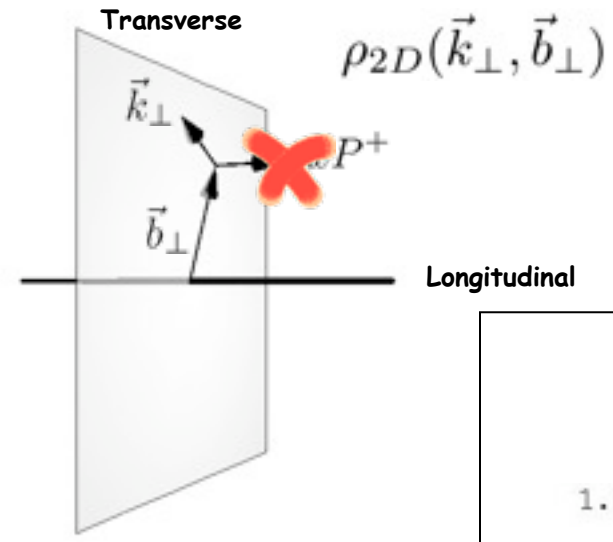
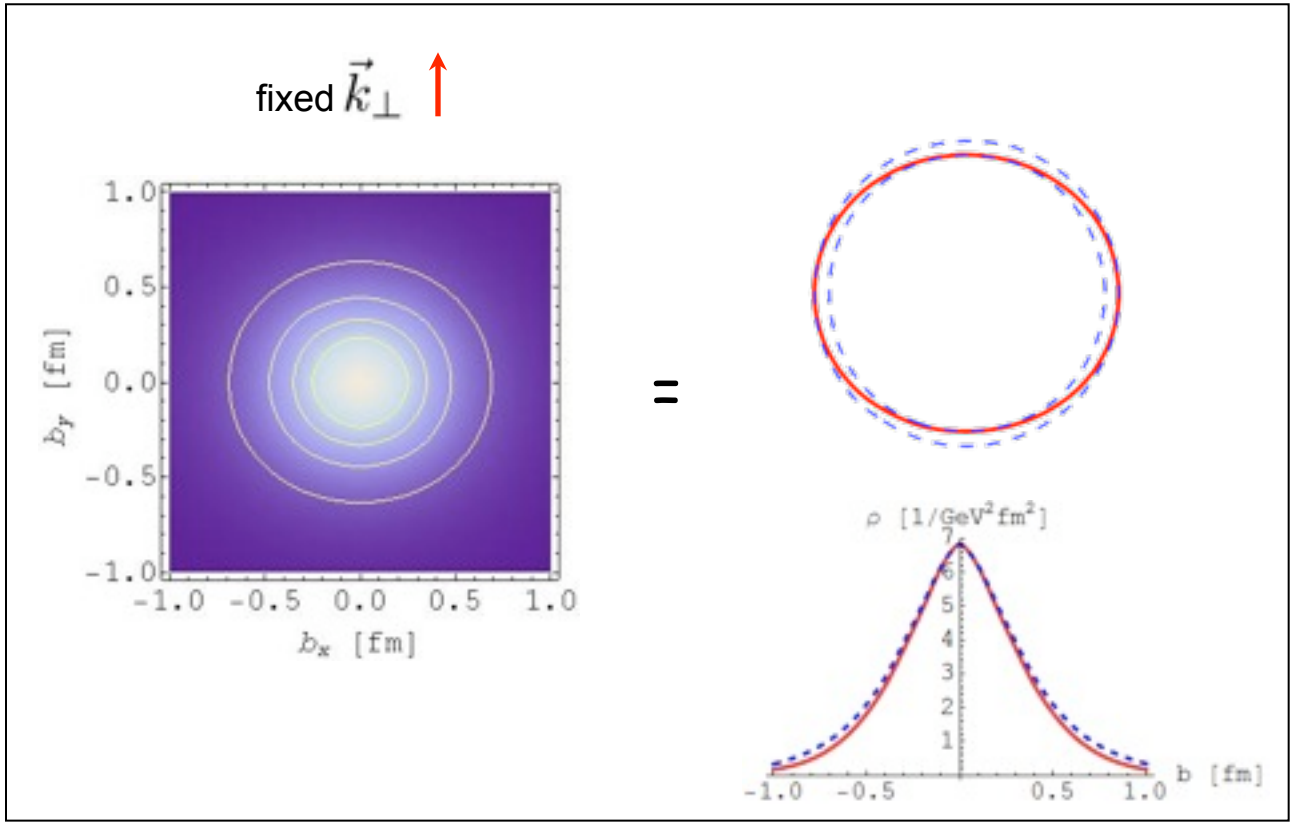
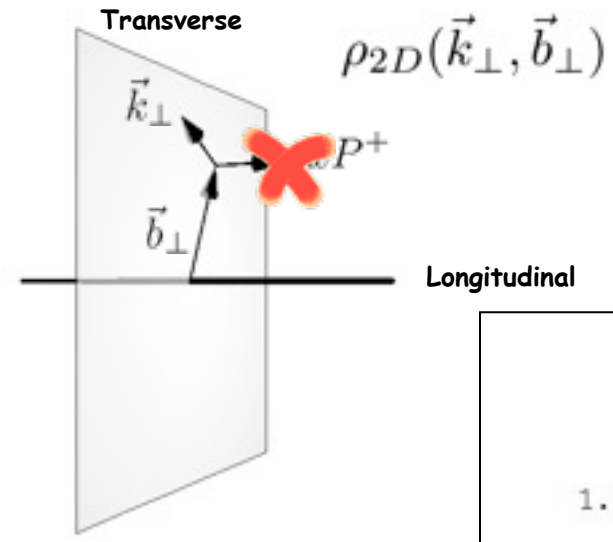


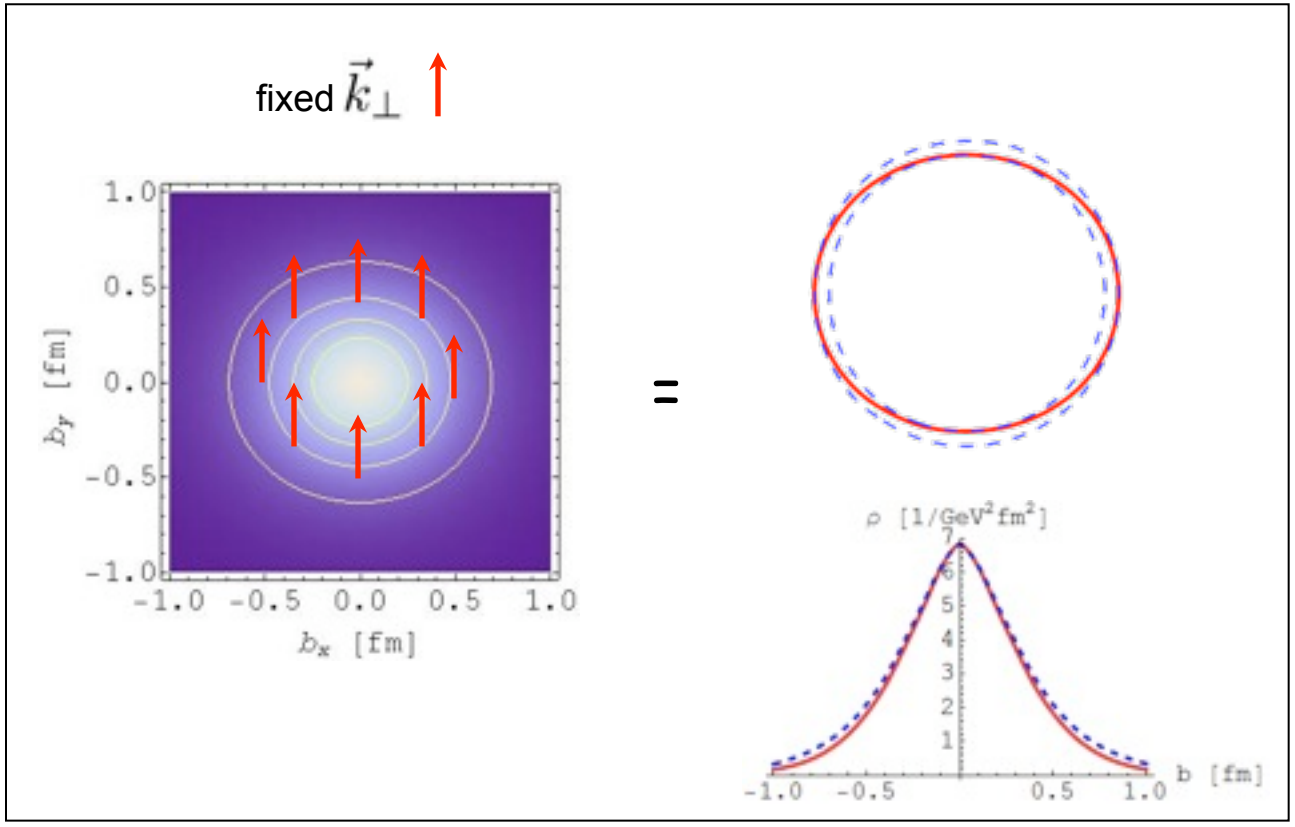


3Q light-cone model

[Lorce', BP, in preparation]

Example: Unpol. up Quark in Unpol. Proton

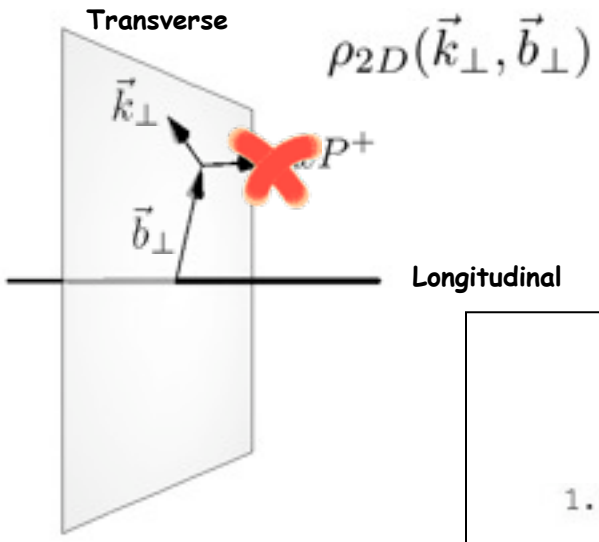


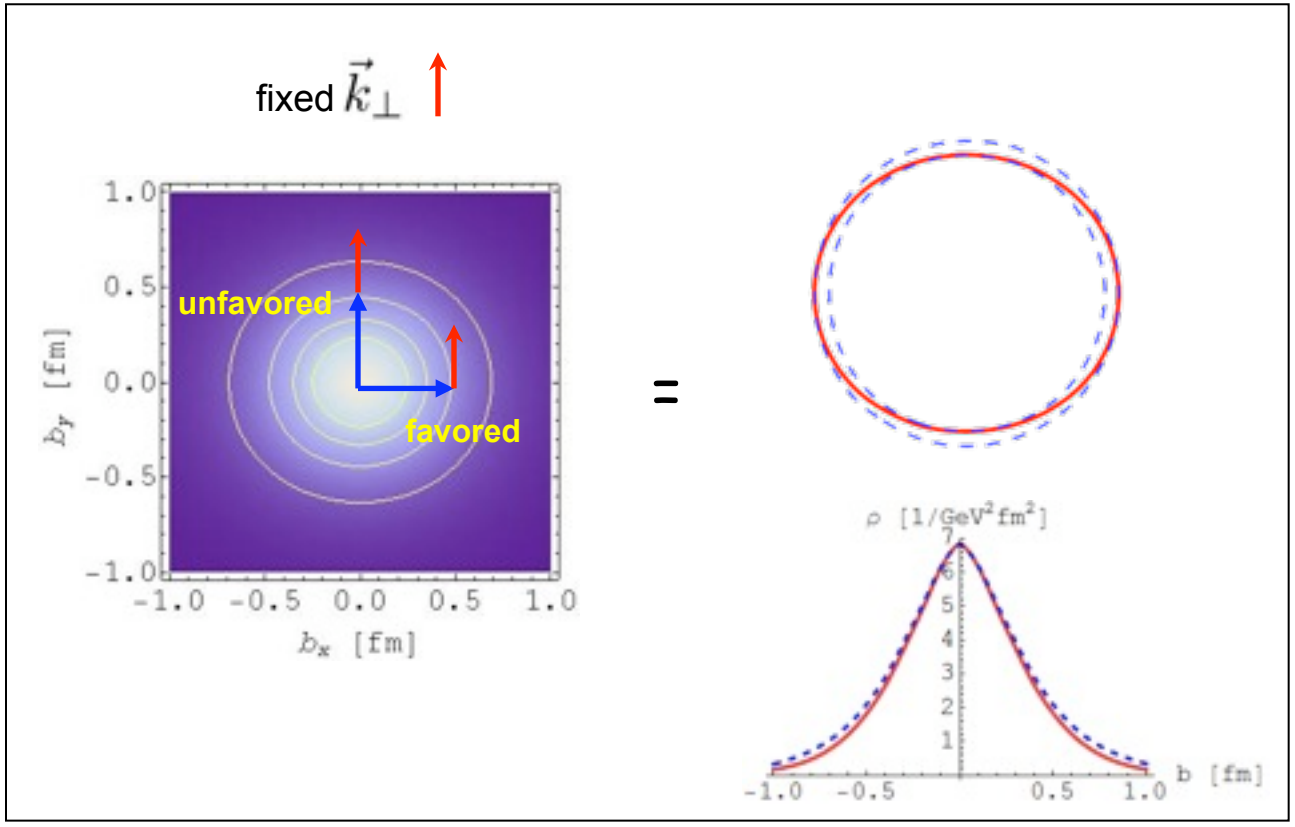


3Q light-cone model

[Lorce', BP, in preparation]

Example: Unpol. up Quark in Unpol. Proton

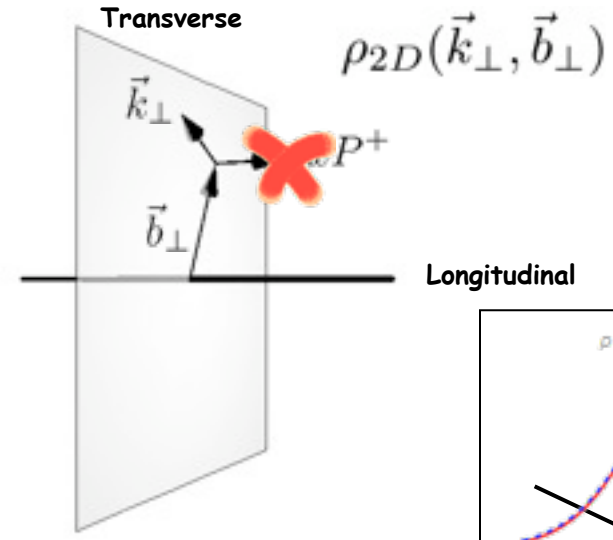




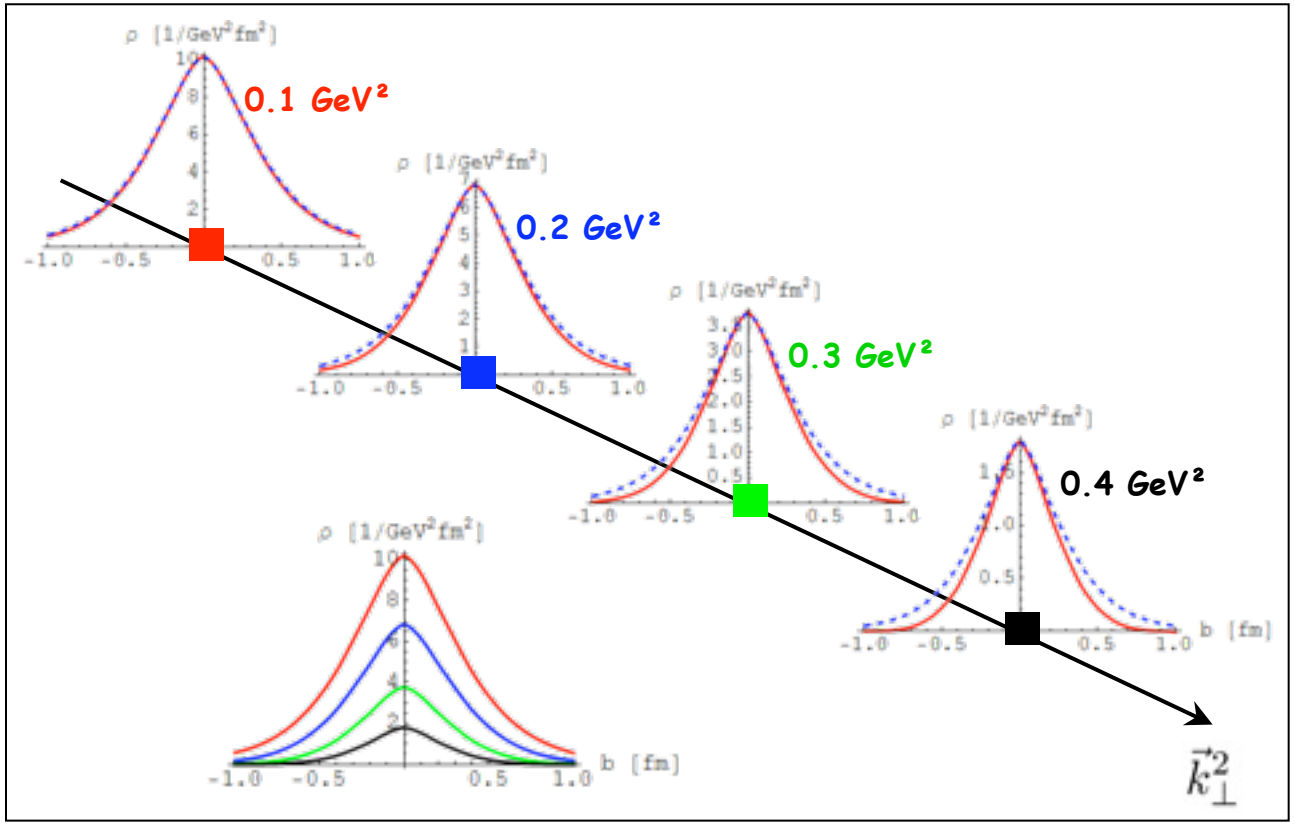
3Q light-cone model

[Lorce', BP, in preparation]

Example: Unpol. up Quark in Unpol. Proton



$$\rho_{2D}(\vec{k}_\perp, \vec{b}_\perp)$$



3Q light-cone model

[Lorce', BP, in preparation]

Summary

- GTMDs $ Wigner Distributions
  - the most complete information on partonic structure of the nucleon

- General Formalism for 3-quark contribution to GTMDs
  - applicable for large class of models: LCQMs, ÂQSM, Bag model

- Results for Wigner distributions in the transverse plane
  - anisotropic distribution in $\vec{k}_?$ for unpolarized quarks in unpolarized nucleon
  - non-trivial correlations between $\vec{b}_?$ and $\vec{k}_?$ due to orbital angular momentum

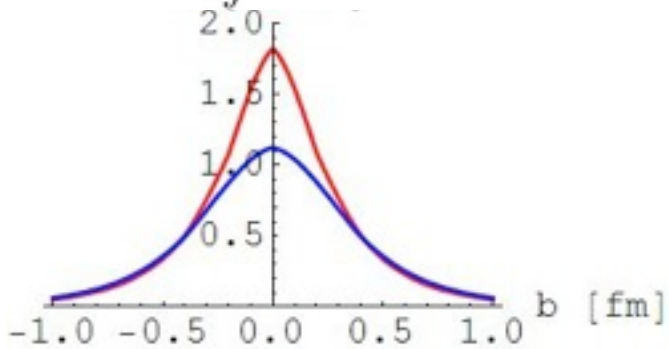
❖ Integrating over $\vec{b}_?$ ➡ $f_1^{(1)}(k_\perp^2) = \int \mathrm{d}x\, f_1(x, k_\perp^2)$

- Integrating over $\vec{b}_?$ ➡ $f_1^{(1)}(k_\perp^2) = \int \mathrm{d}x\, f_1(x, k_\perp^2)$
- Integrating over $\vec{k}_?$ ➡ charge density in the transverse plane $\vec{b}_?$

$$\rho^q(b_\perp^2) = e^q \int \mathrm{d}^2\Delta_\perp e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp} F_1^q(\Delta_\perp^2)$$

- Integrating over $\vec{b}_?$ ➡ $f_1^{(1)}(k_\perp^2) = \int \mathrm{d}x\, f_1(x, k_\perp^2)$
- Integrating over $\vec{k}_?$ ➡ charge density in the transverse plane $\vec{b}_?$

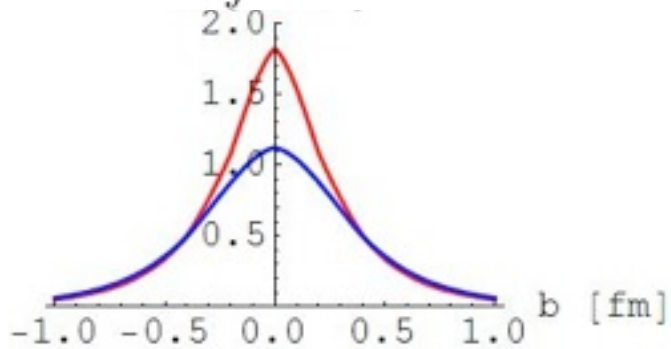
$$\rho^q(b_\perp^2) = e^q \int \mathrm{d}^2\Delta_\perp e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp} F_1^q(\Delta_\perp^2)$$



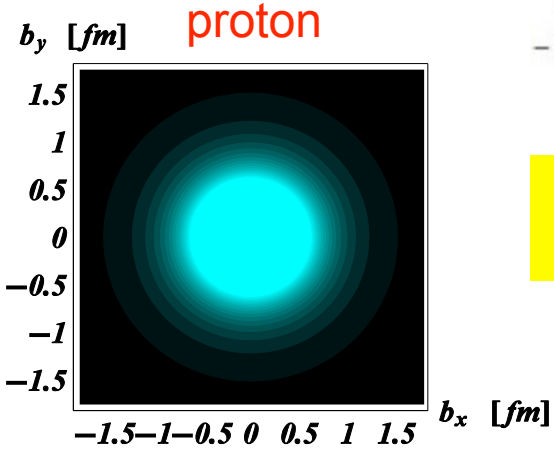
❖ Integrating over $\vec{b}_?$ ➡ $f_1^{(1)}(k_\perp^2) = \int \mathrm{d}x\, f_1(x, k_\perp^2)$

❖ Integrating over $\vec{k}_?$ ➡ charge density in the transverse plane $\vec{b}_?$

$$\rho^q(b_\perp^2) = e^q \int \mathrm{d}^2\Delta_\perp e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp} F_1^q(\Delta_\perp^2)$$

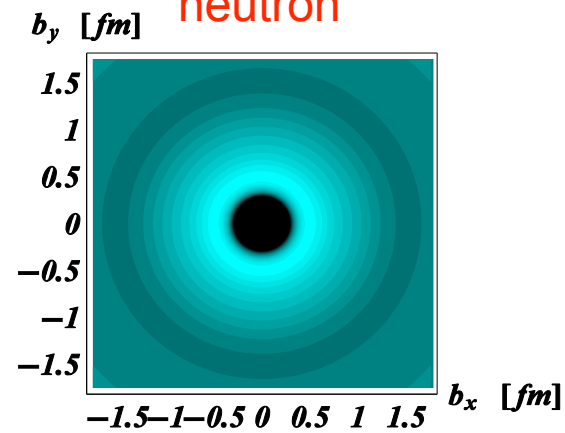


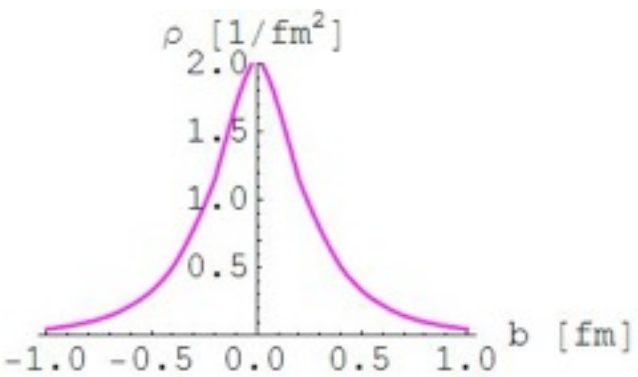
proton

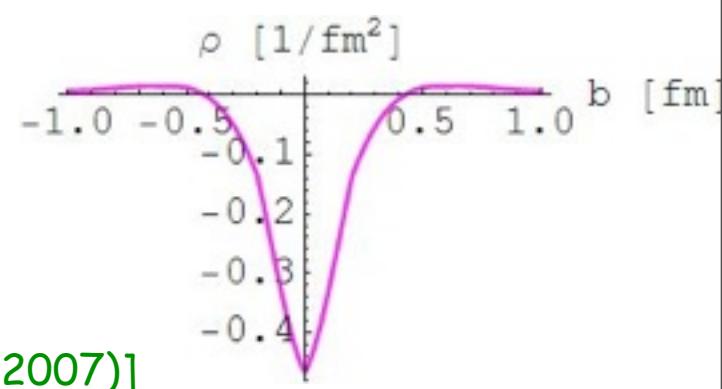


charge distribution in the transverse plane

neutron







[Miller (2007); Burkardt (2007)]

LC helicity amplitudes

nucleon (¤' ¤)

quark (¸' ¸)

$$A_{(\Lambda'\Lambda),\,(\lambda'\lambda)} = \langle \Lambda' | q^\dagger_{\lambda'} q_\lambda | \Lambda \rangle$$

$$U = (++) + (--) \quad T_x = (-+) + (+-) \quad T_y = i[(-+) - (+-)] \quad L = (++) - (--)$$

LC helicity $ canonical spin

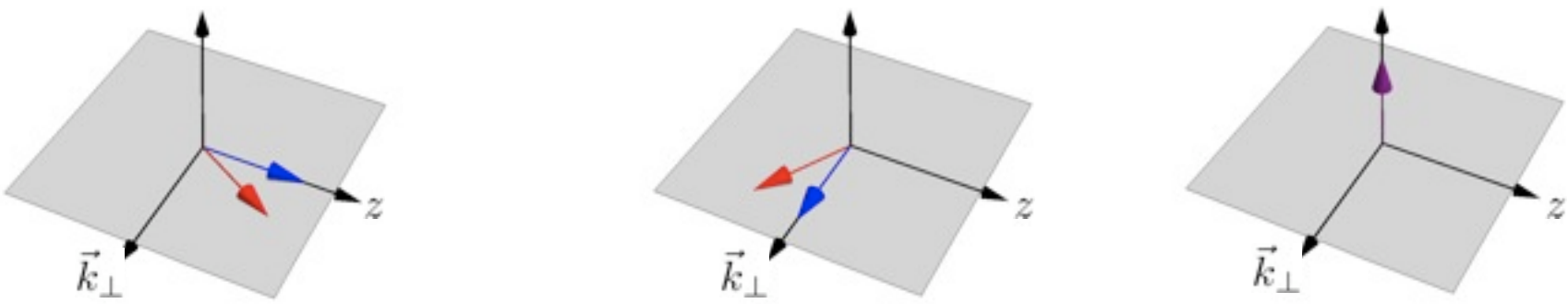
Independent quarks →

$$q^{LC}_\lambda(k) = D^{(1/2)*}_{\lambda s}\, q^C_s(k) \qquad D^{(1/2)*}_{\lambda s}(k) = \frac{1}{|\vec{K}|}\begin{pmatrix} K_z & K_L \\ -K_R & K_z \end{pmatrix}$$

LC helicity canonical spin

rotation around an axis orthogonal to z and k

$$K_{R,L} \to 0 \text{ for } k_\perp = 0$$



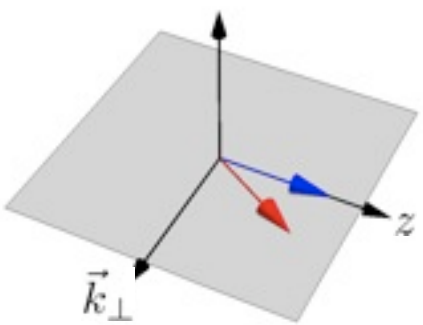
Light-Cone Helicity and Canonical Spin

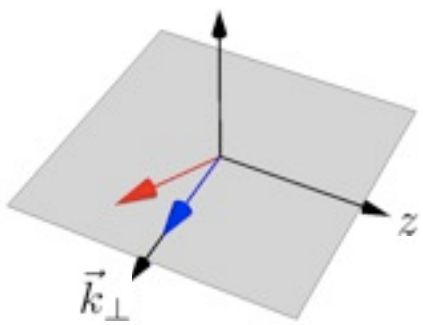
$$q_\lambda^{LC}(k) = D_{\lambda s}^{(1/2)*}\, q_s^C(k)$$

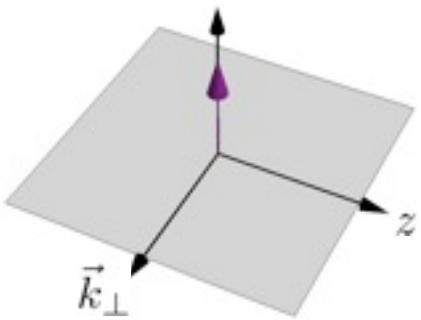
LC helicity canonical spin

$$D_{\lambda s}^{(1/2)*}(k) = \frac{1}{|\vec{K}|}\begin{pmatrix} K_z & K_L \\ -K_R & K_z \end{pmatrix}$$

rotation around an axis orthogonal to z and k







Light-Cone CQM

$$K_z = m + x\mathcal{M}_0$$

$$\vec{K}_\perp = \vec{k}_\perp$$

$$k_z = x\mathcal{M}_0 - \sqrt{\vec{k}^2 + m^2}$$

(Melosh rotation)

Chiral Quark-Soliton Model

$$K_z = h(|\vec{k}|) + \frac{k_z}{|\vec{k}|}\, j(|\vec{k}|)$$

$$\vec{K}_\perp = \frac{\vec{k}_\perp}{|\vec{k}|}\, j(|\vec{k}|)$$

$$k_z = x\mathcal{M}_N - E_{\rm lev}$$

Bag Model

$$K_z = t_0(|\vec{k}|) + \frac{k_z}{|\vec{k}|}\, t_1(|\vec{k}|)$$

$$\vec{K}_\perp = \frac{\vec{k}_\perp}{|\vec{k}|}\, t_1(|\vec{k}|)$$

$$k_z = x\mathcal{M}_N - \omega/R_0$$

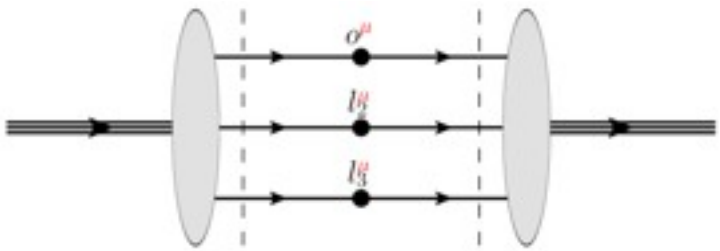
° ! quark polarization $W^{\nu}_{\Lambda'\Lambda} = \left(W^{[\gamma^+]}_{\Lambda'\Lambda}, W^{[i\sigma^{+1}\gamma_5]}_{\Lambda'\Lambda}, W^{[i\sigma^{+2}\gamma_5]}_{\Lambda'\Lambda}, W^{[\gamma^+\gamma_5]}_{\Lambda'\Lambda}\right)$

[1]! nucleon polarization $W^{\mu\nu} = \sum_{\Lambda',\Lambda} (\bar{\sigma}^{\mu})_{\Lambda'\Lambda} W^{\nu}_{(\Lambda'\Lambda)}$ $\quad (\ \sigma_\mu = \overline{\sigma}^{\mu} = (\mathbb{1}, \vec{\sigma}))$

$S \cdot W \cdot s$ ⟶ 16 GTMDs

$$W^{\mu\nu}(P, x, \vec{k}_\perp, \Delta, N; \eta) = \frac{1}{\sqrt{1-\xi^2}} \int [\mathrm{d}x]_3 \, [\mathrm{d}^2 k_\perp]_3 \, \psi^*(k') \, \psi(k) \, \mathcal{A}^{\mu\nu}(k', k)$$

$$\mathcal{A}^{\mu\nu}(r', r) = A \, O_1^{\mu\nu} \, (l_2 \cdot l_3) + B \, [l_2^\mu \, (l_3 \cdot O_1)^\nu + l_3^\mu \, (l_2 \cdot O_1)^\nu]$$



Active quark : $O_1^{\mu\nu}$

Spectator quarks : $l_i^\mu = O_i^{\mu 0}$

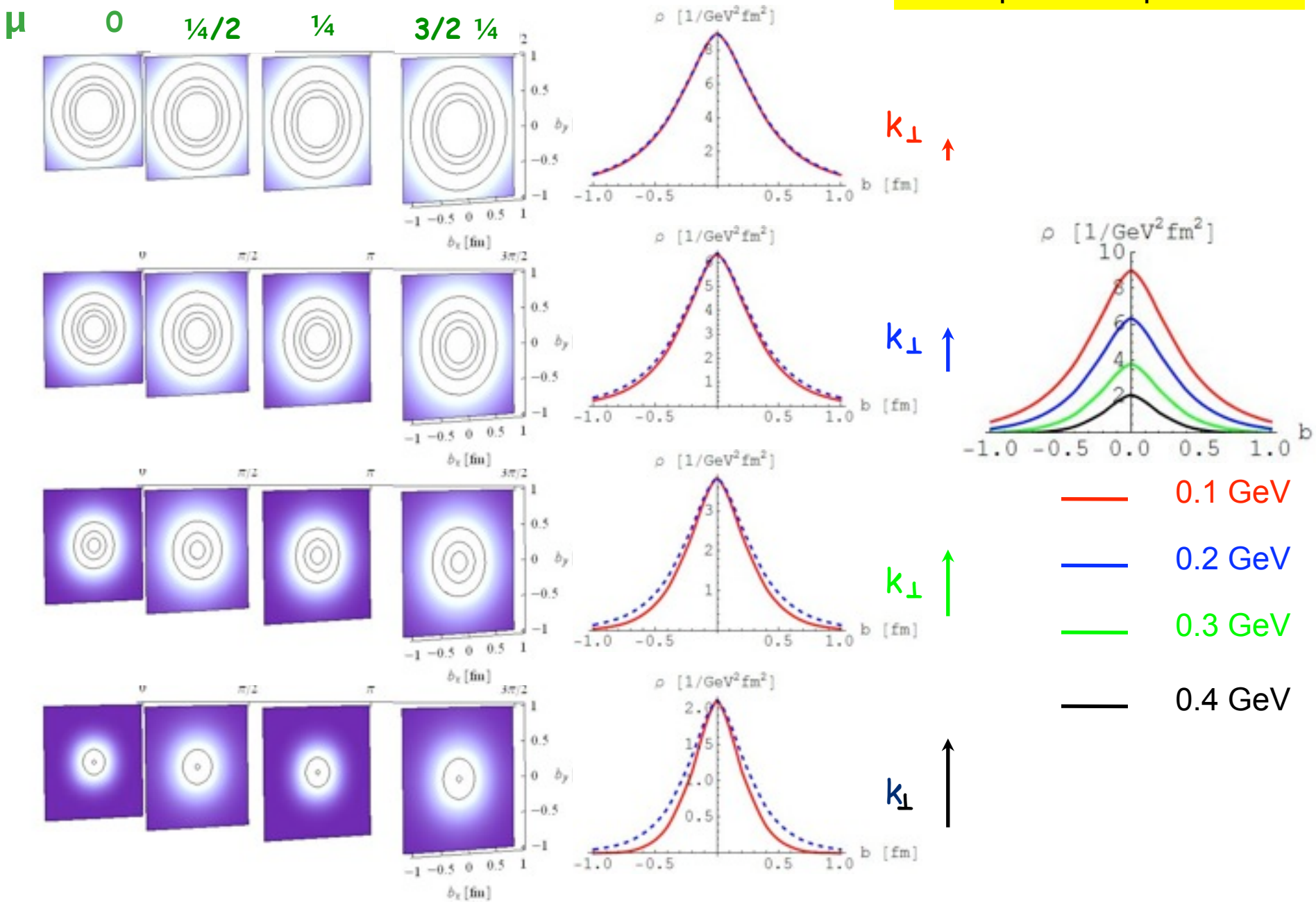
Model Independent Spin Structure

$$O^{\mu\nu} = \frac{1}{|\vec{K}'||\vec{K}|} \begin{pmatrix} \vec{K}' \cdot \vec{K} & i\left(\vec{K}' \times \vec{K}\right)_x & i\left(\vec{K}' \times \vec{K}\right)_y & -i\left(\vec{K}' \times \vec{K}\right)_z \\ i\left(\vec{K}' \times \vec{K}\right)_x & \vec{K}' \cdot \vec{K} - 2K'_x K_x & -K'_x K_y - K'_y K_x & K'_x K_z + K'_z K_x \\ i\left(\vec{K}' \times \vec{K}\right)_y & -K'_y K_x - K'_x K_y & \vec{K}' \cdot \vec{K} - 2K'_y K_y & K'_y K_z + K'_z K_y \\ i\left(\vec{K}' \times \vec{K}\right)_z & -K'_z K_x - K'_x K_z & -K'_z K_y - K'_y K_z & -\vec{K}' \cdot \vec{K} + 2K'_z K_z \end{pmatrix}$$

Backup

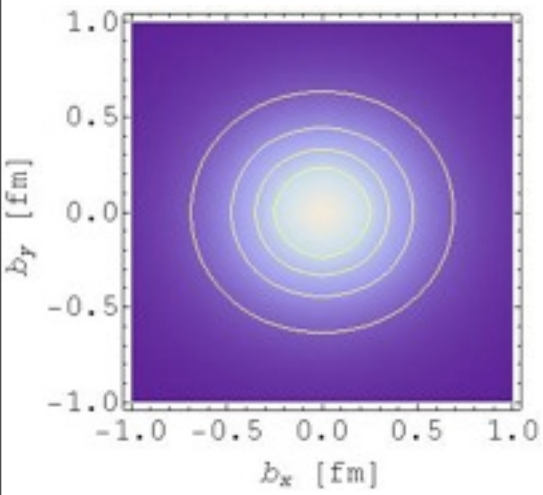
$|\vec{k_\perp}|$, θ fixed

Unpolarized u quark in unpolarized proton



Generalized Transverse Charge Densities

$\vec{k}_\perp$ ↑



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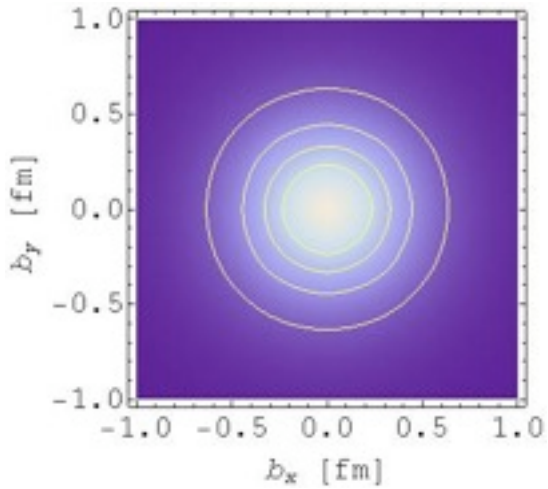

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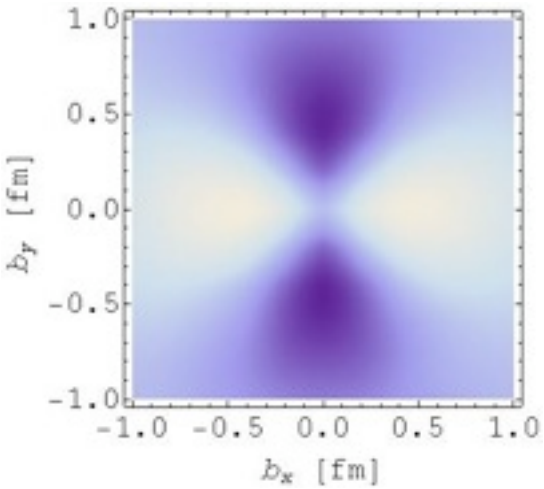
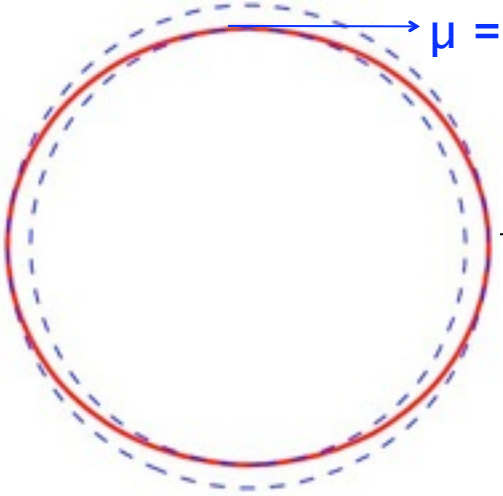
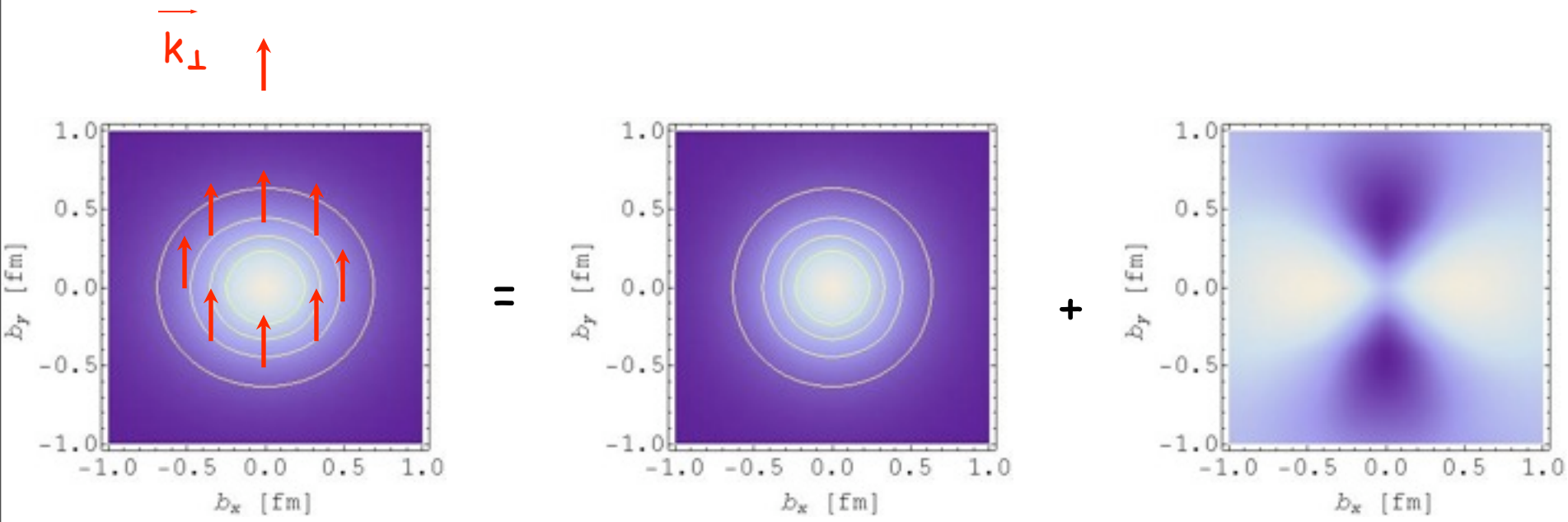


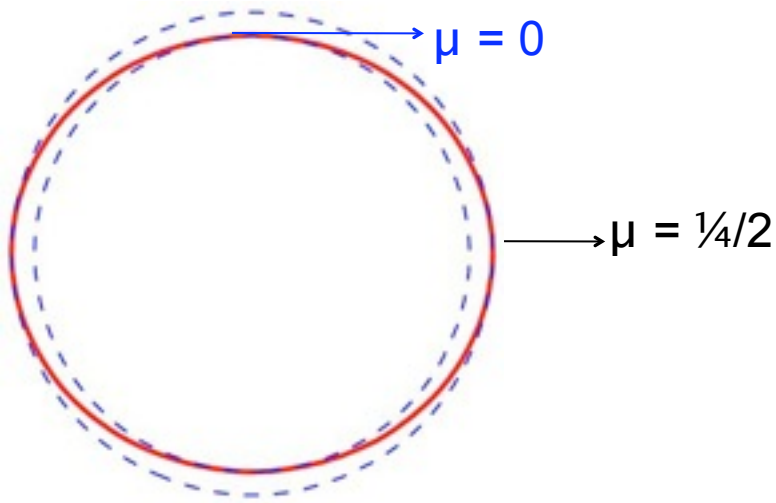
$\vec{k}_\perp$ fixed



Unpolarized u quark in unpolarized proton

Generalized Transverse Charge Densities





$\vec{k}_\perp$ fixed

Unpolarized u quark in unpolarized proton

Wigner function
for transversely pol. quark in longitudinally pol. nucleon

$$\rho^{\Lambda,T}(\vec{b}^2, \vec{k}_\perp^2, \vec{b}\cdot\vec{k}_\perp) = \frac{1}{2}\left(F_{11} + \Lambda\,\vec{s}_T\cdot\vec{k}_\perp\frac{H_{17}}{M} + i\Lambda\,\vec{s}_T\cdot\vec{b}_\perp\frac{H'_{18}}{M}\right)$$

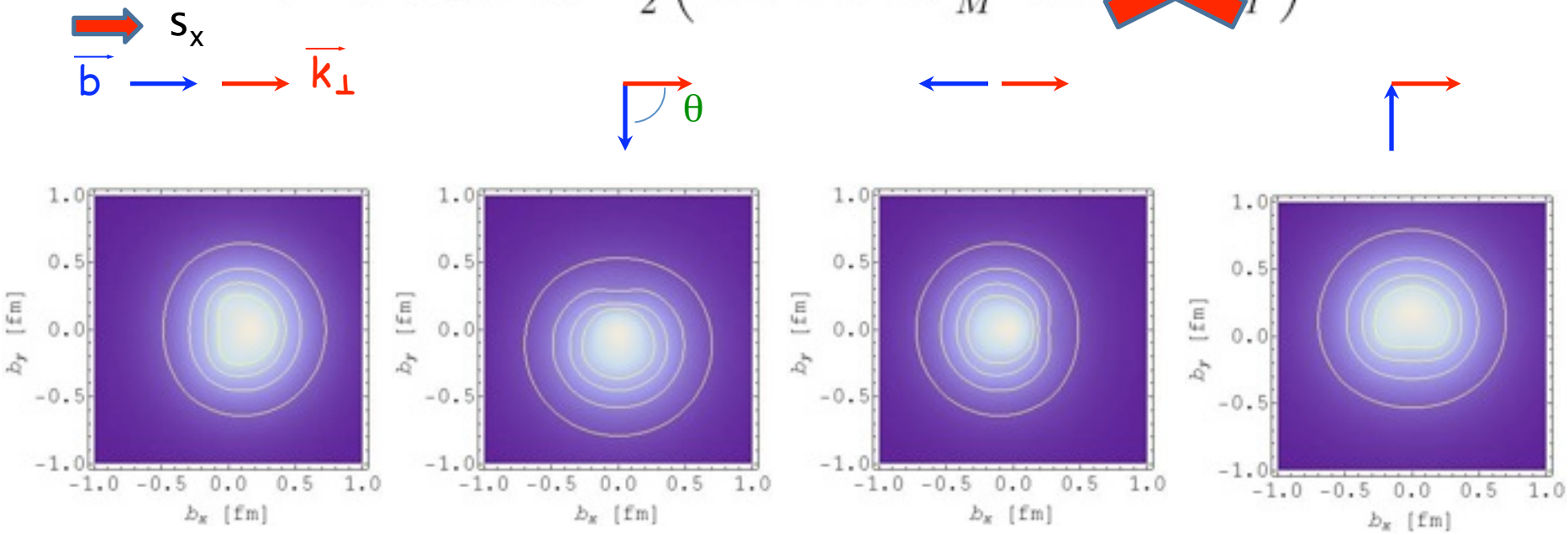
Wigner function
for transversely pol. quark in longitudinally pol. nucleon

$$\rho^{\Lambda,T}(\vec{b}^2,\vec{k}_\perp^2,\vec{b}\cdot\vec{k}_\perp) = \frac{1}{2}\left(F_{11} + \Lambda\,\vec{s}_T\cdot\vec{k}_\perp\frac{H_{17}}{M} + i\Lambda\,\vec{s}_T\cdot\vec{b}\,\frac{H'_{18}}{M}\right)$$

T-odd

Wigner function
for transversely pol. quark in longitudinally pol. nucleon

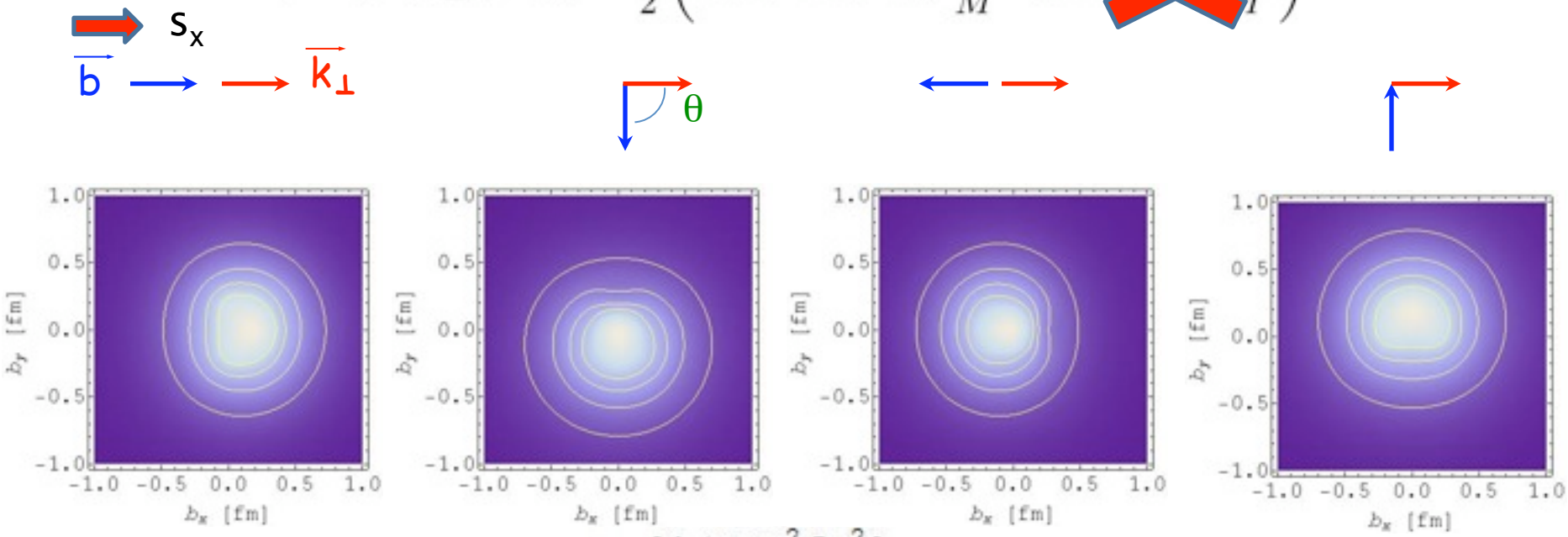
$$\rho^{\Lambda,T}(\vec{b}^2,\vec{k}_\perp^2,\vec{b}\cdot\vec{k}_\perp) = \frac{1}{2}\left(F_{11} + \Lambda\,\vec{s}_T\cdot\vec{k}_\perp\frac{H_{17}}{M} + i\Lambda\,\vec{s}_T\cdot\vec{k}_\perp\frac{H'_{18}}{M}\right)$$ T-odd



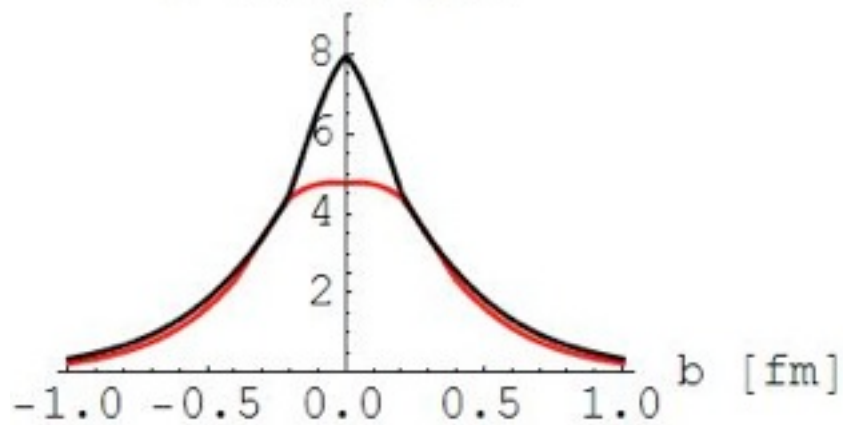
$|\vec{k}_\perp|$,μ fixed

Wigner function
for transversely pol. quark in longitudinally pol. nucleon

$$\rho^{\Lambda,T}(\vec{b}^2,\vec{k}_\perp^2,\vec{b}\cdot\vec{k}_\perp) = \frac{1}{2}\left(F_{11} + \Lambda\,\vec{s}_T\cdot\vec{k}_\perp\frac{H_{17}}{M} + i\Lambda\,\vec{s}_T\cdot\vec{k}_\perp\frac{H'_{18}}{M}\right)$$ T-odd



$|\vec{k}_\perp|$, μ fixed



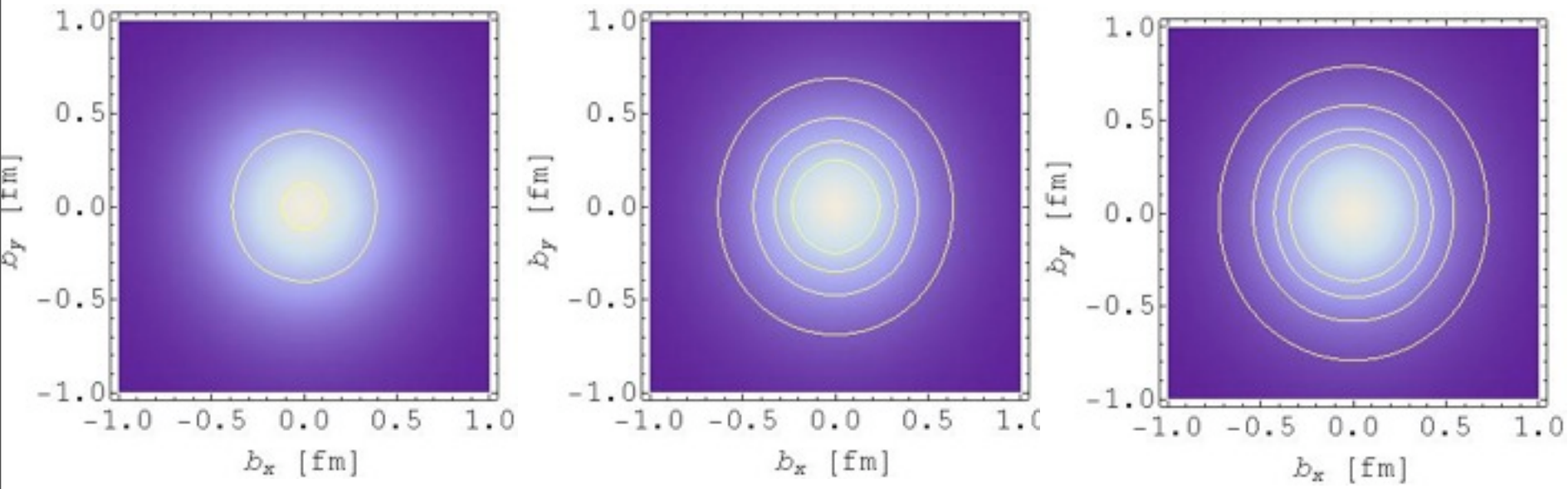
μ = ¼/2

μ = 0

Wigner function
for transversely pol. quark in longitudinally pol. nucleon

s_x

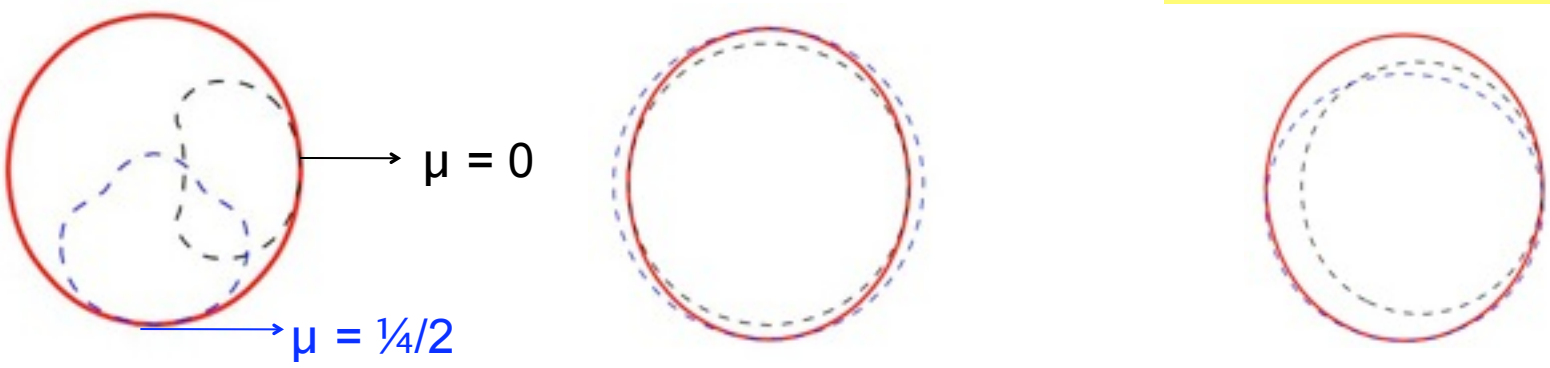
$\vec{k}_\perp \longrightarrow$ fixed



Dipole

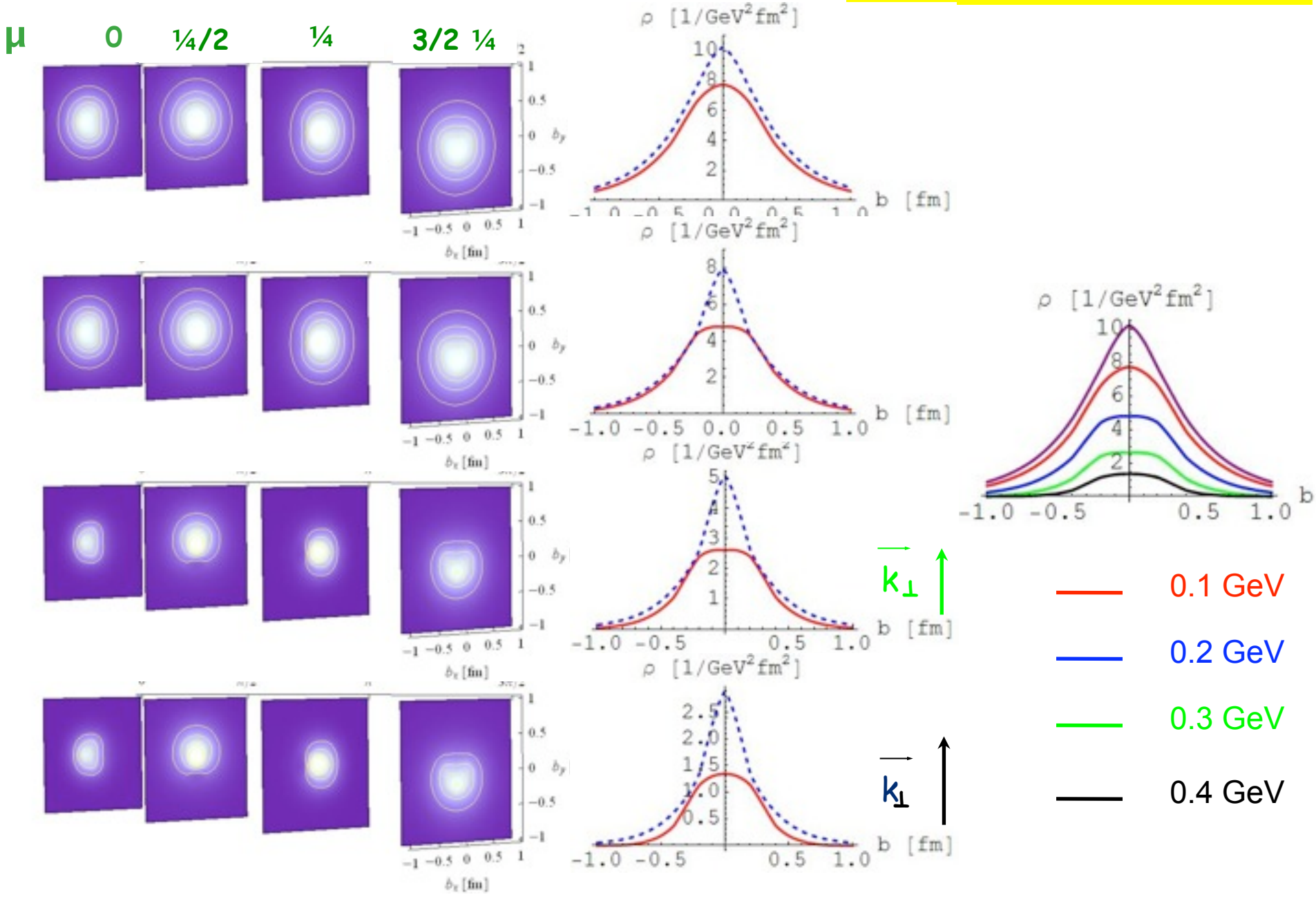
Monopole

Monopole + Dipole



$|\vec{k}_\perp|$, θ fixed

u quark pol. in x direction
in longitudinally pol.proton



❖ Integrating over $\vec{k}_?$ ➡ $\int \mathrm{d}\vec{k}_\perp \, H_{17}(\vec{b}_\perp^2, \vec{k}_\perp^2, \vec{b}_\perp \cdot \vec{k}_\perp) = 0$

- Integrating over $\vec{k}_?$ ➡ $\int d\vec{k}_\perp \, H_{17}(\vec{b}_\perp^2, \vec{k}_\perp^2, \vec{b}_\perp \cdot \vec{k}_\perp) = 0$
- Integrating over $\vec{b}_?$ ➡ density in the transverse plane $\vec{k}_?$
  of transversely pol. u and d quark in longitudinally pol nucleon

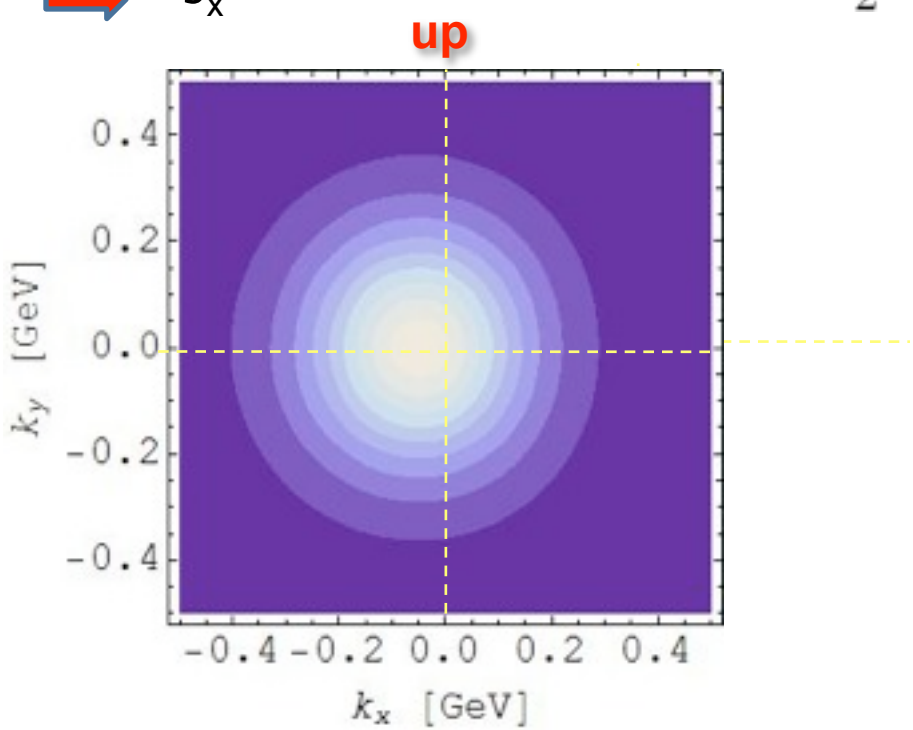
$$\rho(k_\perp, \Lambda, s_\perp) = \frac{1}{2}[f_1 + \Lambda \, s^i \, k^i \frac{1}{M} \, h_{1L}^\perp]$$

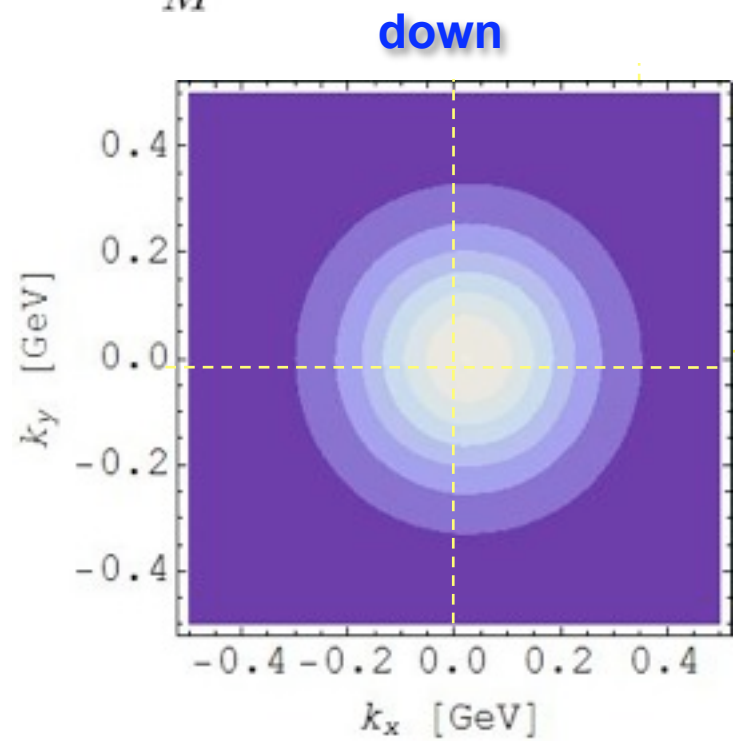
❖ Integrating over $\vec{k}_\perp$ ➡ $\int d\vec{k}_\perp \, H_{17}(\vec{b}_\perp^2, \vec{k}_\perp^2, \vec{b}_\perp \cdot \vec{k}_\perp) = 0$

❖ Integrating over $\vec{b}_\perp$ ➡ density in the transverse plane $\vec{k}_\perp$ of transversely pol. u and d quark in longitudinally pol nucleon

$$\rho(k_\perp, \Lambda, s_\perp) = \frac{1}{2}[f_1 + \Lambda \, s^i \, k^i \frac{1}{M} \, h_{1L}^\perp]$$

➡ s_x





BP, Cazzaniga, Boffi, PRD78 (2008)
Lorce`, BP, in preparation

Haegler, Musch, Negele, Schaefer, Europhys. Lett. 88 (2009)

$\langle k_x^u \rangle = -55.8$ MeV $\quad \langle k_x^d \rangle = 27.9$ MeV

$\langle k_x^u \rangle = -60(5)$ MeV $\quad \langle k_x^d \rangle = 16(5)$ MeV