A k_T-dependent sea-quark density for the CASCADE Monte Carlo event generator

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Abstract. Parton-shower event generators that go beyond the collinear-ordering approximation at small x have so far included only gluon and valence quark channels at transverse momentum dependent level. We describe results of recent work to include effects of the sea-quark distribution with explicit dependence on the transverse quark-momentum. This sea-quark density is then applied to the description of forward Z -production. The $qq^* \to Z$ matrix element (with one off-shell quark) is calculated in an explicit gauge invariant way, making use of high energy factorization. The k_T-factorized result has been implemented into the CCFM Monte-Carlo CASCADE and a numerical comparison with the $qg^* \to Zq$ matrix element has been carried out.

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INTRODUCTION

Transverse momentum dependent parton distribution arise at small x naturally as a consequence of high energy factorization and BFKL-evolution [1]. A formulation of high energy factorization which is in accordance with conventional collinear factorization is provided by k_T-factorization as defined in [2, 3]: at high center of mass energies, the resummation of high energy logarithms (BFKL) can be brought into a form consistent with conventional collinear resummation (DGLAP). The CCFM evolution equation [4], which interpolates between DGLAP and BFKL evolution, allows then for a Monte Carlo implementation of k_T-factorization, which is realized by the Monte Carlo event generator CASCADE [5].

Based on the principle of color coherence, the CCFM parton shower describes only the emission of gluons, while real quarks emissions are left aside. Note that such an approximation is in principle justified as enhanced regions of phase space for $x \to 1$ and $x \to 0$ are dominated by gluonic dynamics at leading logarithmic level. For the description of unintegrated parton densities this implies that the CCFM evolution describes only the distinct evolution of unintegrated gluon and valence quarks [6] while non-diagonal transitions between quarks and gluons are absent. On the other hand, it is necessary to include quark emissions into the parton shower to take into account subleading effects. Also the determination of k_T-dependent hard matrix elements is affected by the absence of quark emissions: the gluon-to-quark splitting which is needed for the generation of seaquarks must be included into the matrix element which complicates their determination, see *e.g.* [7]. As a first step to supplement the CCFM evolution with quarks we present in this contribution a definition of a partonic matrix element involving off-shell quarks and the definition of an off-shell sea-quark density. For the latter we restrict here to the case where the gluon-to-quark splitting occurs as the last evolution step. To test our procedure numerically, we apply it to the production of a Z-boson in the forward direction at LHC energies. For a detailed discussion we refer to [8].

DEFINITION OF A q_T-DEPENDENT SEA-QUARK

Let us take the approach of generalizing k_T-factorization to the case of quarks by mimicking the already existing gluonic case. There off-shell gauge invariance is ensured through a reformulation of QCD at high center of mass energies in terms of effective degrees of freedom, reggeized gluons. The latter coincide in their on-shell limit with conventional collinear QCD gluon, while one uses for the general off-shell case effective vertices which contain additional induced terms which ensure off-shell gauge invariance[1]. In complete analogy one can construct a reggeized quark formalism for the description of the high energy limit of scattering amplitudes with quark exchange in the t-channel [10]. As (reggeized) quark exchanges are in comparison to (reggeized) gluon exchanges suppressed by powers of s, they generally do not occur in the high energy resummation of total cross-sections. They can however be used as a starting point for the construction of an off-shell factorization of matrix elements which are limited to quark exchange in the t-channel. Note that this is the case for the $g^*q \to Zq$ matrix element for which we will construct an off-shell factorization into an off-shell matrix element $q^*q \to Z$ and a corresponding gluon-to-quark splitting function. Making use of the reggeized quark formalism, it is straightforward to arrive at the off-shell partonic cross-section $qq^* \to Z$,

$$\hat{\sigma}(x_1 x_2 s, M_Z^2, \mathbf{q}^2) = \sqrt{2} G_F M_Z^2 (V_q^2 + A_q^2) \frac{\pi}{N_c} \delta(x_1 x_2 s - \mathbf{q}^2 - M_Z^2). \qquad (1)$$

Here $x_1\sqrt{s}$ and $x_2\sqrt{s}$ are the light-cone momenta of the two incoming quarks, while $\mathbf{q}$ denotes the transverse momentum of the off-shell quark; M_Z denotes the mass of the Z-boson, while $\sqrt{2} G_F M_Z^2 (V_q^2 + A_q^2)$ describes the coupling of the Z-boson to the quark. Note that eq. (1) agrees for $\mathbf{q}^2 \to 0$ with the $qq \to Z$ cross-section. The splitting function is on the other hand obtained as a pure color factor, if the reggeized quark formalism is taken literally. In comparison to the collinear splitting function, this is a rather crude approximation which can be traced back to non-conservation of energy at the level of high-energy leading logarithms. In the present case it is however possible to supply energy conservation, while maintaining all desired gauge invariance properties. Doing so we arrive at the following k_T-dependent splitting function

$$P_{qg}\left(z, \frac{\mathbf{k}^2}{\mathbf{q}^2}\right) = T_R \left(\frac{\mathbf{q}^2}{\mathbf{q}^2 + z(1-z)\mathbf{k}^2}\right)^2 \left[(1-z)^2 + z^2 + 4z^2(1-z)^2 \frac{\mathbf{k}^2}{\mathbf{q}^2}\right], \qquad (2)$$

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[1] For an effective action approach to the reggeized gluon formalism see [9].

which can be identified as the k_T-dependent quark-gluon splitting function of [3]. Here $\mathbf{k}$ and $\mathbf{q}$ denote transverse momentum of the off-shell gluon and quark respectively, while z denotes the fraction of gluon light cone momentum which is carried on by the t-channel quark. Note that in the on-shell limit $\mathbf{k}^2 \to 0$ eq. (2) reduces to the conventional DGLAP splitting function $T_R[z^2 + (1-z)^2]$. Restricting the gluon-to-quark splitting to the last evolution step, a q_T-dependent sea-quark density is then defined by

$$\mathscr{Q}^{\text{sea}}(x, \mathbf{q}^2, \mu^2) = \frac{1}{\mathbf{q}^2} \int_x^1 \frac{dz}{z} \int_0^{\mathbf{k}^2_{\text{max}}} d\mathbf{k}^2 \frac{\alpha_s(\mu^2)}{2\pi} P_{qg}\left(z, \frac{\mathbf{k}^2}{\mathbf{q}^2}\right) \mathscr{G}\left(\frac{x}{z}, \mathbf{k}^2, \bar{\mu}^2\right). \tag{3}$$

with $\mathbf{k}^2_{\text{max}} = \mu^2/z - \mathbf{q}^2/(z(1-z))$. For the scale $\bar{\mu}$ of the unintegrated gluon density, we will investigate two possible choices. One choice, closer in spirit to inclusive calculations [3], is to set simply $\bar{\mu}^2 = \mu^2$. A choice based on angular ordering, which is more natural from the point of view of the CCFM evolution, is given by $\bar{\mu}^2 = \mathbf{q}^2/(1-z)^2 + \mathbf{k}^2/(1-z)$. The complete q_T-factorized cross-section for forward Z-production is given by

$$\sigma^{q_T-\text{fact.}}_{pp\to Z}(s, M_Z^2) = \sum_f \int_0^1 dx_1 \int_0^\infty d\mathbf{q}^2 \int_0^1 dx_2\, \hat{\sigma}^f_{qq^*\to Z}\, \mathscr{Q}^{\text{sea}}(x_1, \mathbf{q}^2, \mu^2) Q^f(x_2, \mu^2) \tag{4}$$

with $Q^f(x_2, \mu^2)$ the valence quark distribution, implemented as in the first paper of [6].

NUMERICAL TEST OF THE OFF-SHELL FACTORIZATION

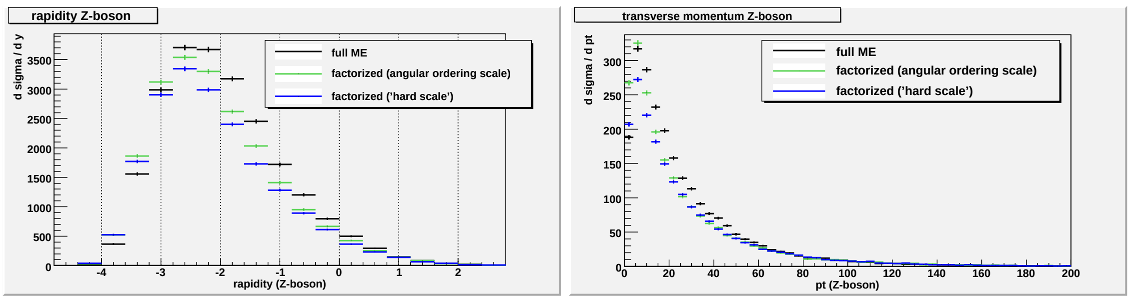

FIGURE 1.

There exists already a result for the cross-section eq. (4) which uses the $g^*q \to Zq$ off-shell cross-section of [11]. In the following paragraph we compare this “unfactorized" matrix element, which is already implemented in the CASCADE Monte Carlo generator, with the factorized expression eq. (4). The results are summarized in fig. 1 and fig. 2. For the factorized result both choices for the hard scale $\bar{\mu}^2$ have been implemented. The differences between the matrix element (ME) and the factorized results are discussed in [8]. We note that the factorized result contains the t-channel contribution, while the s-channel contribution is sub-leading both in the collinear and high energy limit. The factorized expression also contains a kinematical approximation [8, 10] in comparison to the full $g^*q \to Zq$ cross-section, which leads to a small, but finite correction.

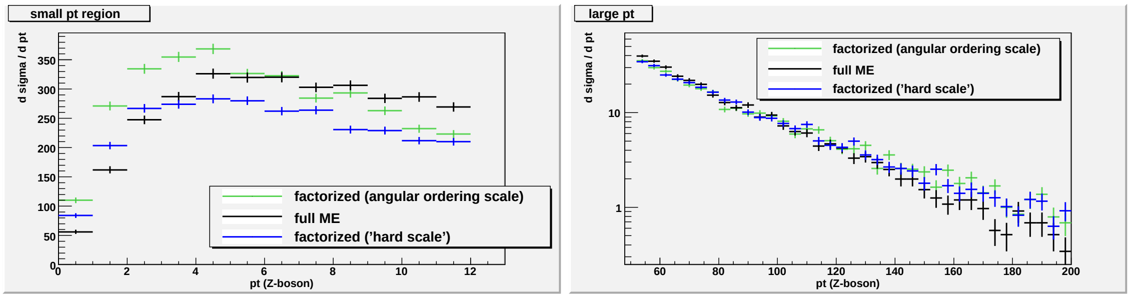

FIGURE 2.

In summary, we achieved an off-shell factorization of the $g^*q \to Zq$ process, where the factorized expression can be both analytically and numerically be shown to coincide with the full $g^*q \to Zq$ cross-section up to a finite remainder, sub-leading in the collinear and the high energy limit.

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REFERENCES

1. L. N. Lipatov, Sov. J. Nucl. Phys. **23** (1976) 338 [Yad. Fiz. **23** (1976) 642]; V. S. Fadin, E. A. Kuraev and L. N. Lipatov, Phys. Lett. B **60** (1975) 50; I. I. Balitsky and L. N. Lipatov, Sov. J. Nucl. Phys. **28** (1978) 822 [Yad. Fiz. **28** (1978) 1597].
2. S. Catani, M. Ciafaloni and F. Hautmann, Phys. Lett. B **307** (1993) 147; Nucl. Phys. B **366** (1991) 135; Phys. Lett. B**242** (1990) 97.
3. S. Catani and F. Hautmann, Nucl. Phys. B **427** (1994) 475; Phys. Lett. B**315** (1993) 157.
4. M. Ciafaloni, Nucl. Phys. **B296** (1988) 49, S. Catani, F. Fiorani, G. Marchesini, Phys. Lett. **B234** (1990) 339, S. Catani, F. Fiorani, G. Marchesini, Nucl. Phys. **B336** (1990) 18, G. Marchesini, Nucl. Phys. **B445** (1995) 49-80. [hep-ph/9412327].
5. H. Jung and G. P. Salam, Eur. Phys. J. C **19** (2001) 351 [arXiv:hep-ph/0012143]; H. Jung *et al.*, Eur. Phys. J. C **70**, 1237 (2010) [arXiv:1008.0152 [hep-ph]].
6. M. Deak, F. Hautmann, H. Jung and K. Kutak, arXiv:1012.6037 [hep-ph]; JHEP **0909**, 121 (2009) [arXiv:0908.0538 [hep-ph]].
7. M. Deak and F. Schwennsen, JHEP **0809** (2008) 035 [arXiv:0805.3763 [hep-ph]], S. P. Baranov, A. V. Lipatov and N. P. Zotov, Phys. Rev. D **78** (2008) 014025 [arXiv:0805.4821 [hep-ph]].
8. F. Hautmann, M. Hentschinski, H. Jung, in preparation.
9. L. N. Lipatov, Nucl. Phys. **B452** (1995) 369-400 [arXiv:hep-ph/9502308 [hep-ph]]; L. N. Lipatov, Phys. Rept. **286** (1997) 131-198 [hep-ph/9610276]; M. Hentschinski, [arXiv:0908.2576 [hep-ph]]; M. Hentschinski, Nucl. Phys. Proc. Suppl. **198** (2010) 108-111 [arXiv:0910.2981 [hep-ph]].
10. L. N. Lipatov and M. I. Vyazovsky, Nucl. Phys. B **597** (2001) 399 [arXiv:hep-ph/0009340]; A. V. Bogdan and V. S. Fadin, Nucl. Phys. B **740**, 36 (2006) [arXiv:hep-ph/0601117]; V. A. Saleev, Phys. Rev. D **78**, 114031 (2008) [arXiv:0812.0946 [hep-ph]].
11. S. Marzani and R. D. Ball, Nucl. Phys. B **814**, 246 (2009) [arXiv:0812.3602 [hep-ph]].