S-ACOT Heavy flavor contributions at NNLO in CTEQ-TEA analysis

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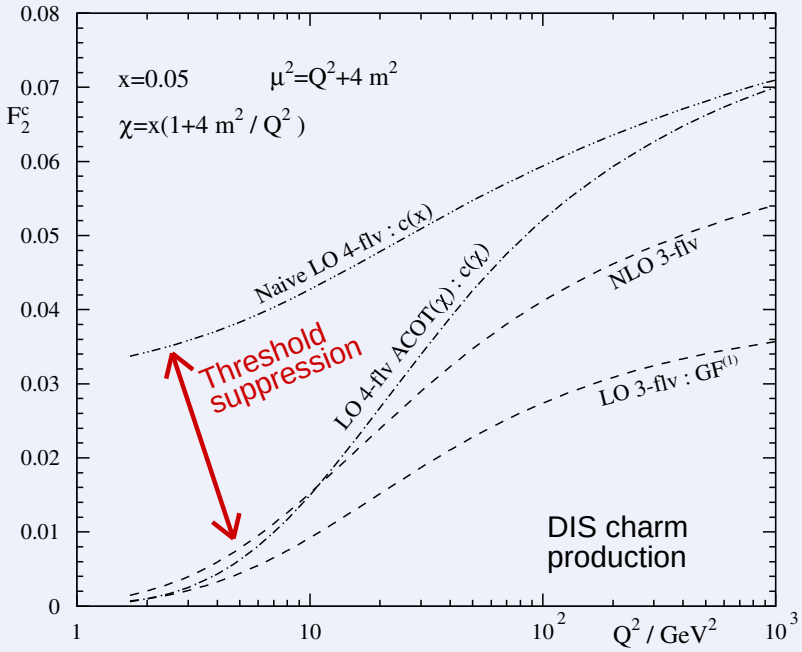
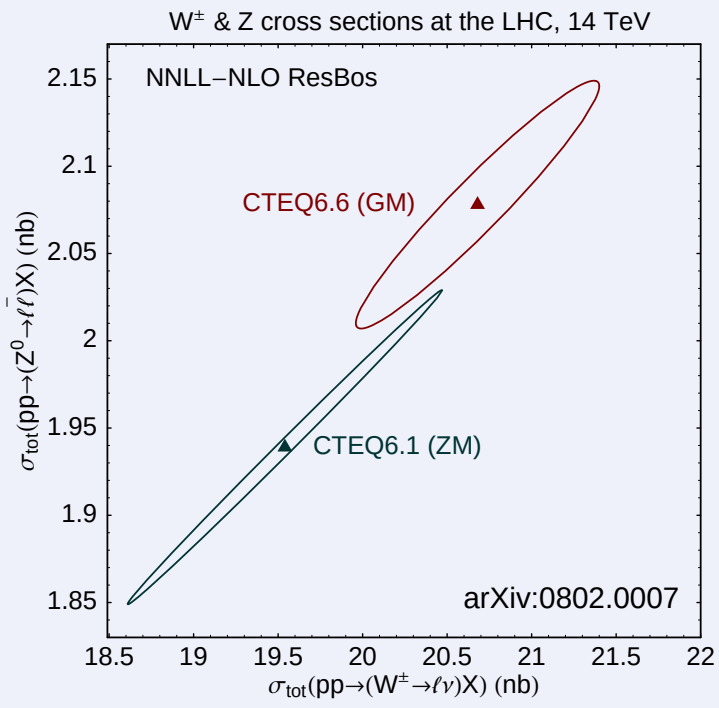
Heavy-quark DIS and LHC observables

Motivation:

General-mass (and not zero-mass of fixed-flavor number) treatment of c, b mass terms in DIS is essential for predicting precision W, Z cross sections at the LHC *(Tung et al., hep-ph/0611254)*

Several quark mass effects are comparable to NNLO radiative contributions, must be included in a consistent way

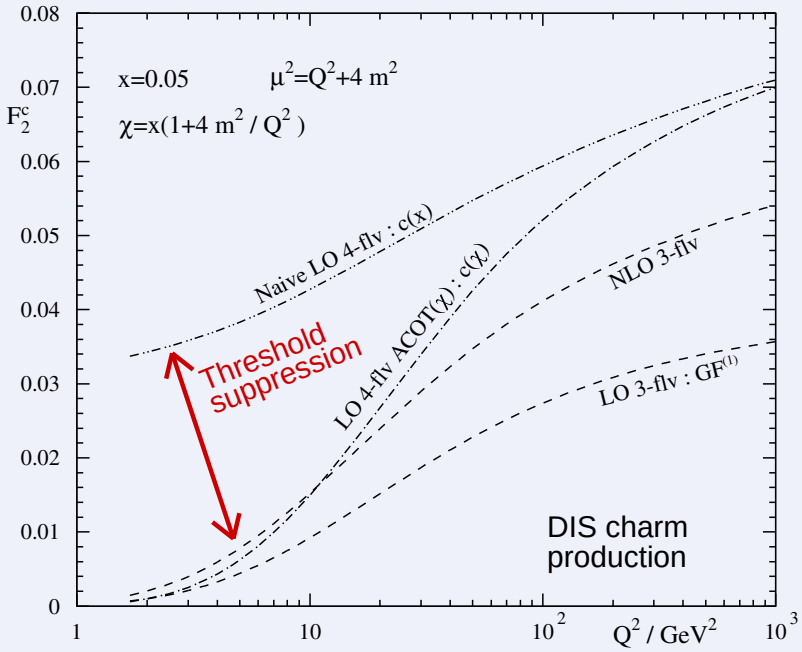


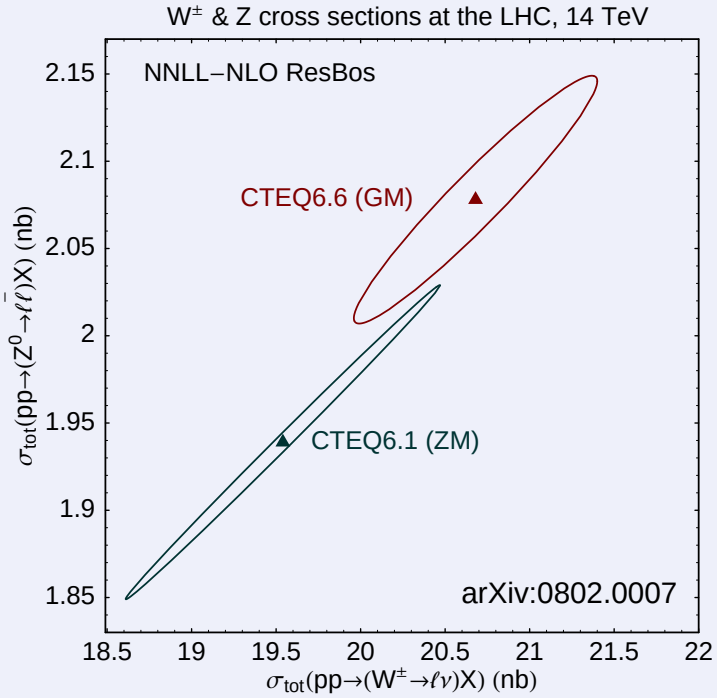


Heavy-quark DIS and LHC observables

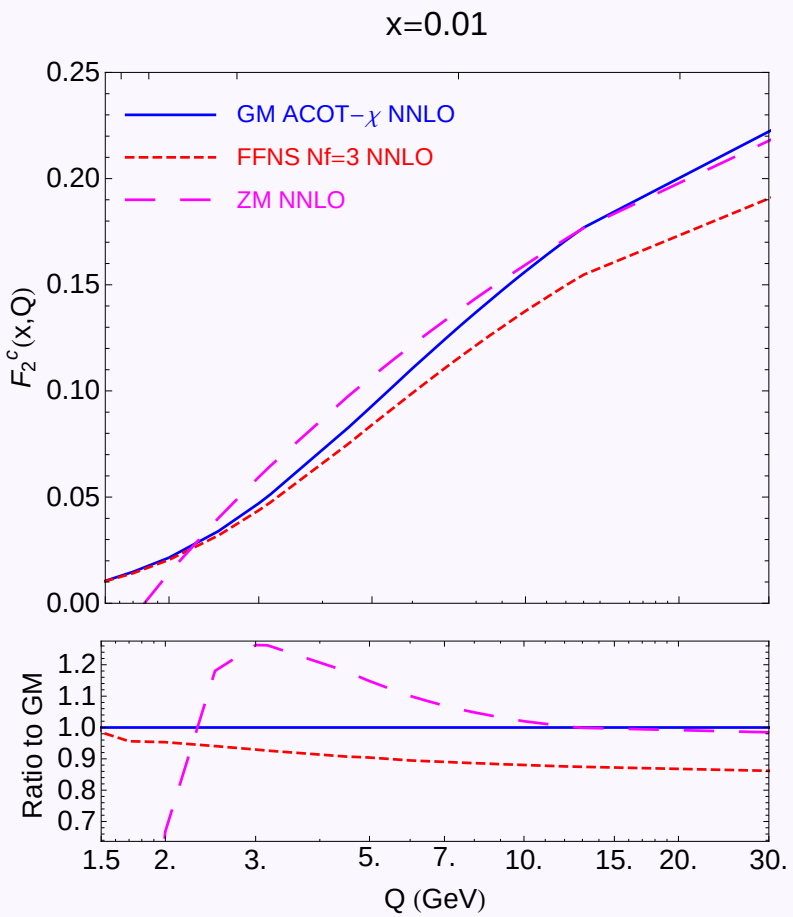
This talk:

- an NNLO computation for heavy-quark DIS structure functions, $F_i^{c,b}(x,Q)$, in a general-mass scheme (S-ACOT)
- focus on consistent treatment of all relevant factors in $F_i^{c,b}(x,Q)$ affecting CTEQ-TEA PDFs at NNLO accuracy





$F_2^c(x, Q^2)$ at NNLO - Preliminary



S-ACOT reduces to FFNS at $Q \approx m_c$ and to ZM at $Q \gg m_c$

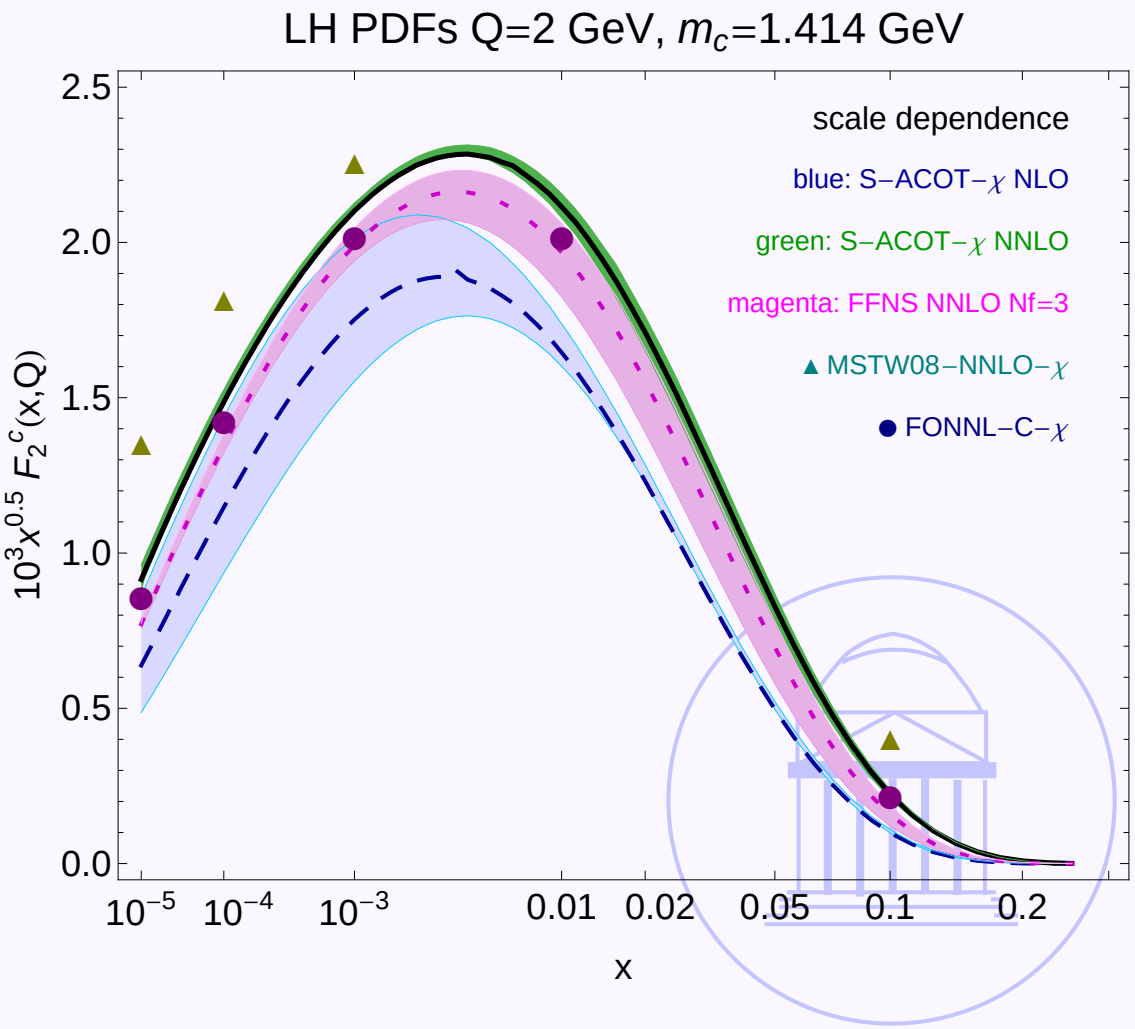
Les Houches toy PDFs, evolved at NNLO with threshold matching terms

NNLO predictions for F_L^c are in the backup slides

Results for $F_2^c(x, Q^2)$ at NLO/NNLO - Preliminary

At NNLO and $Q \approx m_c$:

- S-ACOT-$\chi \approx$ FFN$(N_f = 3)$ without tuning
- It is close to other NNLO schemes
- S-ACOT-χ predictions are for a physically motivated rescaling variable $\zeta = x(1 + 4m_c^2/Q^2)$. Dependence on the form of ζ is also reduced



Details of the computation

- NNLO evolution for α_s and PDFs (HOPPET)
  - matching coefficients relating the PDFs in N_f and N_{f+1} schemes *(Smith, van Neerven, et al.)*
- NNLO Wilson coefficient functions for $F_2^c(x,Q)$, $F_L^c(x,Q)$
- One value of N_f and one PDF set in each Q-range
- $\Rightarrow$ S-ACOT: implementation as an algorithm that follows the proof of QCD factorization for DIS with massive quarks, J. Collins Phys.Rev.D 1998

Components of inclusive $F_{2,L}(x,Q)$.

Components of inclusive $F_{2,L}(x,Q^2)$ are classified according to the quark couplings to the photon

$$F = \sum_{l=1}^{N_l} F_l + F_h \tag{1}$$

$$F_l = e_l^2 \sum_a \left[C_{l,a} \otimes f_{a/p}\right](x,Q), \quad F_h = e_h^2 \sum_a \left[C_{h,a} \otimes f_{a/p}\right](x,Q). \tag{2}$$

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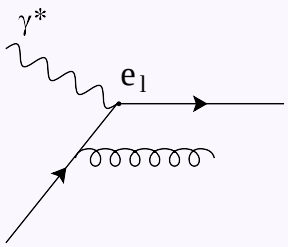

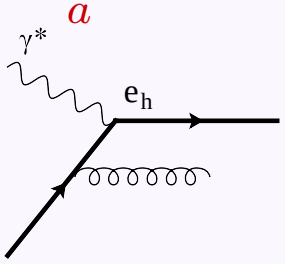

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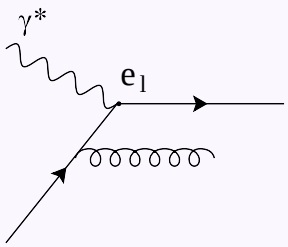

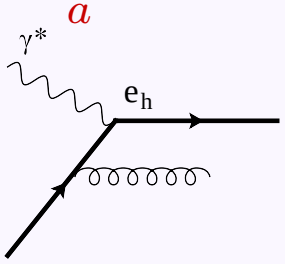

At $\mathcal{O}(\alpha_s^2)$:

$$F_h^{(2)} = e_h^2 \left\{ c_{h,h}^{NS,(2)} \otimes (f_{h/p} + f_{\bar{h}/p}) + C_{h,l}^{(2)} \otimes \Sigma + C_{h,g}^{(2)} \otimes f_{g/p} \right\}$$

$$F_l^{(2)} = e_l^2 \left\{ C_{l,l}^{NS,(2)} \otimes (f_{l/p} + f_{\bar{l}/p}) + c^{PS,(2)} \otimes \Sigma + c_{l,g}^{(2)} \otimes f_{g/p} \right\}. \tag{3}$$

Components of inclusive $F_{2,L}(x,Q)$.

Components of inclusive $F_{2,L}(x,Q^2)$ are classified according to the quark couplings to the photon

The "light-quark" F_l contains some subgraphs with heavy-quark lines, denotd by "$G_{l,l,heavy}$". The "heavy-quark" $F_h \neq F_2^c$,

$$F_2^c = F_h + (G_{l,l,heavy})_{real}, \tag{1}$$

where $G_{i,j} = C_{i,j}^{(2)}, \;\; F_{i,j}^{(2)}$, and $A_{i,j}^{(2)}$

Notations

- Lower case $c^{(2)}_{a,b}$, $\hat{f}^{(k)}_{a,b}$ $\rightarrow$ ZM epxressions
  Zijlstra and Van Neerven PLB272 (1991), NPB383 (1992)
  S. Moch, J.A.M. Vermaseren and A. Vogt, NPB724 (2005)

- Upper case $C^{(2)}_{a,b}$, $F^{(k)}_{a,b}$ $A^{(k)}_{a,b}$ $\rightarrow$ coeff. functions, structure functions and subtractions with $m_c \neq 0$,
  Buza *et al.*, NPB 472 (1996); EPJC1 (1998);
  Riemersma, *et al.* PLB 347 (1995); Leanen *et al.* NPB392 (1993)

A parton level calculation of $C_{a,b}^{(k)}$

Coefficient functions $C_{a,b}^{(k)}$ are found from parton-level structure functions by considering $F(e+b \to e+X) \equiv \sum_{i=1}^{N_f} e_i^2 F_{i,b}$ for DIS on an initial-state parton b

$$F(e+b \to e+X) = \sum_{i=1}^{N_f} e_i^2 \sum_{a=-N_f}^{N_f} \left[C_{ia} \otimes f_{a/b}\right](x,Q), \tag{2}$$

$f_{a/p}$ are parton-level PDFs.

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$f_{a/p}$ are parton-level PDFs. Perturbative expansions as a series of $a_s \equiv \alpha_s(\mu, N_f)/(4\pi)$

$$\begin{aligned} f_{a/b}(x) &= \delta_{ab}\delta(1-x) + a_s\, A_{ab}^{(1)} + a_s^2 A_{ab}^{(2)} + \ldots, \\ C_{i,a} &= C_{i,a}^{(0)} + a_s\, C_{i,a}^{(1)} + a_s^2 C_{i,a}^{(2)} + \ldots, \\ F_{i,b} &= F_{i,b}^{(0)} + a_s\, F_{i,b}^{(1)} + a_s^2\, F_{i,b}^{(2)} + \ldots, \end{aligned} \tag{3}$$

$A_{ab}^{(m)}$ $(m = 0, 1, 2, \ldots)$ are perturbative coefficients of operator matrix elements defining the parton-level PDFs.

α_s^2-Coefficient functions in F_h

Perturbative contributions $C_{i,a}^{(k)}$ to the Wilson functions can be found as

$$\begin{aligned} C_{i,b}^{(0)} &= F_{i,b}^{(0)}, \\ C_{i,b}^{(1)} &= F_{i,b}^{(1)} - C_{i,a}^{(0)} \otimes A_{ab}^{(1)}, \\ C_{i,b}^{(2)} &= F_{i,b}^{(2)} - C_{i,a}^{(0)} \otimes A_{ab}^{(2)} - C_{i,a}^{(1)} \otimes A_{ab}^{(1)}, \end{aligned} \tag{4}$$

S-ACOT prescription: use ZM expressions for $F_{a,b}^{(k)}$ and $C_{a,b}^{(k)}$ with incoming heavy-quark lines.

α_s^2-Coefficient functions in F_h

This leads to the following F_h

$$F_h^{(2)} = e_h^2 \left\{ c_{h,h}^{NS,(2)} \otimes (f_{h/p} + f_{\bar{h}/p}) + C_{h,l}^{(2)} \otimes \Sigma + C_{h,g}^{(2)} \otimes f_{g/p} \right\} \tag{5}$$

where

$$\begin{aligned} c_{h,h}^{NS,(2)} &= \widehat{f}_{h,h}^{NS,(2)}; \\ C_{h,l}^{(2)} &= \widehat{F}_{h,l}^{PS,(2)} - A_{hl}^{PS,(2)}; \\ C_{h,g}^{(2)} &= \widehat{F}_{h,g}^{(2)} - A_{hg}^{(2)} - c_{h,h}^{(1)} \otimes A_{hg}^{(1)}; \end{aligned} \tag{6}$$

$c_{h,h}^{(1)} = \widehat{f}_{h,h}^{(1)}$ and $\Sigma(x,\mu) = \sum_{i=1}^{N_f} \left[f_{i/p}(x,\mu) + \bar{f}_{i/p}(x,\mu) \right]$,
Available from literature!

α_s^2-Coefficient functions in F_h

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Available from literature!

Zijlstra, Van Neerven, PLB 272 (1991), NPB383 (1992), M.Buza *et al.*, NPB472 (1996), J.Smith *et al.* EPJC1 (1998)

...Similarly for F_l

while the function F_l is given by

$$F_l^{(2)} = e_l^2 \left\{ C_{l,l}^{NS,(2)} \otimes (f_{l/p} + f_{\bar{l}/p}) + c^{PS,(2)} \otimes \Sigma + c_{l,g}^{(2)} \otimes f_{g/p} \right\}. \tag{7}$$

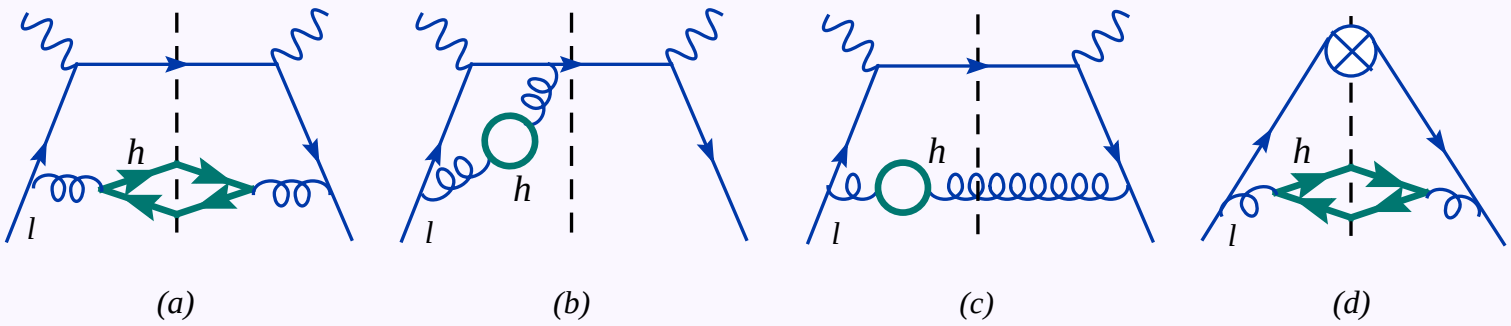
where

$$C_{l,l}^{NS,(2)} = \widehat{f}_{l,l,light}^{NS,(2)} + F_{l,l,heavy}^{NS,(2)} - A_{ll,heavy}^{NS,(2)}; \tag{8}$$

$c^{PS,(2)}$ and $c_{l,g}^{(2)}$: ZM expressions by Zijlstra, Van Neerven, PLB 272 (1991), NPB383 (1992)

S. Moch, J.A.M. Vermaseren and A. Vogt, NPB724 (2005)

Light-quark component of $F(x,Q)$



Among all terms, only non-singlet contributions $F_{l,l}^{NS,(2)}$ and $A_{ll}^{NS,(2)}$ to $C_{l,l}^{(2)}$ include disconnected heavy-quark lines in (cut) fermion loops and should be evaluated with full dependence on m_h. These heavy-quark contributions can be explicitly identified inside the non-singlet functions as

$$G = g_{light}(\{m\}=0) + G_{heavy}(m_h) \qquad \text{for } G = F_{l,l}^{NS,(2)},\ A_{ll}^{NS,(2)},$$

Experimentally defined HQ structure function

The heavy-quark component F_h of inclusive $F(x,Q)$:
Not directly measurable!
At Q accessible at HERA the observable semi-inclusive $F_{h,SI} \approx F_2^c$ is related to F_h by

$$F_{h,SI}(x,Q) = F_h(x,Q) + \sum_{l=1}^{N_l} e_l^2 (F^{NS,(2)}_{l,l,heavy})_{real} \otimes (f_{l/p} + f_{\bar{l}/p}), \quad (9)$$

$(F^{NS,(2)}_{i,i,heavy})_{real}$ is a part of the non-singlet contribution with the incoming light quark that is contributed by real emission diagrams.

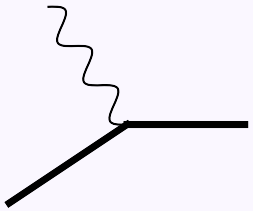
At higher Q the experimental $F_{h,SI}$ can be regularized in the collinear region as shown in Chuvakin *et al.* Phys. Rev. D 61, (2000).

Experimentally defined HQ structure function

We use $F_{h,SI}$ to compute $F^c_{2,L}$ at moderate Q.

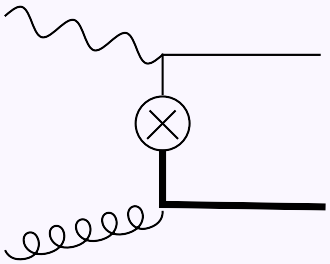
We use F_l and F_h to compute inclusive F_2 and F_L.

Classes of Feynman diagrams I



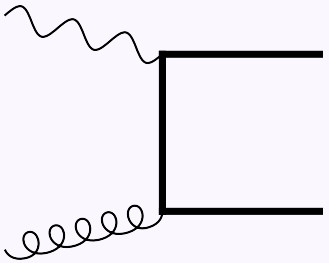
LO γ* c

−

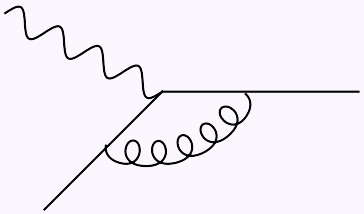


NLO Subtraction

+



NLO γ* g



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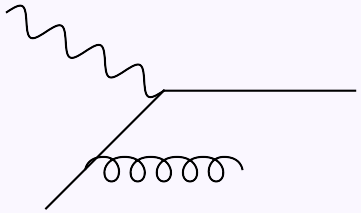
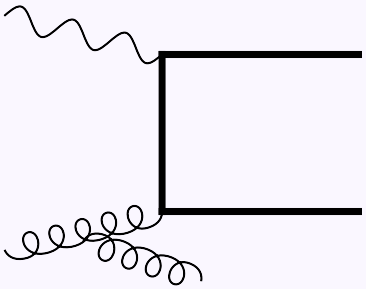


NLO γ* c



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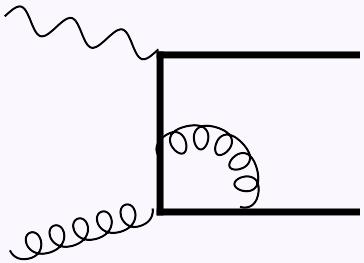
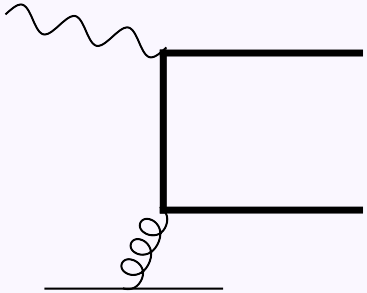


NNLO: γ* g

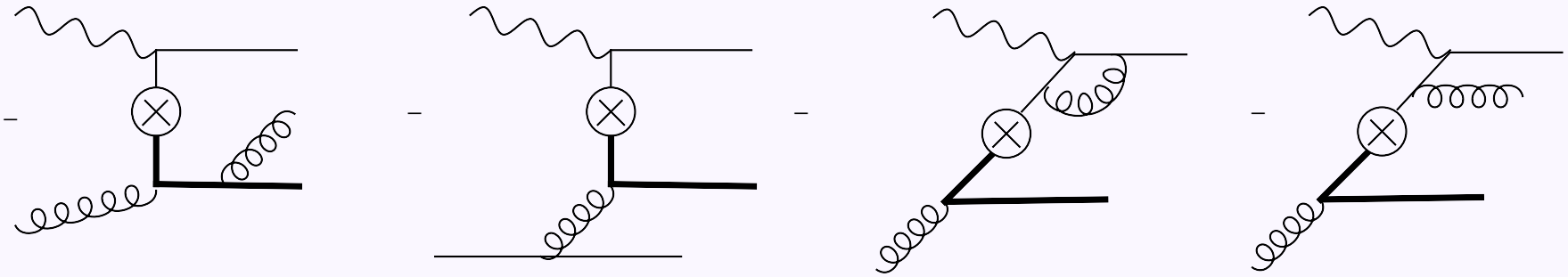
+



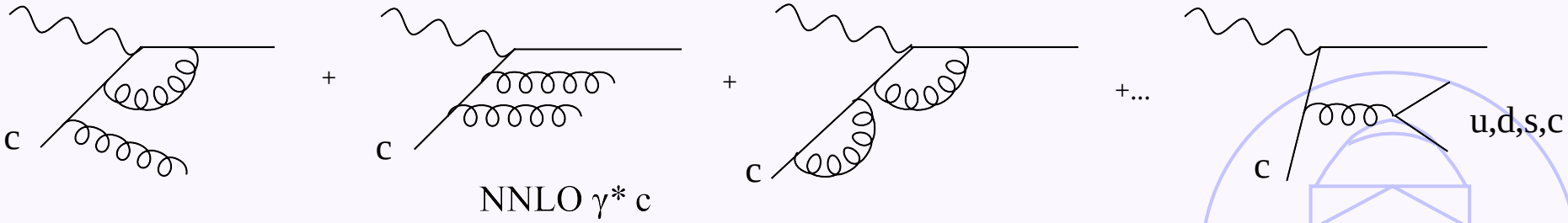
NNLO: γ* Σ

+ ....

Classes of Feynman Diagrams II



NNLO Subtractions



NNLO γ* c

Cancellations between Feynman diagrams

Validity of the S-ACOT calculation was verified by checking for certain cancelations at $Q \approx m_c$ and $Q \gg m_c$

- $Q \approx m_c$:

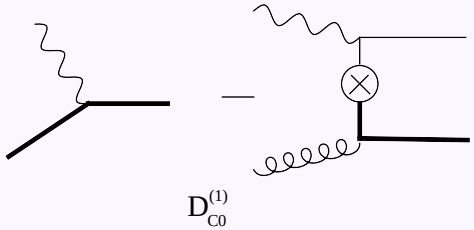
$$D_{C1}^{(2)} \ll D_{C0}^{(2)} \ll D_{C0}^{(1)} \leq F_2^c(x, Q)$$

- $Q \gg m_c$:

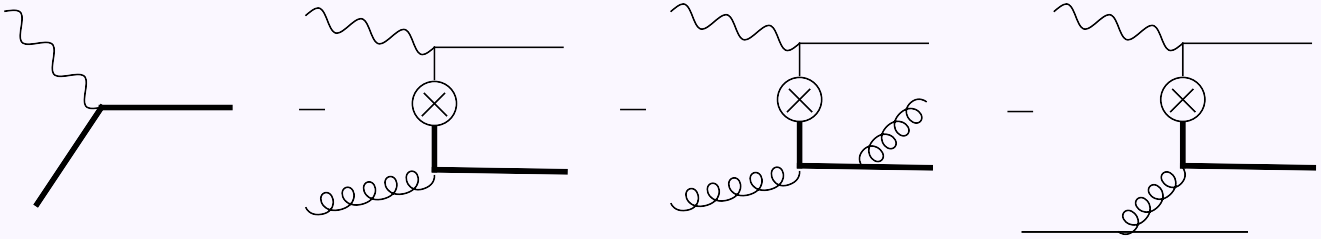
$$D_g^{(2)} \ll D_g^{(1)} < F_2^c(x, Q)$$

These cancellations are indeed observed in our results

NNLO: Cancellations at $Q^2 \approx m_c^2$

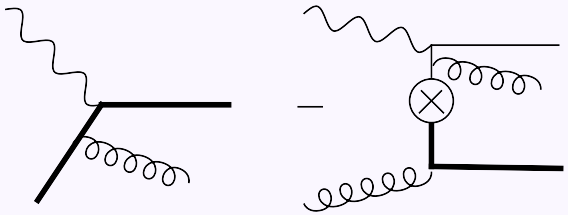


$$D_{C0}^{(1)} = c_{h,h}^{(0)} \otimes c - a_s\, c_{h,h}^{(0)} \otimes A_{hg}^{(1)} \otimes g; \qquad a_s = \frac{\alpha_s}{(4\pi)} \tag{10}$$



$D_{C0}^{(2)}$

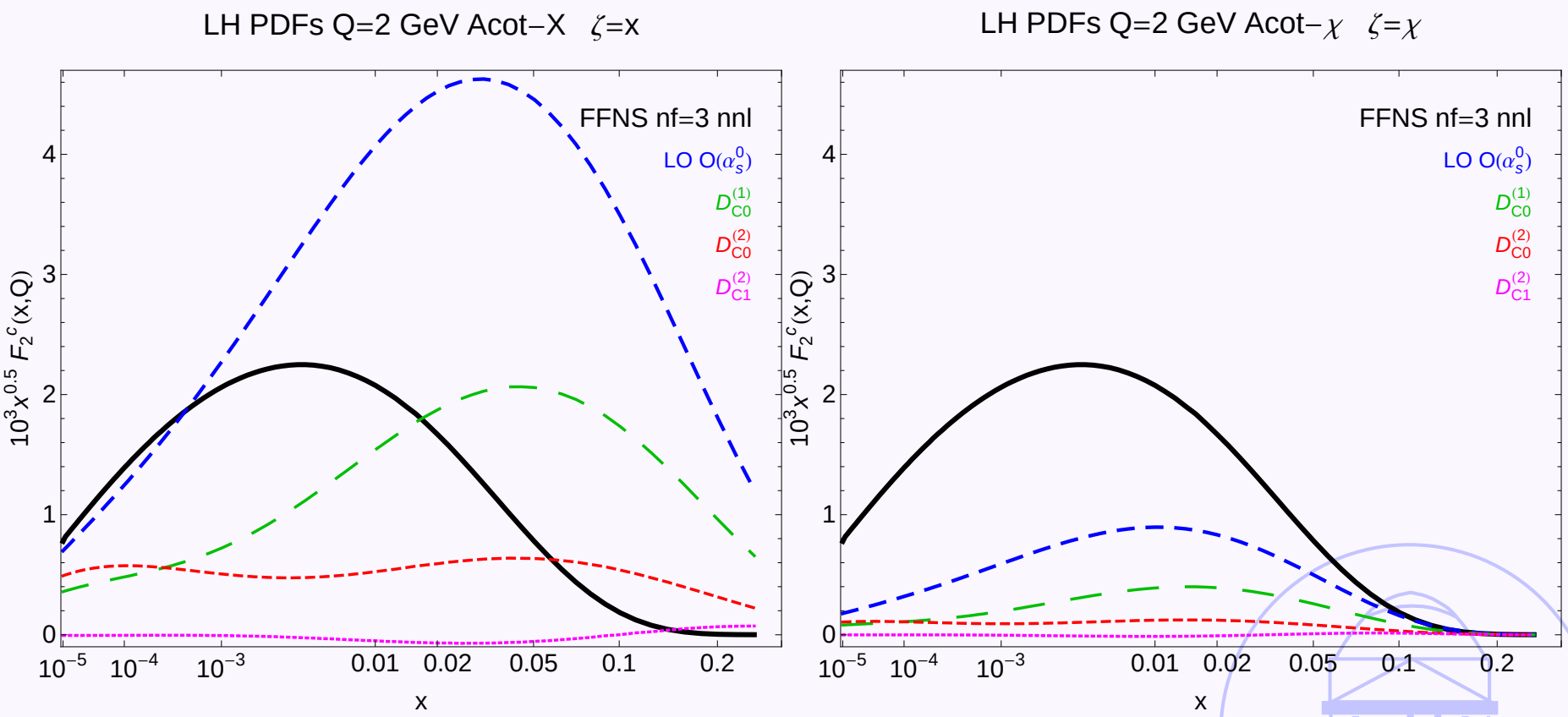
$$D_{C0}^{(2)} = D_{C0}^{(1)} - a_s^2\, c_{h,h}^{(0)} \otimes A_{hg}^{(2)} \otimes g - a_s^2\, c_{h,h}^{(0)} \otimes A_{hl}^{PS,(2)} \otimes \Sigma \tag{11}$$



$D_{C1}^{(2)}$

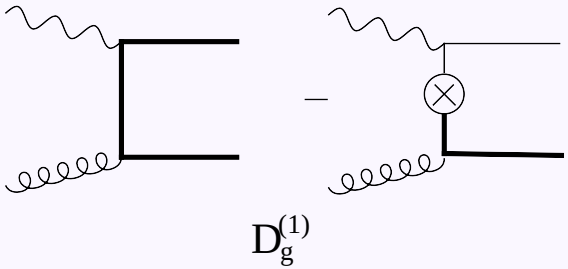
$$D_{C1}^{(2)} = a_s\, c_{h,h}^{(1)} \otimes c - a_s^2\, c_{h,h}^{(1)} \otimes A_{hg}^{(1)} \otimes g \tag{12}$$

NNLO: Cancellations at $Q^2 \approx m_c^2$

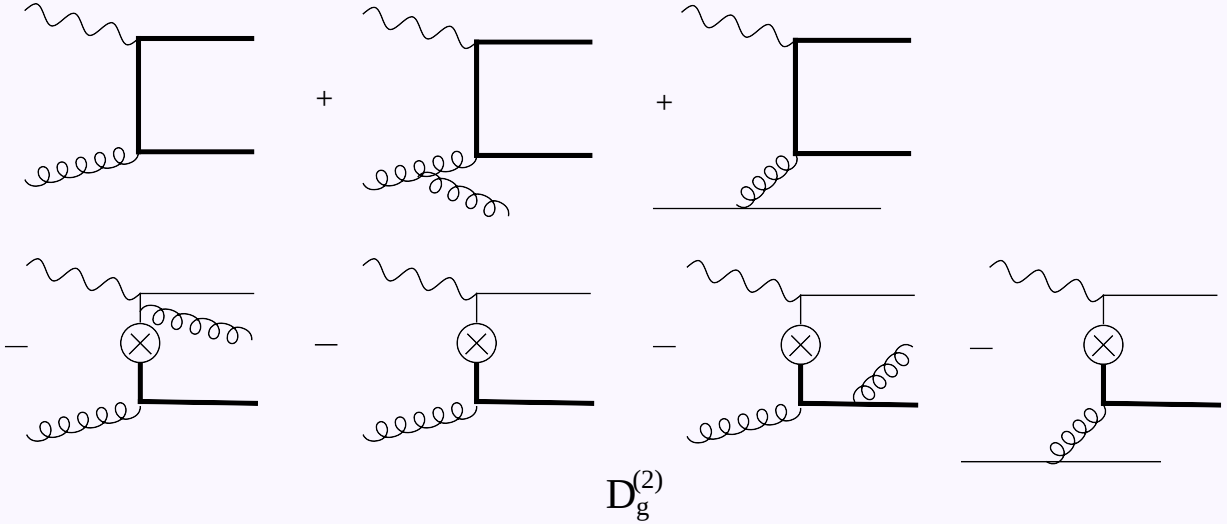


$D_{C1}^{(2)} << D_{C0}^{(2)} << D_{C0}^{(1)} \leq$ FFN at NNLO both for $\zeta = x$ and $\zeta = \chi$.

NNLO: Cancellations at $Q >> m_c$



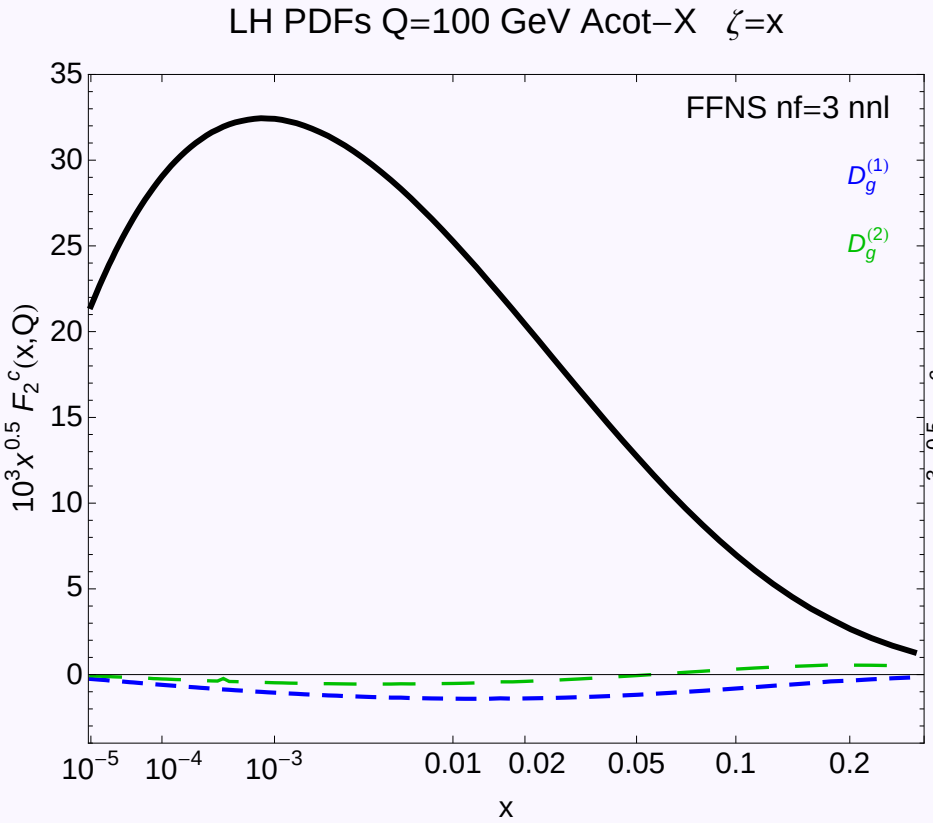
$$D_g^{(1)} \equiv C_{h,g}^{(1)} = a_s \left(F_{h,g}^{(1)} \otimes g - c_{h,h}^{(0)} \otimes A_{hg}^{(1)} \otimes g \right) \tag{13}$$

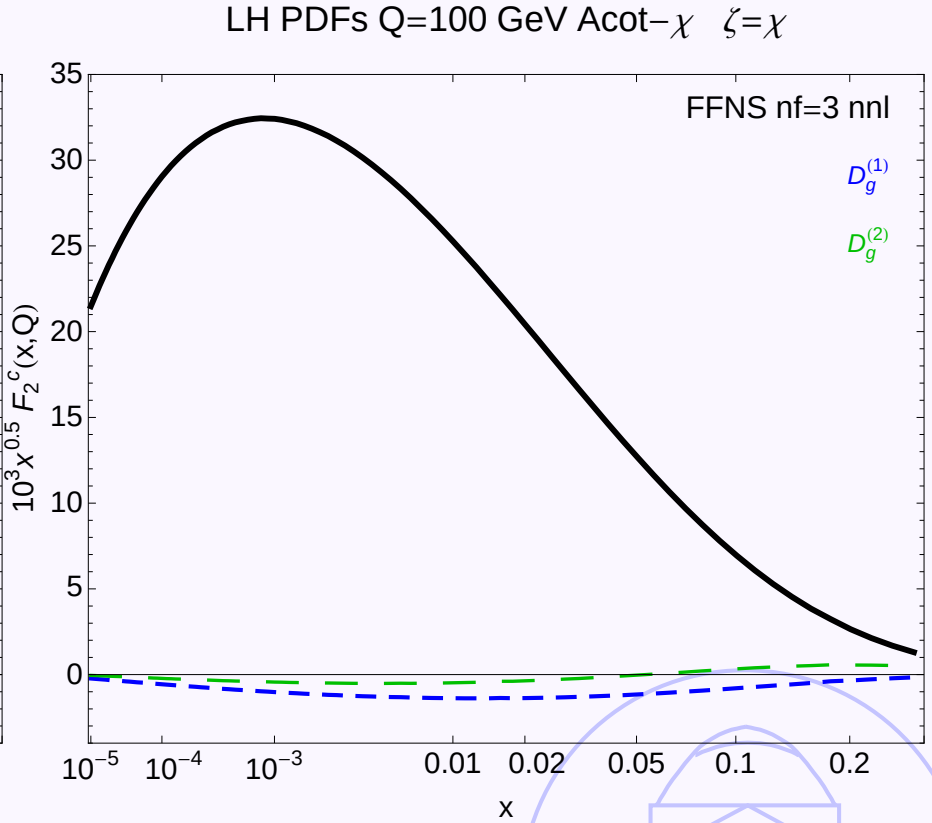


$$D_g^{(2)} = D_g^{(1)} + a_s^2 \left[\widehat{F}_{h,g}^{(2)} \otimes g + \widehat{F}_{h,l}^{PS,(2)} \otimes \Sigma - c_{h,h}^{(1)} \otimes A_{hg}^{(1)} \otimes g \right.$$
$$\left. - c_{h,h}^{(0)} \otimes A_{hg}^{(2)} \otimes g - c_{hh}^{(0)} \otimes A_{hl}^{PS,(2)} \otimes \Sigma \right] \tag{14}$$

$D_g^{(1)}$ is of order of α_s^2 while $D_g^{(2)}$ is of order of α_s^3.

F_2^c at NNLO: Cancellations at $Q = 100$ GeV





$D_g^{(2)} << D_g^{(1)} <$ FFN at NNLO $<$ ACOT

$\log \frac{Q^2}{m_c^2}$ terms in FFN are canceled well by subtractions.

Main messages

- An NNLO calculation for $F_2^{c,b}$ and $F_L^{c,b}$ in the S-ACOT scheme is proven to be viable
- This is the most challenging component of the NNLO CTEQ PDF analysis, which will be made available soon.
- NNLO predictions are stable and show a remarkable reduction in the dependence on free parameters, compared to NLO.
- They will help us to reduce tuning of m_c and scale parameters, currently used by NLO CT10 PDFs to achieve excellent agreement with the HERA DIS data

BACK UP SLIDES

Simplified Aivazis-Collins-Olness-Tung scheme

ACOT, PRD 50 3102 (1994); Collins, PRD 58 (1998) 094002; Kramer, Olness, Soper, PRD (2000) 096007

- The default mass scheme of CTEQ6.6 and CT10 PDFs
- Based upon, and closely follows, the proof of QCD factorization for DIS with massive quarks *(Collins, 1998)*
- Relatively simple, compared to BMSN or TR schemes
  - One value of N_f (and one PDF set) in each Q range
  - Straightforward matching based on kinematical rescaling
  - Sets $m_Q = 0$ in ME with incoming c or b
- Reduces to the ZM $\overline{MS}$ scheme at $Q^2 \gg m_Q^2$, without additional renormalization
- Reduces to the FFN scheme at $Q^2 \approx m_Q^2$
  - has reduced dependence on tunable parameters at NNLO

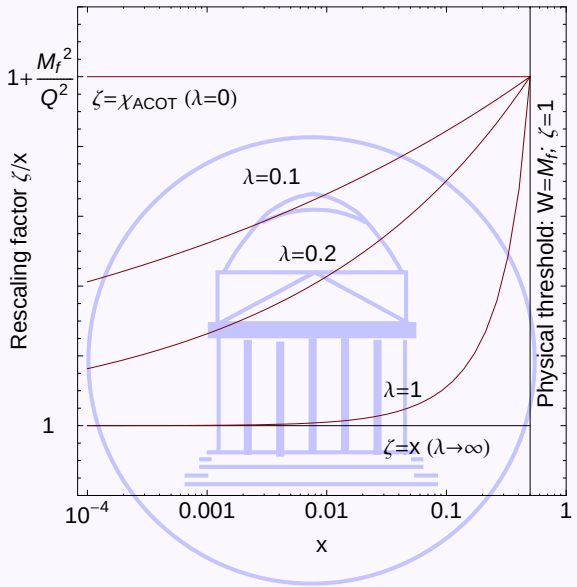
S-ACOT input parameters

At $Q \approx m_c$, F_2^c depends significantly on

1. **Charm mass:** $m_c = 1.3$ GeV in CT10
2. **Factorization scale:** $\mu = \sqrt{Q^2 + \kappa m_c^2}$; $\kappa = 1$ in CT10
3. **Rescaling variable** $\zeta(\lambda)$ for matching in $\gamma^* c$ channels
   (*Tung et al., hep-ph/0110247; Nadolsky, Tung, PRD79, 113014 (2009)*)

$$F_i(x, Q^2) = \sum_{a,b} \int_\zeta^1 \frac{d\xi}{\xi}\, f_a(\xi, \mu)\, C^a_{b,\lambda}\left(\frac{\zeta}{\xi}, \frac{Q}{\mu}, \frac{m_i}{\mu}\right)$$

$$x = \zeta \Big/ \left(1 + \zeta^\lambda \cdot (4m_c^2)/Q^2\right), \text{ with } 0 \le \lambda \lesssim 1$$

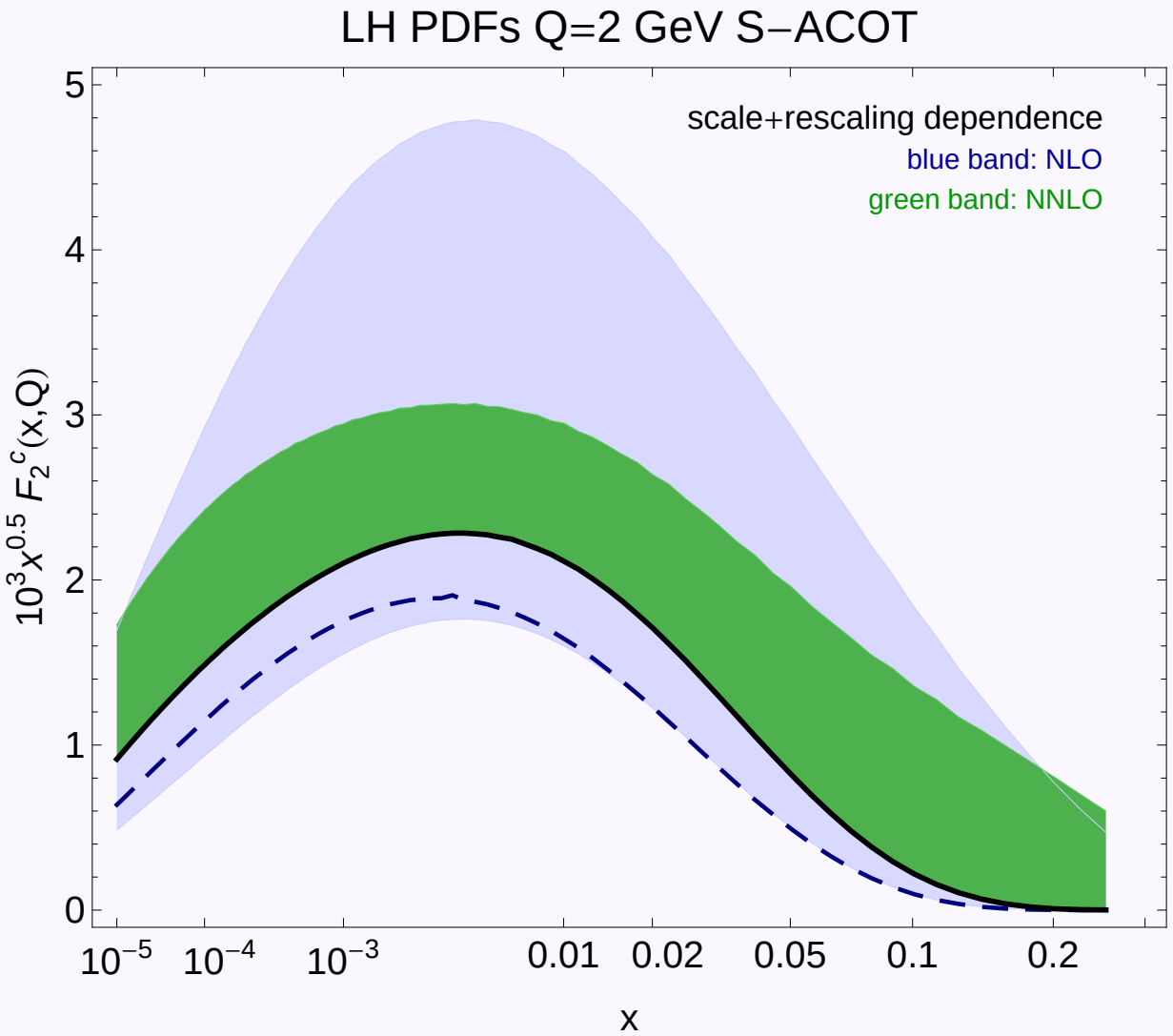


CT10 uses

$$\zeta(0) \equiv \chi \equiv x\left(1 + 4m_c^2/Q^2\right),$$

motivated by momentum conservation

Results for $F_2^c(x, Q^2)$ at NLO/NNLO - Preliminary



Plots for $Q^2 = 10\ GeV^2$ and $Q^2 = 100\ GeV^2$ are in the backup.

PS and NS structure of $F(x,Q)$

Using PS and NS decomposition we write

$$F(x,Q) = \sum_{i,a} e_i^2 c_{i,a} \otimes f_{a/p} = \sum_i e_i^2 \left\{ \sum_j \left[c^{PS} + \delta_{ij} c^{NS} \right] \otimes f_{j/p} + c_g \otimes f_{g/p} \right\}$$

$$= \sum_i e_i^2 c^{NS} \otimes f_{i/p} + \left(\sum_i e_i^2 \right) \left\{ c^{PS} \otimes f_{\Sigma/p} + c_g \otimes f_{g/p} \right\}$$

$$= c^{NS} \otimes \Sigma^{+,NS} + \frac{\left(\sum_i e_i^2\right)}{N_f} \left\{ c^S \otimes \Sigma + N_f c_g \otimes f_{g/p} \right\}. \tag{15}$$

Here $\Sigma^{+,NS}$ and Σ are the non-singlet and singlet sums of (anti-)quark PDFs,

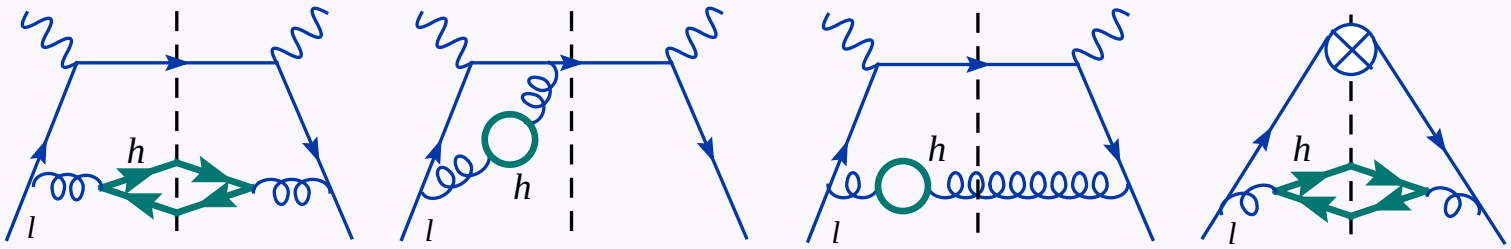
$$\Sigma(x,\mu) = \sum_{i=1}^{N_f} \left[f_{i/p}(x,\mu) + \bar{f}_{i/p}(x,\mu) \right],$$

$$\Sigma^{+,NS}(x,\mu) = \sum_{i=1}^{N_f} e_i^2 \left(f_{i/p}(x,\mu) + \bar{f}_{i/p}(x,\mu) - \frac{1}{N_f} \Sigma(x,\mu) \right),$$

and

$$c^S = c^{NS} + N_f c^{PS}.$$

Light-quark component of $F(x,Q)$



(a) (b) (c) (d)

The non-singlet heavy-quark coefficient function,

$$F^{NS,(2)}_{l,l,heavy}(x,Q^2/m_h^2) = \left(L^{NS,(2)}_{I,q}(x,Q^2/m_h^2)\right)_+ + \frac{2}{3}\ln\left(\frac{Q^2}{m_h^2}\right) c^{(1)}_{l,l}(x), \tag{16}$$

is composed of contributions of several classes shown in Figs (a)-(c). The real emission of heavy-quark pair as in Fig. (a) are accounted from $L^{NS,(2)}_{I,q}(x,Q^2/m_h^2)$, Buza *et al.* Nucl.Phys.B472 1996, for which Adler's sum rule holds, so that

$$\int_0^1 F^{NS,(2)}_{l,l,heavy}(x,Q^2/m_h^2)\,dx = 0. \tag{17}$$

Light-quark component of $F(x,Q)$

In the $Q^2 \gg m_h^2$ limit for the inclusive F_2

- $F^{NS,(2)}_{l,l,heavy} \Rightarrow$ large terms $\propto \ln(Q^2/m_h^2)$ + finite,
- Those coincide with $A^{NS,(2)}_{l,l,heavy}$ to the heavy-quark PDF from light quark flavors.

After we subtract from the coeff. function as shown before

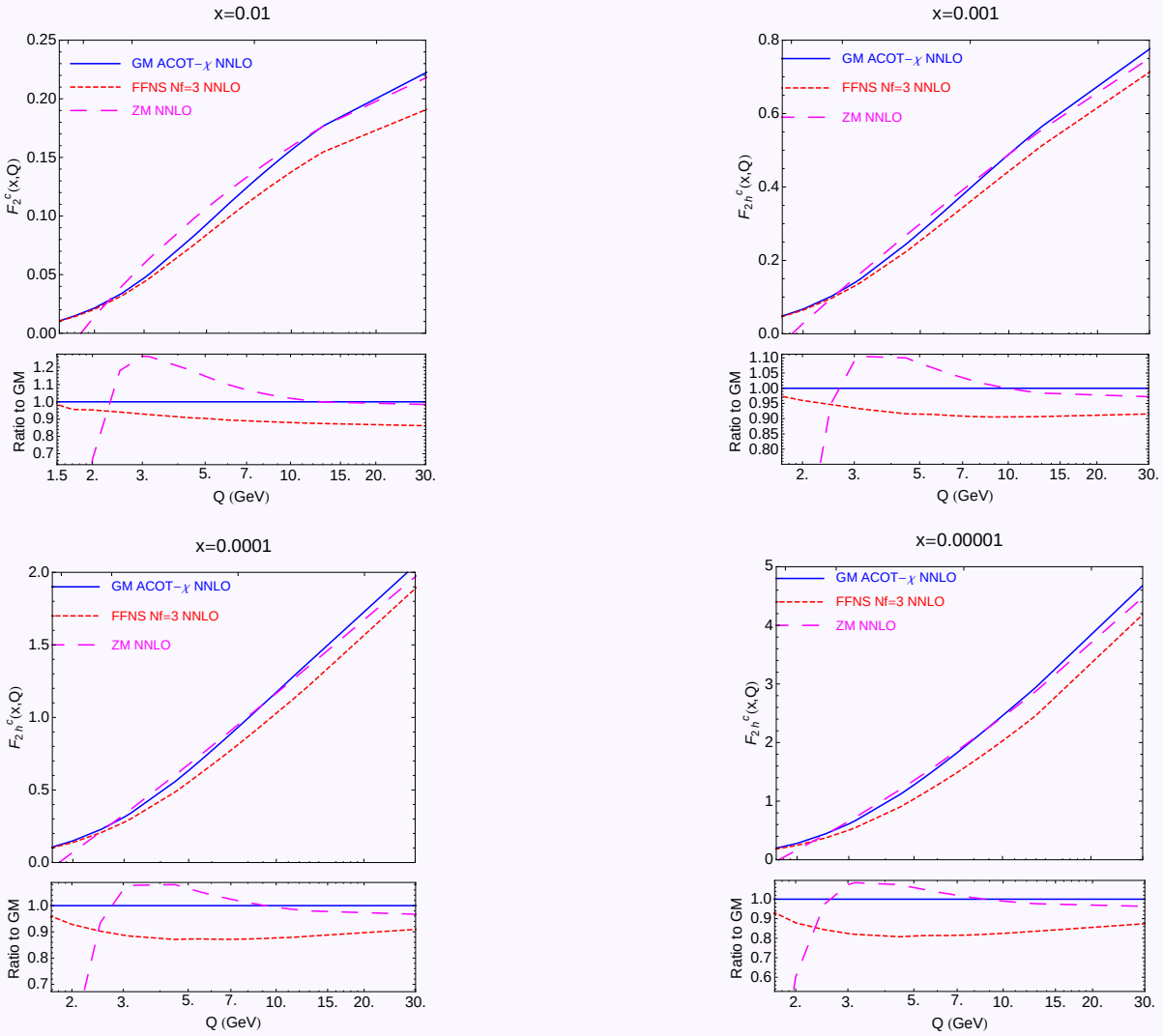
$$C^{(2),NS}_{l,l} = \widehat{f}^{NS,(2)}_{l,l,light} + F^{NS,(2)}_{l,l,heavy} - A^{NS,(2)}_{ll,heavy}, \tag{18}$$

- $C^{(2),NS}_{l,l}$ does not contain heavy quark lines,
- for $m_h^2/Q^2 \to 0$, $C^{(2),NS}_{l,l} \approx$ the ZM expression computed by Zijlstra, Van Neerven, PLB272 1991

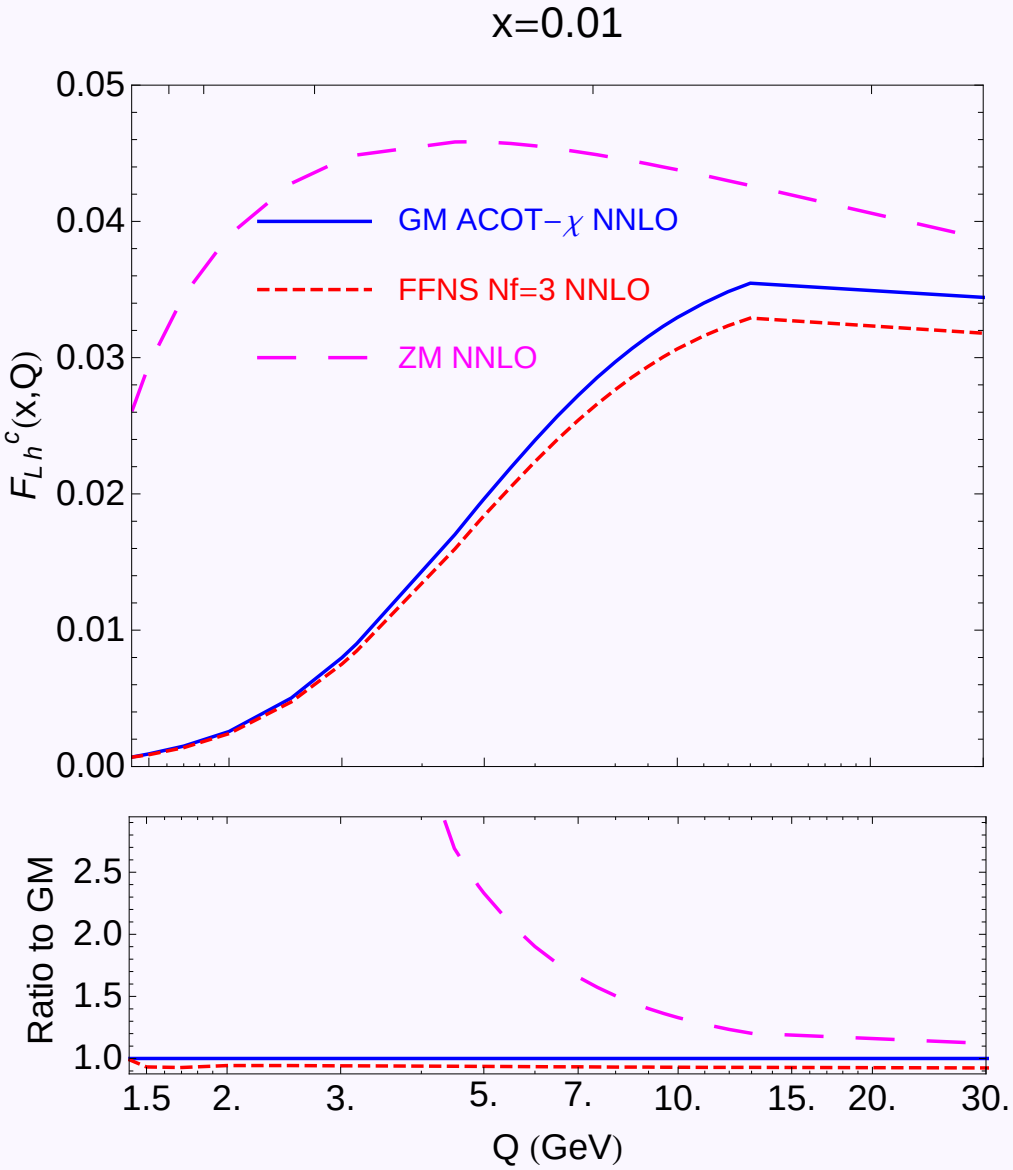
Finally we obtain

$$F^{(2)}_l = e_l^2 \left\{ C^{NS,(2)}_{l,l} \otimes (f_{l/p} + f_{\bar{l}/p}) + c^{PS,(2)} \otimes \Sigma + c^{(2)}_{l,g} \otimes f_{g/p} \right\}. \tag{19}$$

$F_2^c(x, Q^2)$ at NNLO, other x bins - Preliminary

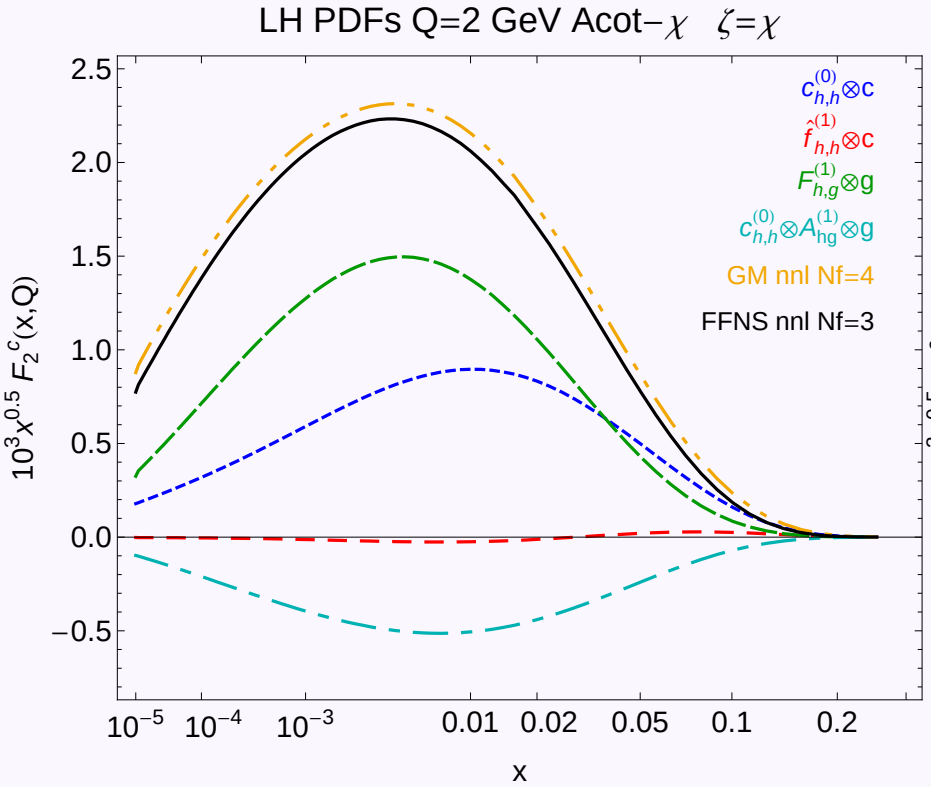


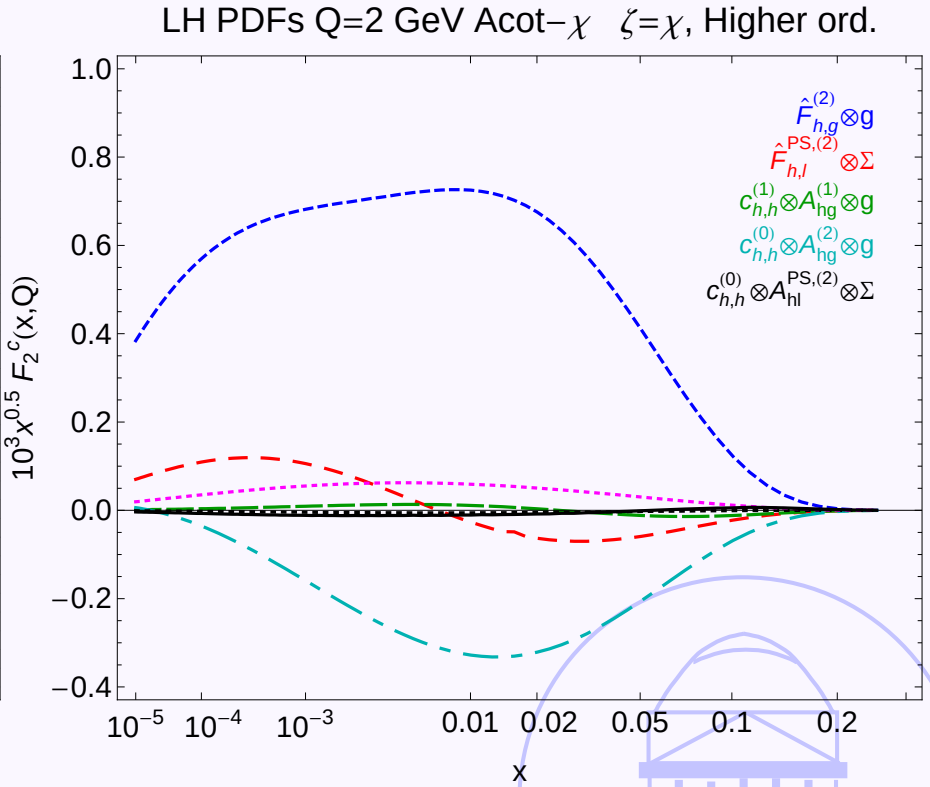
$F_L^c(x, Q^2)$ at NNLO - Preliminary



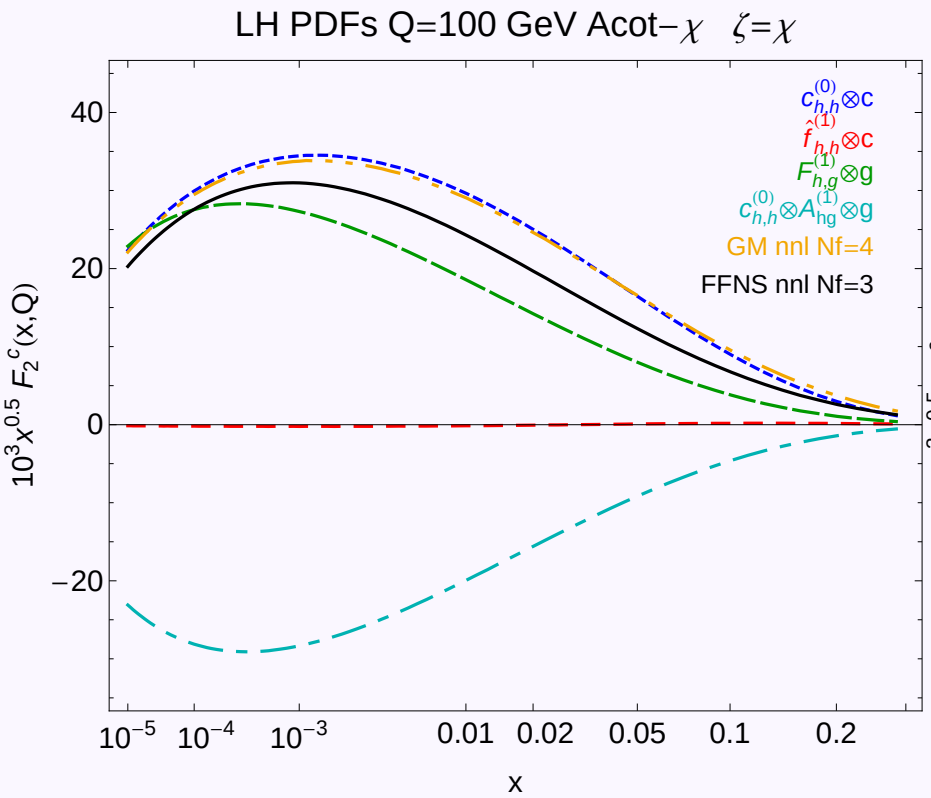
S-ACOT is close to FFNS at all Q. ZM over-estimates S-ACOT everywhere

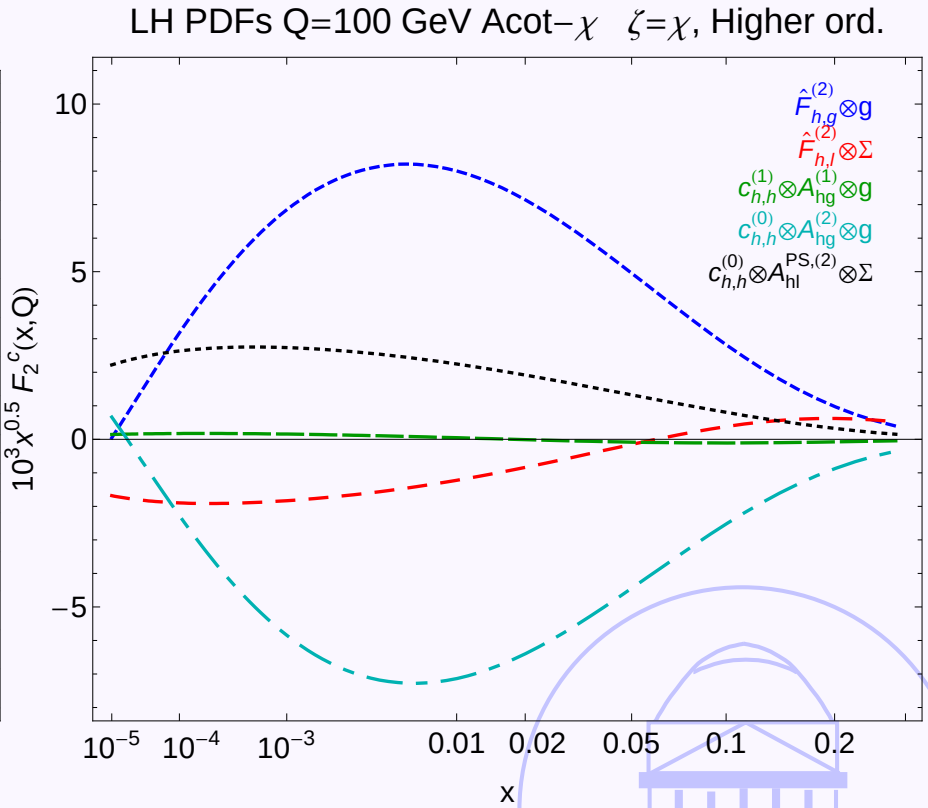
F_2^c: Anatomy of the contributions $Q = 2$ GeV





F_2^c: Anatomy of the contributions $Q = 100$ GeV





FFNS expression for $F^c_{2,L}(x,Q)$

- Riemersma, Smith, Van Neerven, Phys. Lett. B347 (1995) 143-151- The structure functions are given by

$$
\begin{aligned}
F_k(x,Q) &= \frac{Q^2\alpha_s}{4\pi^2 m^2}\int_x^{z_{max}}\frac{dz}{z}\left[e_H^2\, g\left(\frac{x}{z},\mu^2\right)c^{(0)}_{k,g}\right] \\
&+ \frac{Q^2\alpha_s^2}{\pi m^2}\int_x^{z_{max}}\frac{dz}{z}\left\{e_H^2\, g\left(\frac{x}{z},\mu^2\right)\left(c^{(1)}_{k,g}+\bar{c}^{(1)}_{k,g}\,\ln\frac{\mu^2}{m^2}\right)\right. \\
&+ \sum_{i=q,\bar{q}}\left[e_H^2\, f_i\left(\frac{x}{z},\mu^2\right)\left(c^{(1)}_{k,i}+\bar{c}^{(1)}_{k,i}\,\ln\frac{\mu^2}{m^2}\right)\right. \\
&+ \left.\left. e^2_{L,i}\, f_i\left(\frac{x}{z},\mu^2\right)\left(d^{(1)}_{k,i}+\bar{d}^{(1)}_{k,i}\,\ln\frac{\mu^2}{m^2}\right)\right]\right\}, \qquad (20)
\end{aligned}
$$

Here e_H is the charge of the heavy quark while e_L refers to the light quark. Furthermore $k=2,L$, $z_{max}=Q^2/(Q^2+4m^2)$ and $i=g,q,\bar{q}$.