# Accessing the distribution of linearly polarized gluons in unpolarized hadrons *

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- Safe extraction of $h_1^{\perp\, g}$ in $e\, h \to e'\, Q\, \bar{Q}\, X$ and $e\, h \to e'\, \mathrm{jet}\, \mathrm{jet}\, X$
- Azimuthal asymmetries measurable at EIC or LHeC
- Comparison with $h_1\, h_2 \to Q\, \bar{Q}\, X$ and $h_1\, h_2 \to \mathrm{jet}\, \mathrm{jet}\, X$
- Summary and conclusions

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* PRL 106 (2011) 132001
In coll. with D. Boer (U. Groningen), S. Brodsky (SLAC), P. Mulders (VU Amsterdam)

## *Gluon distributions*

- Experimental and theoretical investigations of gluons inside hadrons focussed so far on their momentum and helicity distributions:
  - $g(x)$: *unpolarized* gluons with collinear momentum fraction $x$ in *unp.* hadrons
  - $\Delta g(x)$: *circularly polarized* gluons with mom. fraction $x$ in *polarized* hadrons

- Taking into account the transverse momentum $\boldsymbol{p}_T$ of the gluon:

$$(\Delta)g(x) \longrightarrow (\Delta)g(x, \boldsymbol{p}_T^2)$$

  and other transverse momentum dependent gluon pdfs (TMDs) can be nonzero

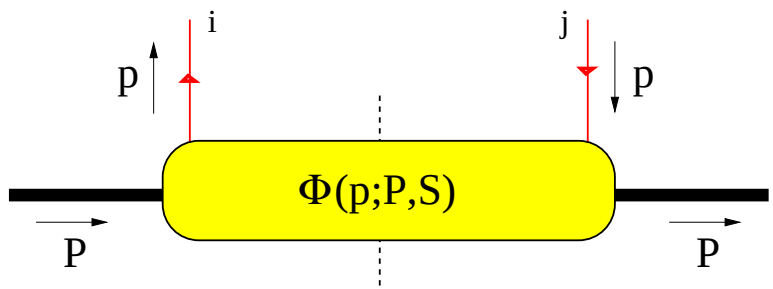
- In this framework, gluons do not have to be unpolarized, even if the parent hadron itself is unpolarized (different polarization mode compared to $\Delta g$)!

- Nontrivial property that has received much more attention in the quark sector, where it is also supported by first-principle lattice QCD calculations

D. Brömmel it et al., PRL 101 (2008) 122001

## *Quark and antiquark correlators*

- Parton correlators describe the hadron $\longrightarrow$ parton transitions and can be parameterized in terms of transverse momentum dependent (TMD) pdfs




- For an unp. hadron with momentum $P$, at leading twist (LT), omitting gauge links

$$\Phi_q(x,p_T;P) = \frac{1}{2}\left\{ f_1^q(x,\boldsymbol{p}_T^2)\ \not{P} + i h_1^{\perp\, q}(x,\boldsymbol{p}_T^2)\ \frac{[\not{p}_T, \not{P}]}{2M} \right\}$$

  - $f_1^q(x,\boldsymbol{p}_T^2) \equiv q(x,\boldsymbol{p}_T^2)$ is the unpolarized quark distribution; $M^2 = P^2$
  - $h_1^{\perp q}(x,\boldsymbol{p}_T^2)$ is the $T$-odd, quark transverse spin distribution in an unp. hadron

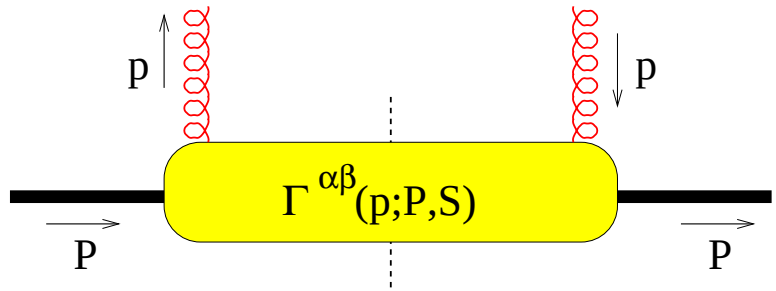
D. Boer, P. Mulders, PRD 57 (1998) 5780

- $h_1^{\perp q}(x,\boldsymbol{p}_T^2) = 0$ in the absence of initial or final state interactions (ISI/FSI)

- The antiquark corr. $\bar{\Phi}_q$ is obtained from $\Phi_q$ by replacing $f_1^q \longrightarrow f_1^{\bar{q}}$ and $h_1^{\perp\, q} \longrightarrow h_1^{\perp\, \bar{q}}$

## Gluon correlator

- We introduce the light-like vector $n$ conjugate to $P$ satisfying $n^2{=}0$ and $n{\cdot}P{=}1$, and define the transverse projector $g_T^{\alpha\beta} \equiv g^{\alpha\beta} - P^\alpha n^\beta / P{\cdot}n - n^\alpha P^\beta / P{\cdot}n$




- For a gluon momentum $p = x\,P + p_T + p^- n$, at LT and omitting gauge links

$$\Phi_g^{\alpha\beta}(x,p_T;P) \equiv \Gamma^{\alpha\beta} = \frac{-1}{2x}\left\{ g_T^{\alpha\beta}\, f_1^g(x,\boldsymbol{p}_T^2) - \left( \frac{p_T^\alpha p_T^\beta}{M^2} + g_T^{\alpha\beta}\,\frac{\boldsymbol{p}_T^2}{2M^2} \right) h_1^{\perp\, g}(x,\boldsymbol{p}_T^2) \right\}$$

  - $f_1^g(x,\boldsymbol{p}_T^2) \equiv g(x,\boldsymbol{p}_T^2)$ is the usual unpolarized gluon distribution; $p_T^2 = -\boldsymbol{p}_T^2$
  - $h_1^{\perp\, g}(x,\boldsymbol{p}_T^2)$ is the $T$-even distribution of linearly pol. gluons in an unp. hadron

P. Mulders, J. Rodrigues, PRD 63 (2001) 094021

- $h_1^{\perp\, g}$ is a helicity-flip distribution, and a second rank tensor in $p_T$ ($p_T$-even)

- $h_1^{\perp g}(x,\boldsymbol{p}_T^2) \neq 0$ in the absence of ISI or FSI, but, as any TMD, it will receive contributions from ISI/FSI $\longrightarrow$ it can be nonuniversal!

## *The function $h_1^{\perp\, g}$: phenomenology*

- So far no experimental studies of the function $h_1^{\perp\, g}$ have been performed

- $h_1^{\perp\, g}$ contributes to the so-called dijet imbalance in hadronic collisions, commonly used to extract the average partonic intrinsic transverse momentum. Complication!

  It is likely too hard to measure the azimuthal asymmetry for $p\,p \;\to\; \mathrm{jet}\,\mathrm{jet}\,X$ ,

$$\mathcal{A} \quad \sim \quad \cos 4\phi \; h_1^{\perp\, g} \otimes h_1^{\perp\, g}$$

D. Boer, P. Mulders, CP, PRD 80 (2009) 094017

- Similar to azimuthal asymmetry appearing in the Drell-Yan process:

$$\nu_{\mathrm{DY}} \quad \sim \quad \cos 2\phi \; h_1^{\perp\, q} \otimes h_1^{\perp\, \bar{q}}$$

D. Boer, PRD 60 (1999) 014012

- In $h\,h$ collisions: problems because of factorization breaking effects

T. Rogers, P. Mulders, PRD 81 (2010) 094006

- Unlike $h_1^{\perp\, q}$, $h_1^{\perp\, g}$ does not need to appear in pairs, hence a safe extraction of $h_1^{\perp\, g}$ in a simpler manner could be possibile, for example from $\mathcal{A} \quad \sim \quad \cos 2\phi \; h_1^{\perp\, g}$

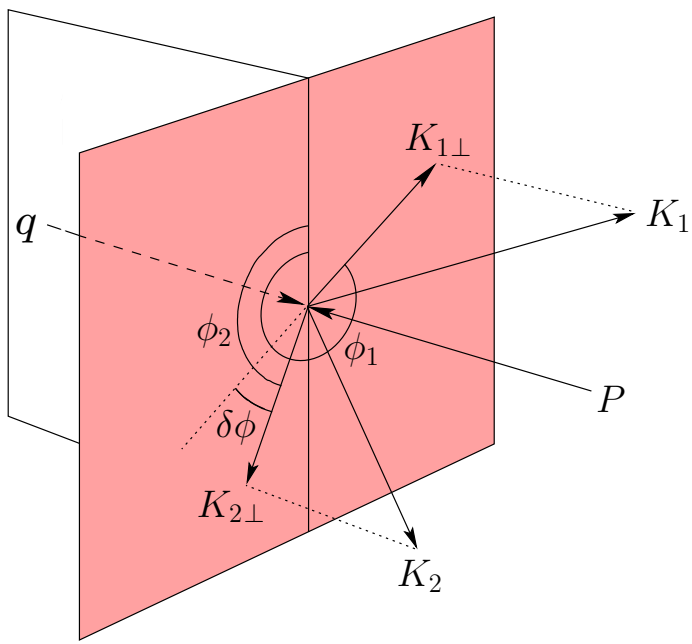
# *Electroproduction of heavy quark and jet pairs*

- In $e\,p$ instead of $p\,p$ collisions: only one TMD is involved; TMD factorization holds for the processes:

$$e(\ell) + h(P) \;\to\; e(\ell') + Q(K_1) + \bar{Q}(K_2) + X$$

$$e(\ell) + h(P) \;\to\; e(\ell') + \mathrm{jet}(K_1) + \mathrm{jet}(K_2) + X$$

  the $Q\bar{Q}$ or jet pairs are almost back-to-back in the plane perpendicular to $q$ and $P$, $q \equiv \ell - \ell'$: four-momentum of the exchanged virtual photon $\gamma^*$




- Perpendicular vectors $a_\perp$: components of the vector $a$ orthogonal to both $q$, $P$

## *Calculation of the cross sections*

- At high energies the cross sections factorize in a leptonic tensor $L$, a soft parton correlator $\Phi$ for the incoming hadron and a hard part $H$ (LO pQCD)

TMD master formula:

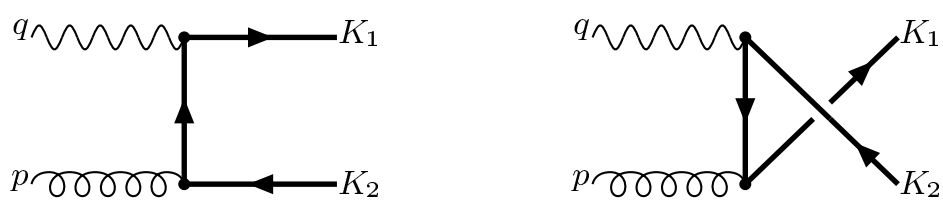
$$d\sigma = \frac{1}{2s}\,\frac{d^3\ell'}{(2\pi)^3\,2E'_e}\,\frac{d^3K_1}{(2\pi)^3\,2E_1}\,\frac{d^3K_2}{(2\pi)^3\,2E_2}\int dx\,d^2\boldsymbol{p}_T\,(2\pi)^4\delta^4(q+p-K_1-K_2)$$
$$\times\sum_{a,b,c}\frac{1}{Q^4}\,L(\ell,q)\otimes\Phi_a(x,\boldsymbol{p}_T)\otimes\,|H_{\gamma^*\,a\to b\,c}(q,p,K_1,K_2)|^2$$

- Leptonic tensor:

$$L^{\mu\nu}(\ell,q) = -g^{\mu\nu}\,Q^2 + 2\,(\ell^\mu\ell'^\nu + \ell^\nu\ell'^\mu)$$

- In $Q\,\bar{Q}$ production: $K_i^2 = M_Q^2$, $|H_{\gamma^*\,a\to b\,c}|^2 = |H_{\gamma^*\,g\to Q\,\bar{Q}}(q,p,K_1,K_2)|^2$

$\gamma^*(q)\,+\,g(p)\,\to\,Q(K_1)\,+\,\bar{Q}(K_2)$:



# Electroproduction of heavy quarks: kinematics

- Utilized DIS variables and relations:

$$
\begin{aligned}
s &\equiv (\ell + P)^2 \\
Q^2 &= -q^2 \\
x_B &\equiv \frac{Q^2}{2P\cdot q} \\
y &\equiv \frac{P\cdot q}{P\cdot \ell} = \frac{Q^2}{s\, x_B}
\end{aligned}
$$

- Approximation: since $Q$ and $\bar{Q}$ are produced almost back-to-back in azimuth, if we define $K_\perp \equiv (K_{1\perp} - K_{2\perp})/2$ , $q_T \equiv K_{1\perp} + K_{2\perp}$ , then $|q_T| \ll |K_\perp|$ and

$$
\begin{aligned}
K_{1\perp} &\approx K_\perp\,, \qquad K_{2\perp} \approx -K_\perp \\
M_{i\perp}^2 &\approx M_\perp^2 = M_Q^2 + \boldsymbol{K}_\perp^2
\end{aligned}
$$

- In the partonic subprocess $q + p = K_1 + K_2$, from which one gets the relations

$$
\begin{aligned}
z_1 + z_2 &= 1 \\
\boldsymbol{p}_T &= \boldsymbol{q}_T
\end{aligned}
$$

where $$z \equiv z_2 = \frac{P\cdot K_2}{P\cdot q}, \qquad 1 - z = z_1 = \frac{P\cdot K_1}{P\cdot q}$$

# Azimuthal asymmetry

- $y_1$, $y_2$: rapidities of $Q$ and $\bar{Q}$ resp., measured along the photon-hadron direction; for $|\boldsymbol{q}_T| \ll |\boldsymbol{K}_T|$, with $\boldsymbol{q}_T = |\boldsymbol{q}_T|(\cos\phi_T, \sin\phi_T)$, $\boldsymbol{K}_\perp = |\boldsymbol{K}_\perp|(\cos\phi_\perp, \sin\phi_\perp)$

$$\frac{d\sigma}{dy_1\ dy_2\, dy\, dx_B\, d^2\boldsymbol{q}_T d^2\boldsymbol{K}_\perp} \sim \delta(1-z_1-z_2)\,\frac{\alpha^2\alpha_s}{sM_\perp^2}\left[A + B\,\boldsymbol{q}_T^2\ \cos 2(\phi_T-\phi_\perp)\right]$$

  - Angular independent part of the cross section:

$$A = e_Q^2\,\mathcal{A}^{eg\to eQ\bar{Q}}\left(y, z, \frac{Q^2}{M_\perp^2}, \frac{M_Q^2}{M_\perp^2}\right) f_1^g(x, \boldsymbol{q}_T^2)$$

  - The term $B$ contains the information about the linear polarization of gluons

$$B = \frac{1}{M^2}\, e_Q^2\,\mathcal{B}^{eg\to eQ\bar{Q}}\left(y, z, \frac{Q^2}{M_\perp^2}, \frac{M_Q^2}{M_\perp^2}\right) h_1^{\perp\, g}(x, \boldsymbol{q}_T^2)$$

    it is not power suppressed if $Q^2$ and/or $M_Q^2$ are $\mathcal{O}(K_\perp^2)$

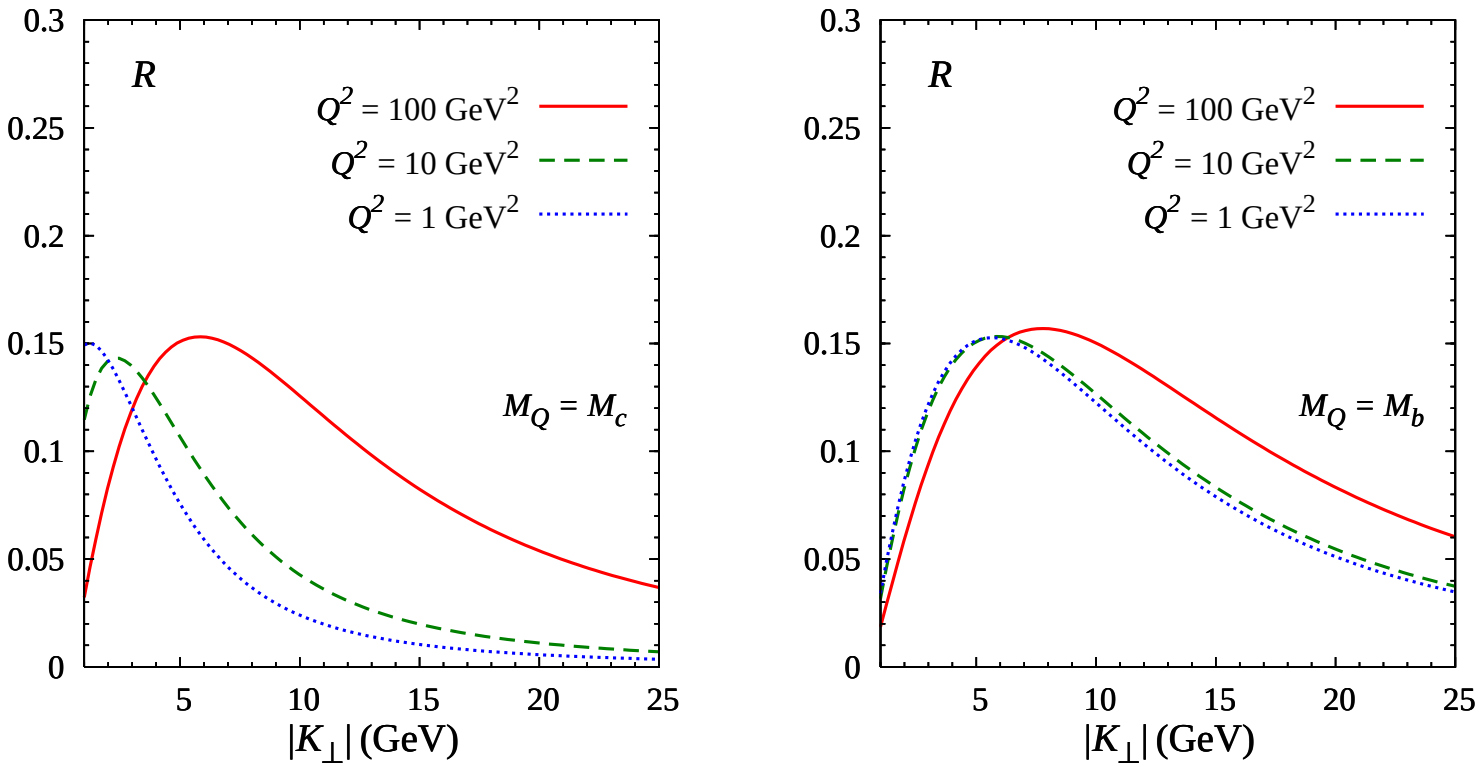
    it can be singled out by weighting $d\sigma$ with $(\boldsymbol{q}_T^2)^n\ \cos 2(\phi_T-\phi_\perp)$

# Upper bound of the asymmetry

- Transverse moments of TMDs: $f^{(n)}(x) \equiv \int d^2\boldsymbol{p}_T \, \left(\boldsymbol{p}_T^2/2M^2\right)^n \, f(x, \boldsymbol{p}_T^2)$

$$|h_1^{\perp\, g(2)}(x)| \leq \frac{\langle p_T^2 \rangle}{2M^2} \, f_1^g(x)$$

$$\Longrightarrow \left| \frac{\int d^2\boldsymbol{q}_T \, \boldsymbol{q}_T^2 \, \cos 2(\phi_T - \phi_\perp) \, d\sigma}{\int d^2\boldsymbol{q}_T \, \boldsymbol{q}_T^2 \, d\sigma} \right| = \frac{\int d\boldsymbol{q}_T^2 \, \boldsymbol{q}_T^4 \, |B|}{2 \int d\boldsymbol{q}_T^2 \, \boldsymbol{q}_T^2 \, A} \leq R \equiv \frac{|\mathcal{B}^{eg \to eQ\bar{Q}}|}{\mathcal{A}^{eg \to eQ\bar{Q}}}$$




- (Large) maximal values of the asymmetry at $z = 0.5$, $y = 0.01$ and different $Q^2$

# Electroproduction of two jets

- The cross section for the process

$$e(\ell) + h(P) \;\to\; e(\ell') + \mathrm{jet}(K_1) + \mathrm{jet}(K_2) + X$$

where the two jets are almost back-to-back in the plane perpendicular to $q$ and $P$, can be calculated in the same way

$$\frac{d\sigma}{d\eta_1\, d\eta_2\, dy\, dx_B\, d^2\boldsymbol{q}_T d^2\boldsymbol{K}_\perp} \sim \delta(1-z_1-z_2)\,\frac{\alpha^2\alpha_s}{s\boldsymbol{K}_\perp^2}\left[A + B\,\boldsymbol{q}_T^2\,\cos 2(\phi_T - \phi_\perp)\right]$$

  - $y_i \to \eta_i = -\ln\left(\tan(\frac{1}{2}\theta_i)\right)$, $\theta_i$ being the polar angles of the final partons in the virtual photon-hadron cms frame

  - The expression for the $B$ term relative to $e\,h \to e\,\mathrm{jet}\,\mathrm{jet}\,X$ can be obtained from the one for $e\,h \to e\,Q\,\bar{Q}\,X$ taking $M_Q = 0$

  - $A$ now also receives a contribution from $\gamma^* q \to gq$, not just from $\gamma^* g \to q\bar{q}$ leading to somewhat smaller asymmetries

## *Comments on the results*

- Reconstruction of the transverse momenta $K_{i\perp}$ of the jets or heavy quarks is essential, accuracy $\delta K_\perp$ has to satisfy $\delta K_\perp \ll |q_T| \ll |K_\perp|$

- Jet and heavy quark production in SIDIS require high energy colliders: the $\cos 2\phi$ asymmetry and $h_1^{\perp\, g}$ could be measured at a future EIC or at the LHeC

- What about hadronic collisions at RHIC, Tevatron, LHC?
  - Two TMDs involved: breaking of TMD factorization becomes relevant
  - ISI/FSI do not allow a separation of gauge links into the matrix elements of the various TMDs

    T. Rogers, P. Mulders, PRD 81 (2010) 094006
  - In simple cases (e.g. single Sivers effect) one can find weighted expressions that allow a factorized result, but with in general different factors for different diagrams in the partonic subprocess

    C. Bomhof, P. Mulders, F. Pijlman, EPJ C 47 (2006) 147
    C. Bomhof, P. Mulders, NP B 795 (2008) 409

## *Hadroproduction of heavy quarks*

- Assuming TMD factorization, the cross section for the process

$$h_1(P_1)+h_2(P_2)\to Q(K_1)+\bar{Q}(K_2)+X\,,$$

is calculated in the hadronic cms frame, similar to hadroproduction of two jets

D. Boer, P. Mulders, CP, PRD 80 (2009) 094017

- For $|\boldsymbol{q}_T| \ll |\boldsymbol{K}_\perp|$, with $\boldsymbol{q}_T = |\boldsymbol{q}_T|(\cos\phi_T,\,\sin\phi_T)$, $\boldsymbol{K}_\perp = |\boldsymbol{K}_\perp|(\cos\phi_\perp,\,\sin\phi_\perp)$

$$\frac{d\sigma}{dy_1 dy_2 d^2\boldsymbol{K}_{1\perp} d^2\boldsymbol{K}_{2\perp}} = \frac{\alpha_s^2}{sM_\perp^2}\left[A + B\,\boldsymbol{q}_T^2\,\cos 2(\phi_T-\phi_\perp) + C\,\boldsymbol{q}_T^4\,\cos 4(\phi_T-\phi_\perp)\right]$$

- The terms $A$, $B$, $C$ depend on $\boldsymbol{q}_T^2$, $z$, $M_Q^2/M_\perp^2$ and on the lightcone momentum fractions of the two incoming partons (carrying momenta $p_1$, $p_2$)

$$x_1 = \frac{1}{\sqrt{s}}\left(M_{1\perp}\,e^{y_1} + M_{2\perp}\,e^{y_2}\right), \quad x_2 = \frac{1}{\sqrt{s}}\left(M_{1\perp}e^{-y_1} + M_{2\perp}\,e^{-y_2}\right)$$

$$\boldsymbol{q}_T \equiv \boldsymbol{K}_{1\perp} + \boldsymbol{K}_{2\perp} = \boldsymbol{p}_{1T} + \boldsymbol{p}_{2T}$$

- $A$, $B$, $C$ contain convolutions of the various pdfs and are calculated at LO in pQCD

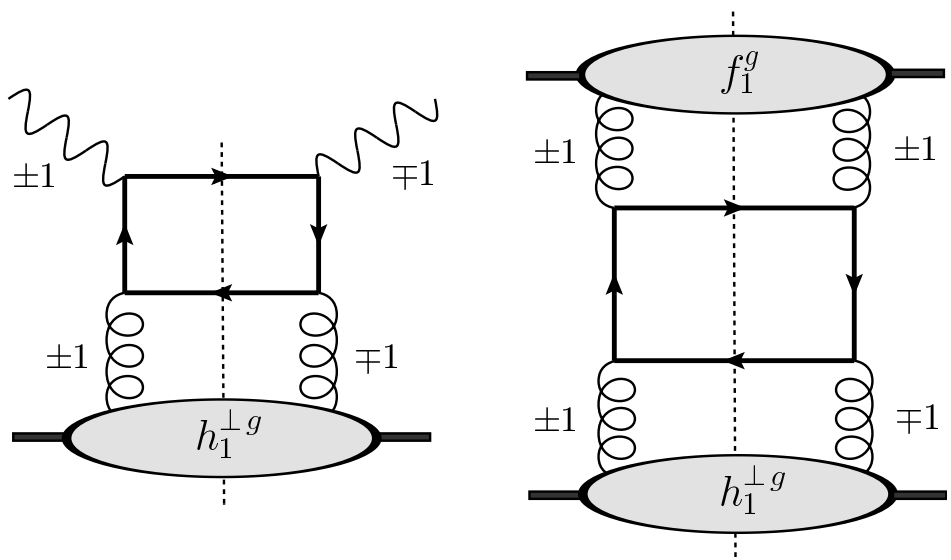
# Cross section and TMDs

- Schematically, leaving out terms with higher powers in $M_Q^2/M_\perp^2$:

$$A: \quad f_1^q \otimes f_1^{\bar q}, \quad f_1^g \otimes f_1^g \qquad\qquad B: \; h_1^{\perp\, q} \otimes h_1^{\perp\, \bar q}, \quad \frac{M_Q^2}{M_\perp^2}\, f_1^g \otimes h_1^{\perp\, g}$$

$$C: \; h_1^{\perp\, g} \otimes h_1^{\perp\, g}$$

- $h_1^{\perp\, g} \otimes h_1^{\perp\, g}$: double suppression, assuming $h_1^{\perp\, g} \ll f_1^g$
- Origin of the $M_Q^2/M_\perp^2$ factor in the contribution of $h_1^{\perp\, g}$ to the $\cos 2\phi$ asymmetry:




$e\, h \to e'\, Q\, \bar Q\, X$ $\qquad$ $h_1\, h_2 \to Q\, \bar Q\, X$

in $h_1\, h_2$ collisions helicity flip is required in quark propagators $\Longrightarrow$ $M_Q^2/M_\perp^2$ factor

## *Weighted cross sections*

- Weighting the cross section with $\cos 2\phi$ allows to project out the term $B$

$$\big\langle \cos 2(\phi_T - \phi_\perp) \big\rangle \propto \int \frac{d\phi_T}{2\pi} \, \cos 2(\phi_T - \phi_\perp) \, d\sigma = \frac{1}{2} \, \frac{\alpha_s^2}{s M_\perp^2} \, \boldsymbol{q}_T^2 \, B$$

but the expression for $B$ does not deconvolute:

$$B = \mathcal{B}^{q\bar{q}\to Q\bar{Q}} \left( z, x_1, x_2, \frac{M_Q^2}{M_\perp^2}, \boldsymbol{q}_T^2 \right) + \frac{M_Q^2}{M_\perp^2} \, \mathcal{B}^{gg\to Q\bar{Q}} \left( z, x_1, x_2, \frac{M_Q^2}{M_\perp^2}, \boldsymbol{q}_T^2 \right)$$

$$\mathcal{B}^{q\bar{q}\to Q\bar{Q}} \sim h_1^{\perp\, q} \otimes h_1^{\perp\, \bar{q}} \qquad \mathcal{B}^{gg\to Q\bar{Q}} \sim f_1^g \otimes h_1^{\perp\, g}$$

- One gets deconvoluted results with additional $\boldsymbol{q}_T^2/M^2$-weighting

$$\pi \int d\boldsymbol{q}_T^2 \left( \frac{\boldsymbol{q}_T^2}{M^2} \right) \big\langle \cos 2(\phi_T - \phi_\perp) \big\rangle = \int d^2\boldsymbol{q}_T \left( \frac{\boldsymbol{q}_T^2}{M^2} \right) \cos 2(\phi_T - \phi_\perp) \, d\sigma$$

(assuming that $h_1$ and $h_2$ have the same mass $M$)

$$\pi \int d\boldsymbol{q}_T^2 \, \left( \frac{\boldsymbol{q}_T^2}{M^2} \right) \, \boldsymbol{q}_T^2 \, \mathcal{B}^{q\bar{q}\to Q\bar{Q}} \quad \propto \quad \sum_{\text{flavors}} h_1^{\perp q(1)}(x_1) \, h_1^{\perp \bar{q}(1)}(x_2) \, ,$$

$$\pi \int d\boldsymbol{q}_T^2 \, \left( \frac{\boldsymbol{q}_T^2}{M^2} \right) \, \boldsymbol{q}_T^2 \, \mathcal{B}^{gg\to Q\bar{Q}} \quad \propto \quad h_1^{\perp g(2)}(x_1) f_1^g(x_2) + f_1^g(x_1) \, h_1^{\perp g(2)}(x_2)$$

# *Photon-jet production in hadronic collisions*

- Suppression of $q\bar{q} \to Q\bar{Q}$ w.r.t. $gg \to Q\bar{Q}$ depends on $h_1$, $h_2$; $x_1$, $x_2$, $M_Q^2/M_\perp^2$; in $p\,\bar{p}$ and for $K_\perp^2 \sim M_Q^2$ the contribution from $h_1^{\perp\,g}$ is expected to be dominant

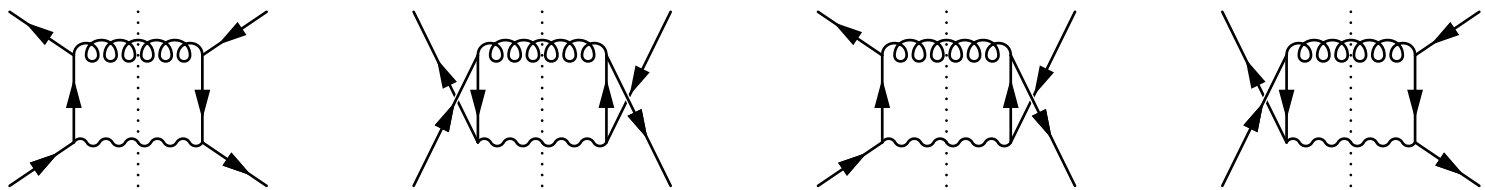
- Importance of the contribution from $h_1^{\perp\,q}$ can be assessed by a comparison to

$$h_1(P_1)+h_2(P_2) \to \gamma(K_\gamma)+\mathrm{jet}(K_j)+X$$

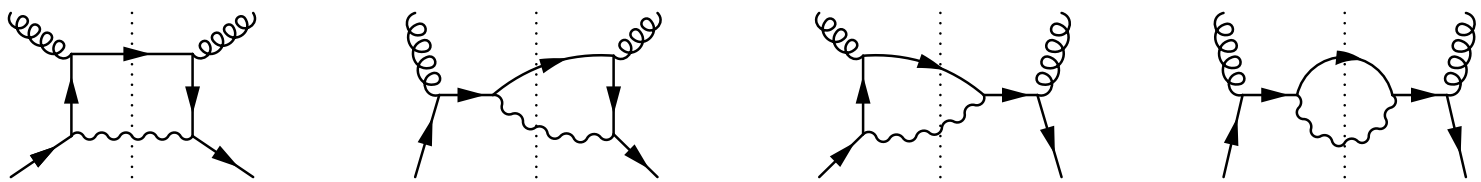
D. Boer, P. Mulders, CP, PLB 660 (2008) 360

- To LO in pQCD the reaction is mediated by the $2 \to 2$ partonic subprocesses

$q\bar{q} \to \gamma g$:



$qg \to \gamma q$:



- $h_1^{\perp\,g}$: power-suppressed contribution; only $\cos 2\phi$ asymm. $\sim h_1^{\perp\,q} \otimes h_1^{\perp\,\bar{q}}$ as in DY

## *Gauge links and factorization breaking*

- Even if weighting allows for factorization, two different $h_1^{\perp g(2)}(x)$ (and $f_1^{g(1)}(x)$) will appear: two gluon operators with the color structures $f_{abe}\, f_{cde}$ and $d_{abe}\, d_{cde}$

  C. Bomhof, P. Mulders, NPB 795 (2008) 409
  F. Dominguez, B-W. Xiao, F. Yuan, PRL 106 (2011) 022301

- Similar to what happens for single transverse spin asymmetries ($A_N$) in heavy quark production processes: two different ($f$ and $d$ type) gluon correlators arise $\Longrightarrow A_N(D) \neq A_N(\bar{D})$

  Z.B. Kang, J. W. Qiu, PRD 78 (2008) 014024
  Z.B. Kang et al., PRD 78 (2008) 114013
  F. Yuan, J. Zhou, PLB 668 (2008) 216

- In unpolarized scattering, simpler situation:
  - in $\gamma^* g \to Q\,\bar{Q}$ only the matrix element with the $f\,f$-structure appears
  - in $g\,g \to Q\,\bar{Q}$ the $d\,d$-structure dominates ( the $f\,f$-one suppressed by $1/N_c^2$)

## *Summary and conclusions*

- Measurements of the $\cos 2\phi$ azimuthal asymmetries of heavy quark and jet pair production in $e\,p$ (EIC, LHeC) and in $p\,p$ or $p\,\bar{p}$ collisions (RHIC, LHC, Tevatron) can probe the distribution of linearly polarized gluons inside unpolarized hadrons $h_1^{\perp\,g}$

- $h_1^{\perp\,g}$ contributes also to diphoton or even Higgs production

  P. Nadolsky, C. Balazs, E. Berger, C. P. Yuan, PRD 76 (2007) 013008
  S. Catani, M. Grazzini, NPB 845 (2011) 297
  J. Qiu, M. Schlegel, W. Vogelsang, arXiv:1103.3861

- From a theoretical viewpoint these asymmetries in $e\,h$ processes are among the simplest TMD observables. Comparison with $h\,h$ case is useful to study expected effects due to breaking of TMD factorization

- Relative simple measurements, polarized beams are not required

- Promising prospect for the extraction of $h_1^{\perp\,g}$ in the future and for the study of its potential process dependence