Time-like small x resummation for Fragmentation Functions

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Outline

1. Motivations

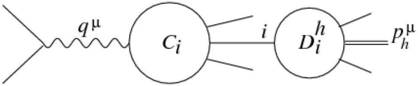
2. The method

3. Analytic Results

4. Conclusions and outlook

Motivations

Motivations for time-like small x resummation



Consider the semi inclusive hadron production $e^+ + e^- \to h + X$ and define the fraction of energy x carried away by h from a jet:

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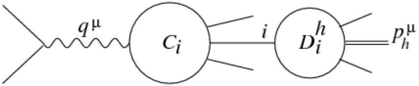

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$$x\, P_T(x, \alpha_s) = \sum_{n,m} \alpha_s^n\, \gamma_{nm}\, [\ln x]^{2n-1-m} \qquad \text{in the time-like splitt. func.}$$

$$x\, C_T(x, \alpha_s) = \sum_{n,m} \alpha_s^n\, c_{nm}\, [\ln x]^{2n-m} \qquad \text{in the coefficient functions}$$

The terms with $m = 1$ are DLs, with $m = 2$ are SLs and so on

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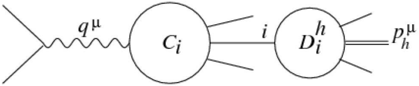

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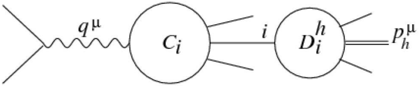

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To have reliable predictions at small x and jet multiplicities large logs have to be resummed

Motivations for the $\overline{MS}$ scheme

- **In the literature resummed expressions for $P_T(x, \alpha_s)$ and $C_T(x, \alpha_s)$ they exist only in the massive gluon (MG) scheme.**

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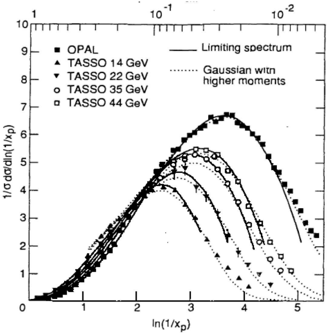
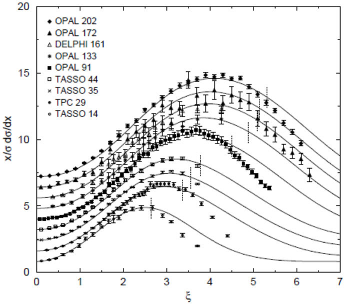
To extend very small x analysis beyond LO the knowledge of resummed expressions in the $\overline{MS}$ scheme for both the anomalous dimensions and the coefficient functions is necessary.

The progresses of the MLLA and of the FO+DLs at LO





In the figure the total hadron multiplicity (x/σ), $d\sigma/dx$ is plotted as a function of $\xi \equiv \ln(1/x)$

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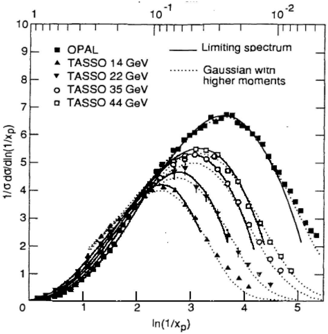

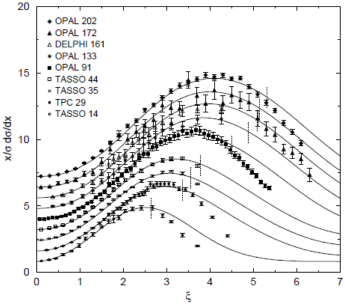

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  $\xi_{peak} \sim 1/4 \ln s$ **[Y. Dokshitzer, V. Khoze, A. Mueller,S. Troyan ('91)]**

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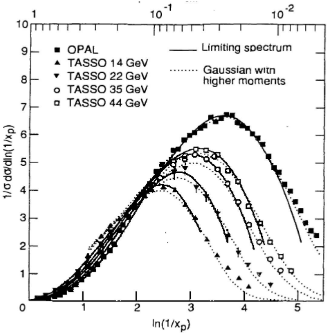

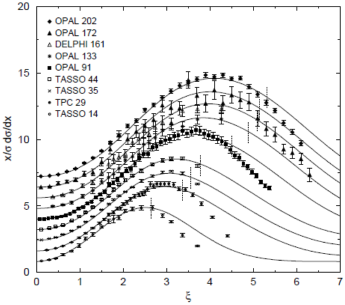

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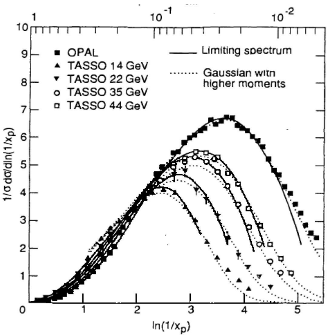

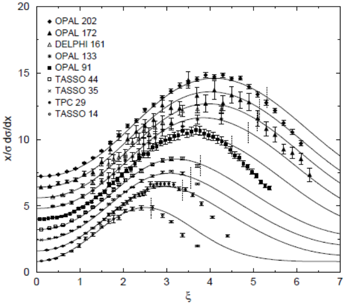

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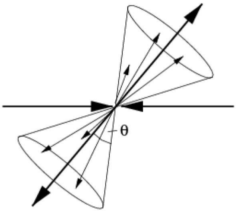
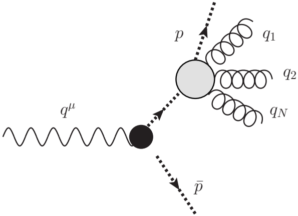
- **The inclusion of DLs also in the gluon coefficient function is expected to produce a significant improvment at larger ξ**
  [S. Albino, P. B., B. Kniehl, A. Kotikov ('11)]

The method

Eikonal verticies and color coherence in jet physics





If q_1 is the softest gluon which is also emitted at large angle with respect to $q_2, q_3, \ldots, q_N, q_{N+1} \equiv p, q_{N+2} \equiv \bar{p}$, the gluons form a jet around either p or $\bar{p}$. Its dominant contribution to the emission current amplitude is given according to the eikonal verticies by

$$J_\mu(q_1) = g\mu^\epsilon \sum_{j=2}^{N+2} \mathbf{T}_j \frac{q_{j\,\mu}}{q_2 \cdot q_1} = -g\mu^\epsilon \mathbf{T}_{\bar{p}} \left(\frac{p_\mu}{p \cdot q_1} - \frac{\bar{p}_\mu}{\bar{p} \cdot q_1} \right),$$

by use of color conservation.

$$\sum_{j=1}^{N+2} \mathbf{T}_j = 0$$

Single soft gluon emission factorization

The factorization of the single gluon probability emission is a direct consequence of the eikonal approximation and color coherence
This implies the folowing simple insertion operator (we put $q_1 \equiv q$)

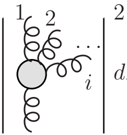
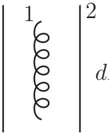
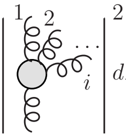
$$I_1(q) = J_\mu(q) J_\mu^\dagger(q) = g^2 \mu^{2\epsilon} C_i \frac{2(p \cdot \bar{p})}{(p \cdot q)(\bar{p} \cdot q)}; \qquad C_i = \mathbf{T}_p^2$$

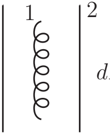
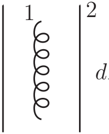
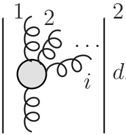
and the following fully factorized gluon probability emission

$$dw(q) = I_1 \, \frac{d^{d-1} q}{(2\pi)^{d-1} 2E_q} \to \frac{\alpha_s C_i}{\pi} \left(\frac{\mu}{Q^2} \right)^{\epsilon} \frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)} \frac{dz}{z^{1+\epsilon}} \frac{dx}{x^{1+2\epsilon}} \equiv dw(x, z)$$

Here $z = (1 - \cos\theta)/2$ with θ the scattering angle of q with respect to p, $Q^2 = (p + \bar{p})^2 = 2(p \cdot \bar{p})$ and $C_i = C_A$ for a glun jet and $C_i = C_F$ for a quark jet

Heuristic proof of the gluon master equation

$d\sigma_g^n = \Sigma_{i=1}^n$  $dPS_i \;=$  $dPS_1 + \Sigma_{i=2}^n$  dPS_i

$=$  $dPS_1 \;+\; \Bigg($  $dPS_1 + \Sigma_{i=2}^{n-1}$  $dPS_i \Bigg) dw(PS_1)$

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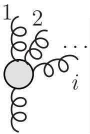
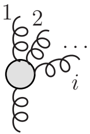
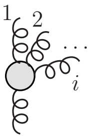
$d\sigma_g^n = \Sigma_{i=1}^n \Big|$  $\Big|^2 dPS_i \;=\; \Big|$  $\Big|^2 dPS_1 + \Sigma_{i=2}^n \Big|$  $\Big|^2 dPS_i$

$= \Big|$  $\Big|^2 dPS_1 \;+\; \Bigg(\Big|$  $\Big|^2 dPS_1 + \Sigma_{i=2}^{n-1} \Big|$  $\Big|^2 dPS_i \Bigg) dw(PS_1)$

$$= d\sigma_g^1 + d\sigma_g^{n-1}\, dw(PS_1)$$

Hence introducing the gluon density distribution $\mathcal{G}(x) = xG(x)$ we can formally write that this quantity satisfies the following master equation

$$\mathcal{G}(x) = \delta(1-x) + \int_{PS_1(x)} \mathcal{G}(x')\, dw(x', z)$$

The gluon master eq. in the MG and in the DR schemes

To extract the leading terms the strong ordering in the momenta ($x \ll x_1 \ll \cdots \ll x_n$) and in the emission angles ($z_{cut-off} \ll z_1 \ll \cdots \ll z_n$) should be imposed to the phase space

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$$xG(x,z) = \delta(1-x) + \frac{\alpha_s C_A}{\pi} \int_x^1 \frac{dx'}{x'} \int_z^1 \frac{dz'}{z'} x' G(x', z'),$$

where $z = m_g^2/x^2 Q^2$ should be put only at the end of the computation

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In the DR scheme

$$x^{1+2\epsilon} G(x,z,\epsilon) = \delta(1-x) + \frac{\alpha_s C_A}{\pi} \frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)} \left(\frac{\mu^2}{Q^2}\right)^\epsilon \int_x^1 \frac{dx'}{x'^{1+2\epsilon}} \int_z^1 \frac{dz'}{z'^{1+\epsilon}} \cdot x'^{1+2\epsilon} G(x', z', \epsilon),$$

where $z = 0$ should be put only at the end of the computation

Analytic Results

Solution to the master equations

We can solve the master equations for G in both schemes applying it iteratively and performing the Mellin transform

$$f(\omega) = \int_0^1 dx\, x^\omega f(x)$$

thus obtaining

In the MG regularization scheme

$$G(\omega, \alpha_s, \frac{m_g^2}{Q^2}) = 1 + \sum_{k=1}^{\infty} \left(\frac{\alpha_s C_A}{\pi} \right)^k \sum_{m=0}^{k} \frac{(-2)^m (k+m-1)! \ln^{k-m} Q^2/m_g^2}{(k-1)! m! (k-m)! \omega^{k+m}}$$

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$$G(x, \alpha_s, \frac{\mu^2}{Q^2}, \epsilon) \quad = \quad 1 + \sum_{k=1}^{\infty} \left[\frac{\alpha_s C_A}{\pi} \frac{(4\pi)^{\epsilon}}{\Gamma(1-\epsilon)} \left(\frac{\mu^2}{Q^2}\right)^{\epsilon}\right]^k \frac{(-1)^k}{\epsilon^k\, k!} \prod_{l=1}^{k} \frac{1}{\omega - 2l\epsilon}$$

Factorization of the mass singlularities

To understand the result we should compare it with the general form of the QCD factorization theorem. For the DR case we choose the $\overline{MS}$ subtraction scheme.

[R.K. Ellis, H. Georgi, M. Machacek, H.D. Politzer, G.G. Ross ('79); G. Curci, W. Furmanski, R. Petronzio ('80)]

In the MG regularization scheme

$$G(\omega, \alpha_s, \frac{m_g^2}{Q^2}) = C^{MG}(\omega, \alpha_s) \exp\left[\gamma^{MG}(\omega, \alpha_s) \ln \frac{Q^2}{m_g^2}\right]$$

In the $\overline{MS}$ scheme

$$G(x, \alpha_s, \frac{\mu^2}{Q^2}) = C^{\overline{MS}}(\omega, \alpha_s) \exp\left[-\frac{1}{\epsilon}\int_0^{\alpha_s(\mu^2/Q^2)^\epsilon S_\epsilon} \frac{d\alpha}{\alpha}\gamma^{\overline{MS}}(\omega, \alpha)\right];$$

$$S_\epsilon = e^{\epsilon(\ln 4\pi - \gamma_E)}$$

The DL anomalous dimension and coefficient function in the $\overline{MS}$ scheme

The extraction of the anomalous dimension γ and of the coefficient function C from this comparison is rather technical. We report here the result. **[S. Albino, P. B., B. Kniehl, A. Kotikov ('11)]**

In the MG regularization scheme

$$\gamma^{MG}(\omega, \alpha_s) = \frac{1}{4}\left(-\omega + \sqrt{\omega^2 + 8\alpha_s C_A/\pi}\right)$$

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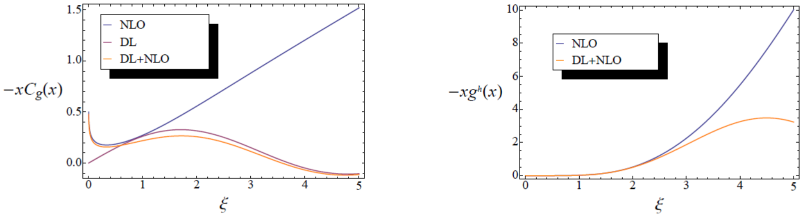
In the $\overline{MS}$ scheme (agrees with NNLO computations)

$$\gamma^{\overline{MS}}(\omega, \alpha_s) = \gamma^{MG}(\omega, \alpha_s)$$

$$C^{\overline{MS}}(\omega, \alpha_s) = \left(\frac{\omega}{\sqrt{\omega^2 + 8\alpha_s C_A/\pi}}\right)^{1/2}$$

Back to the x space



$$g^h(x) = \frac{1}{\sigma_B}\frac{d\sigma_g^h(x)}{dx}(x, Q^2) = \int_x^1 \frac{dx'}{x'} C_g^{\overline{MS}}(x', \alpha_s) D_g^h(x', Q^2); \qquad \xi = \ln\frac{1}{x}$$

All the large logarithms are under control and the correction is well behaved also in the region where tipically the perturbative expansion is spoiled

Conclusions and outlook

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- ✔ We have computed and resummed all the DLs in the gluon coefficient function in the $\overline{MS}$ scheme for the first time
- ✔ This enables the fixed order computations known in the $\overline{MS}$ up to NNLO to be improved by resummation in the same scheme
- ✔ Our formula is in agreement with NNLO full computations
- ✔ Our method provide a direct and simple way to compare with the result in the MG scheme in the literature and perform scheme changes
- ✔ All large logs in the coefficient function are under control at NLO
- ✘ Our results is an essential ingredient also for the resummation of the SLs of the time-like splitting functions; work in progress
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Thanks for your attention!