

# Sivers effect in SIDIS and pp collisions

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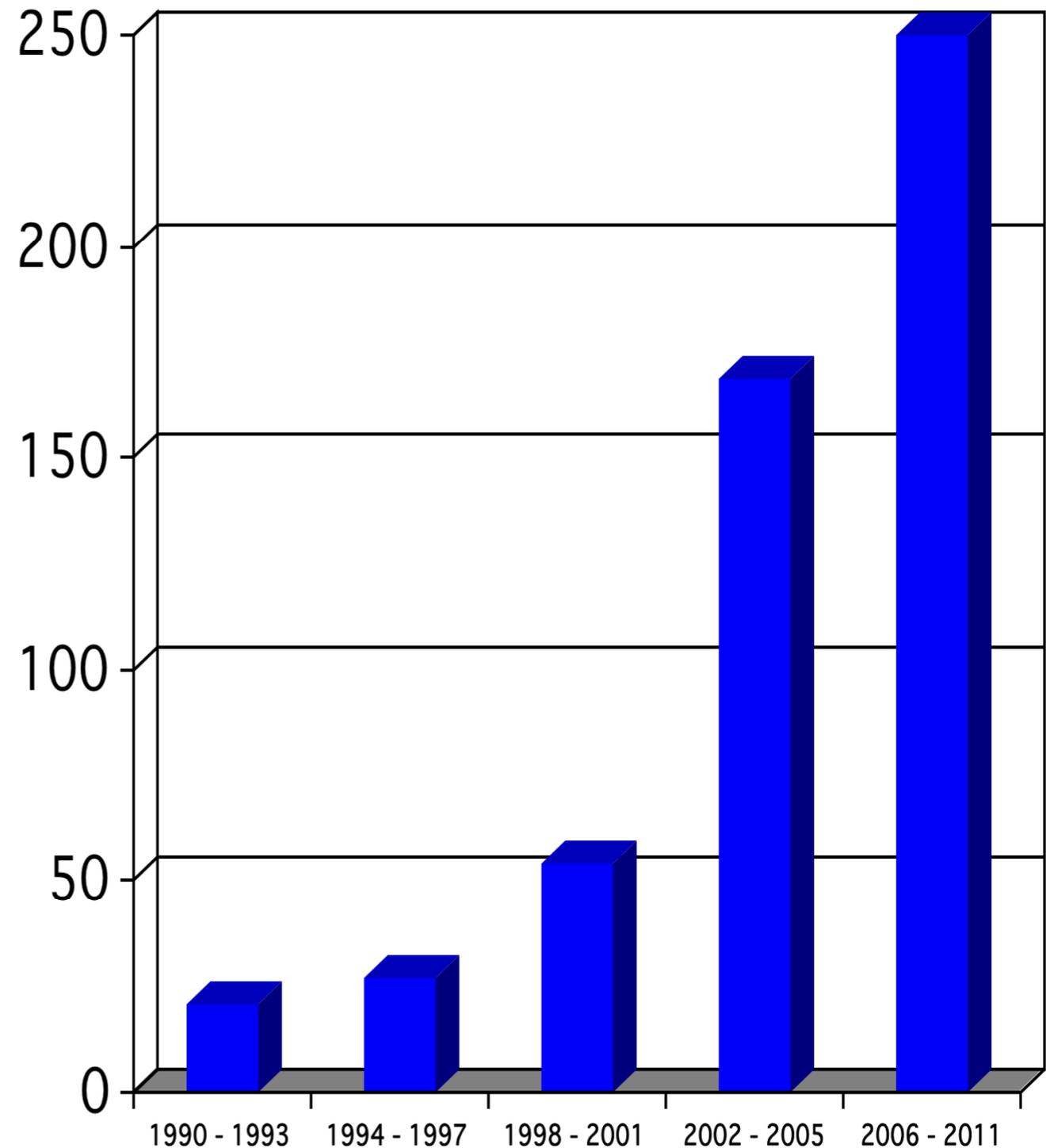
XIX International Workshop on Deep-Inelastic Scattering and  
Related Subjects (DIS 2011)  
Newport News, VA, Apr 12, 2011

Kang, Qiu, Vogelsang, Yuan  
arXiv: 1103.1591, PRD, in press

# Sivers function: birth and growth

- **1990: Sivers**
  - introduce Sivers function (kt-dependent PDF) to explain single transverse spin asymmetry observed in the experiments
- **1993: Collins**
  - shows Sivers function has to vanish due to time-reversal invariance
- **2002: Brodsky, Hwang, Schmidt**
  - the existence of Sivers function relies on the initial- and final-state interactions
- **2002: Ji, Yuan, Belitsky**
  - the initial- and final-state interaction is equivalent to the color gauge links in the definition of the TMD distribution functions

D. Sivers, PRD41, 1990

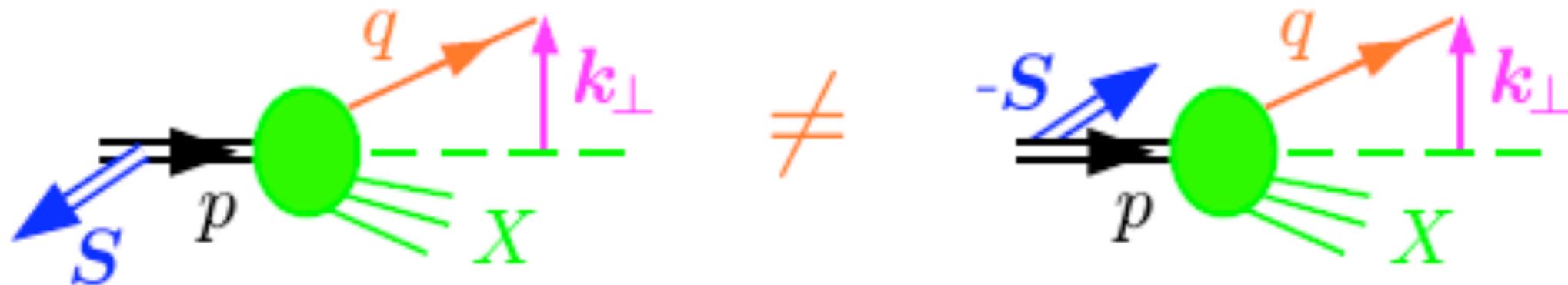


# Transverse momentum dependent distribution (TMD)

- Sivers function: an asymmetric parton distribution in a polarized hadron ( $k_t$  correlated with the spin of the hadron)

$$f_{q/h^\uparrow}(x, \mathbf{k}_\perp, \vec{S}) \equiv f_{q/h}(x, k_\perp) + \frac{1}{2} \Delta^N f_{q/h^\uparrow}(x, k_\perp) \vec{S} \cdot \hat{p} \times \hat{\mathbf{k}}_\perp$$

Spin-independent
Spin-dependent

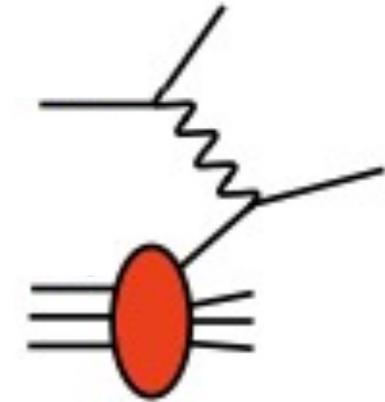
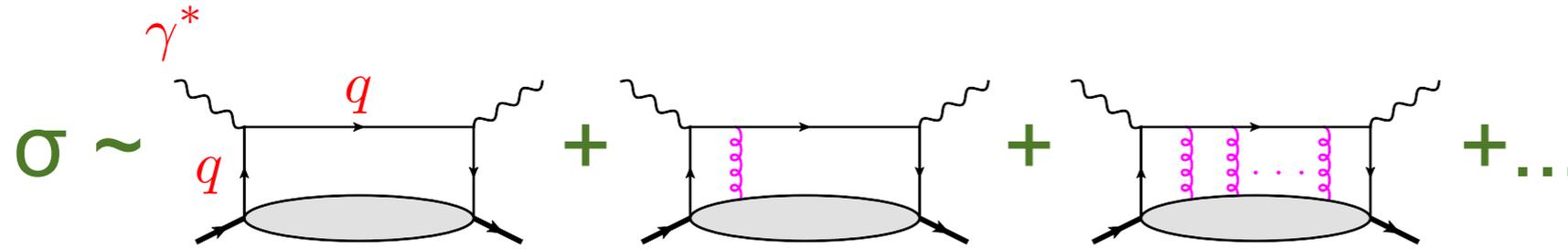


- Where does the phase come from?

# Sivers functions are process-dependent

- Existence of the Sivers function relies on the interaction between the active parton and the remnant of the hadron (process-dependent)

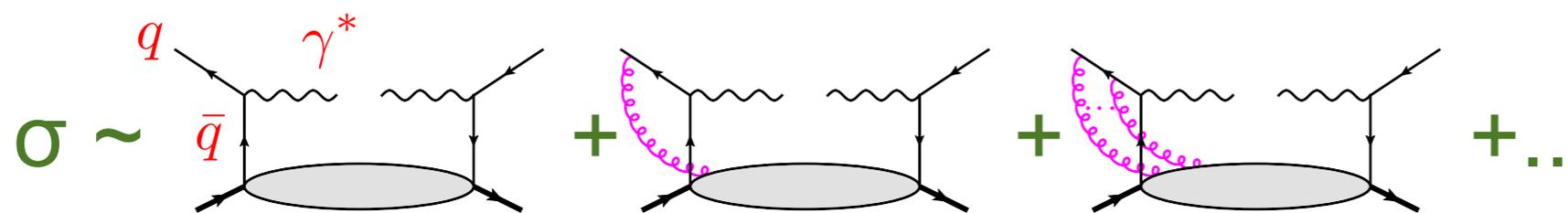
- SIDIS: final-state interaction



PDFs with SIDIS gauge link

$$\mathcal{P} e^{ig \int_y^{\infty} d\lambda \cdot A(\lambda)}$$

- Drell-Yan: initial-state interaction



PDFs with DY gauge link

$$\mathcal{P} e^{ig \int_y^{-\infty} d\lambda \cdot A(\lambda)}$$

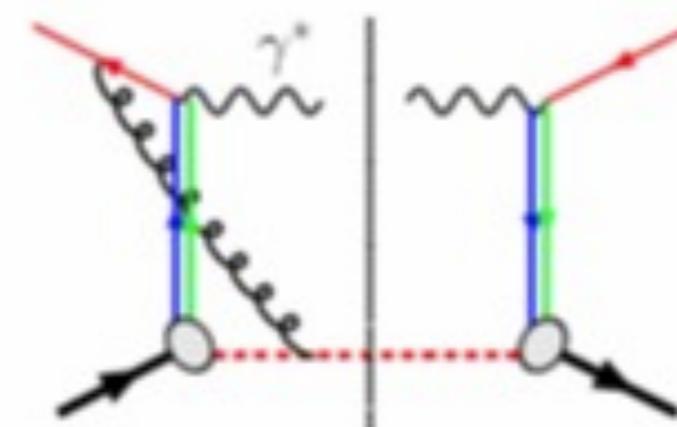
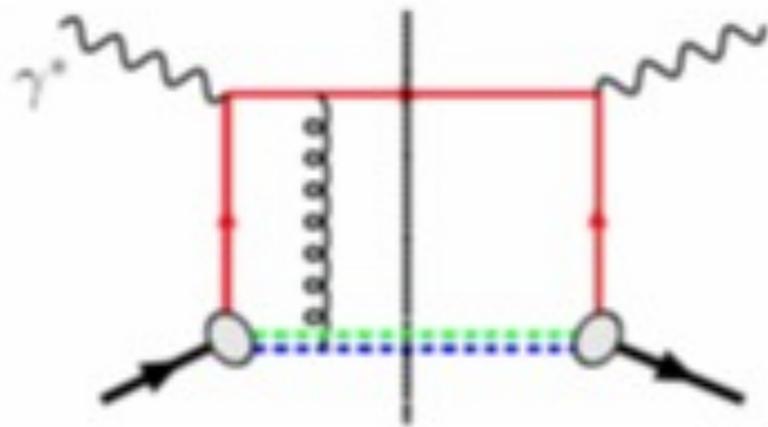
# Time-reversal modified universality of the Sivers function

- Relation between Sivers functions in SIDIS and DY

- From P and T invariance:

Collins 02, Boer-Mulders-Pijlman 03,  
Collins-Metz 04, Kang-Qiu, 09, ...

$$\Delta^N f_{q/h^\uparrow}^{\text{SIDIS}}(x, k_\perp) = \ominus \Delta^N f_{q/h^\uparrow}^{\text{DY}}(x, k_\perp)$$

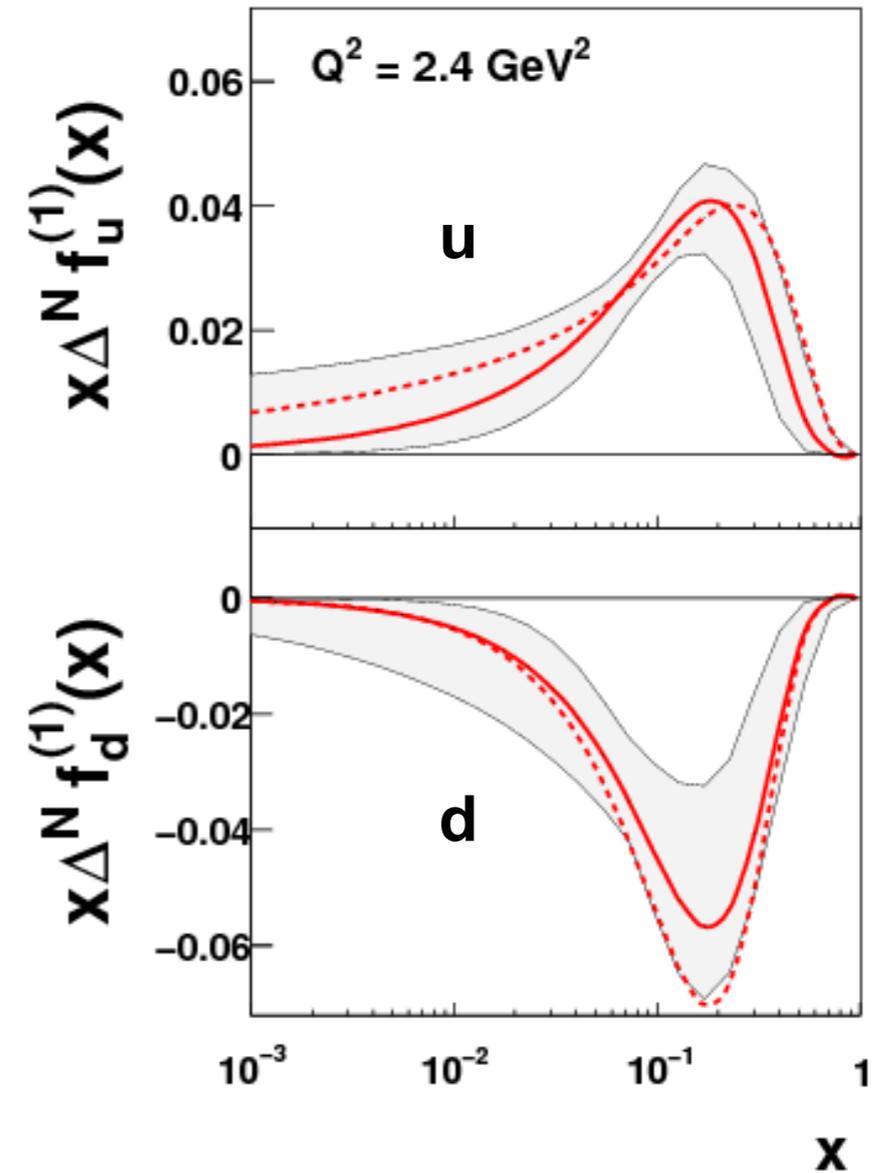
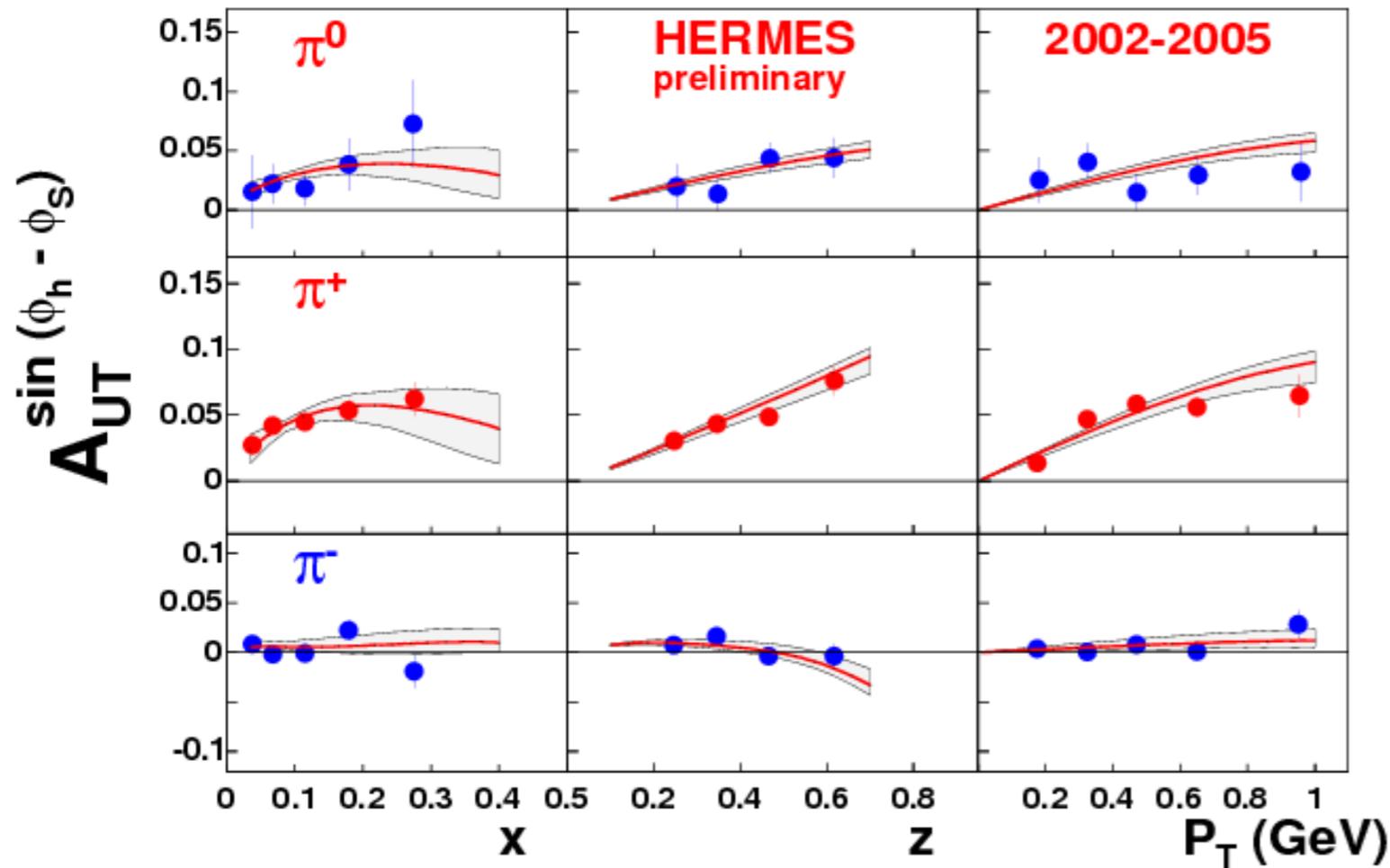


Most critical test for TMD factorization approach to SSA

# Sivers function from SIDIS $\ell + p^\uparrow \rightarrow \ell' + \pi(p_T) + X : p_T \ll Q$

- Extract Sivers function from SIDIS

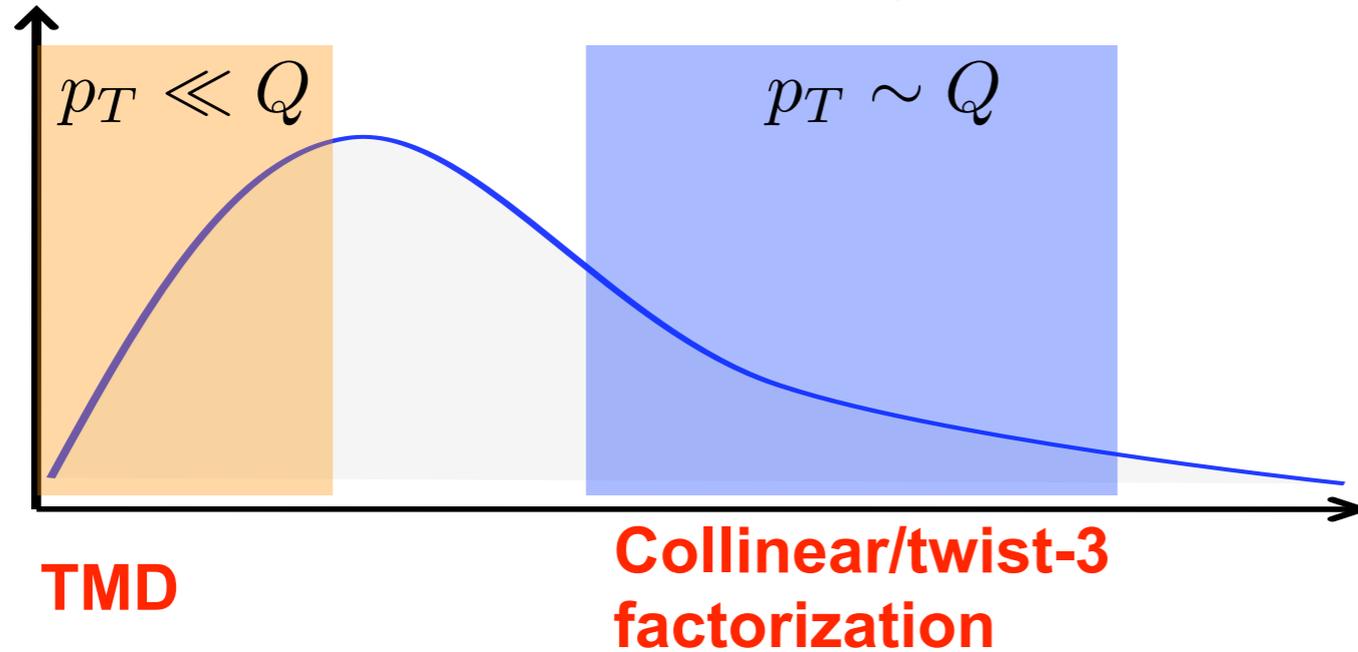
Anselmino, et.al., 2009



- u and d almost equal size, different sign
  - d-Sivers is slightly larger
- Still needs DY results to verify the sign change, thus fully understand the mechanism of the SSAs

# TMD factorization to collinear factorization

- Transition from low  $p_T$  to high  $p_T$



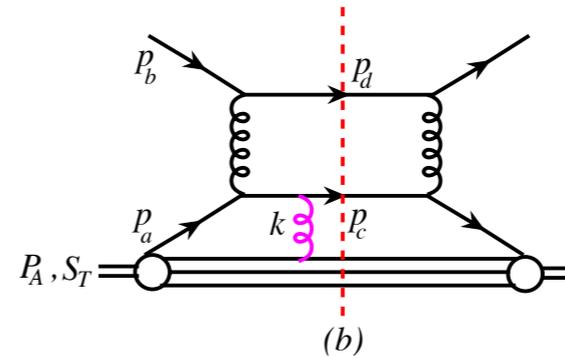
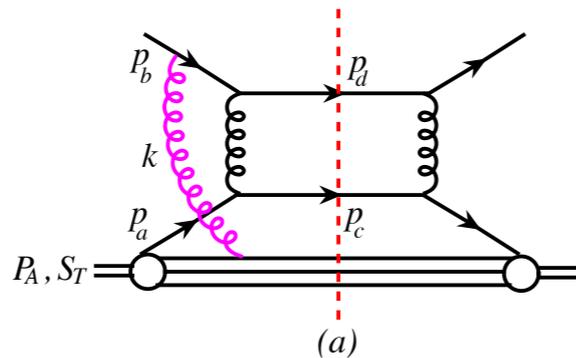
- Collinear twist-3 factorization approach: Efremov-Teryaev 82, 84, Qiu-Sterman 91, 98

$$\sigma(s_T) \sim \left[ \begin{array}{c} \text{Diagram (a)} \\ \text{Diagram (c)} \\ \dots \end{array} \right]^2 \rightarrow \Delta\sigma(s_T) \sim \text{Re}[(a)] \cdot \text{Im}[(c)]$$

The diagram shows two Feynman diagrams, (a) and (c), representing different factorization approaches. Diagram (a) shows a hard scattering process with a collinear parton from a proton (momentum  $p$ , spin  $s_p$ ) and a soft parton (momentum  $k$ ). Diagram (c) shows a similar process but with a twist-3 correction involving a gluon (momentum  $k_1$ ) and a quark (momentum  $k_2$ ). The diagrams are summed and squared to give the cross-section  $\sigma(s_T)$ . The difference between the two approaches is  $\Delta\sigma(s_T) \sim \text{Re}[(a)] \cdot \text{Im}[(c)]$ .

# Both initial- and final-state interactions

- For the process  $pp^\uparrow \rightarrow \pi + X$ , one of the partonic channel:  $qq' \rightarrow qq'$



$$E_h \frac{d\Delta\sigma}{d^3P_h} \propto \epsilon^{P_{hT} S_A n \bar{n}} \sum_{a,b,c} D_{h/c}(z_c) \otimes f_{b/B}(x_b) \otimes T_{a,F}(x, x) \otimes H_{ab \rightarrow c}^{\text{Siv}}$$

Efremov-Teryaev-Qiu-Sterman (ETQS) function

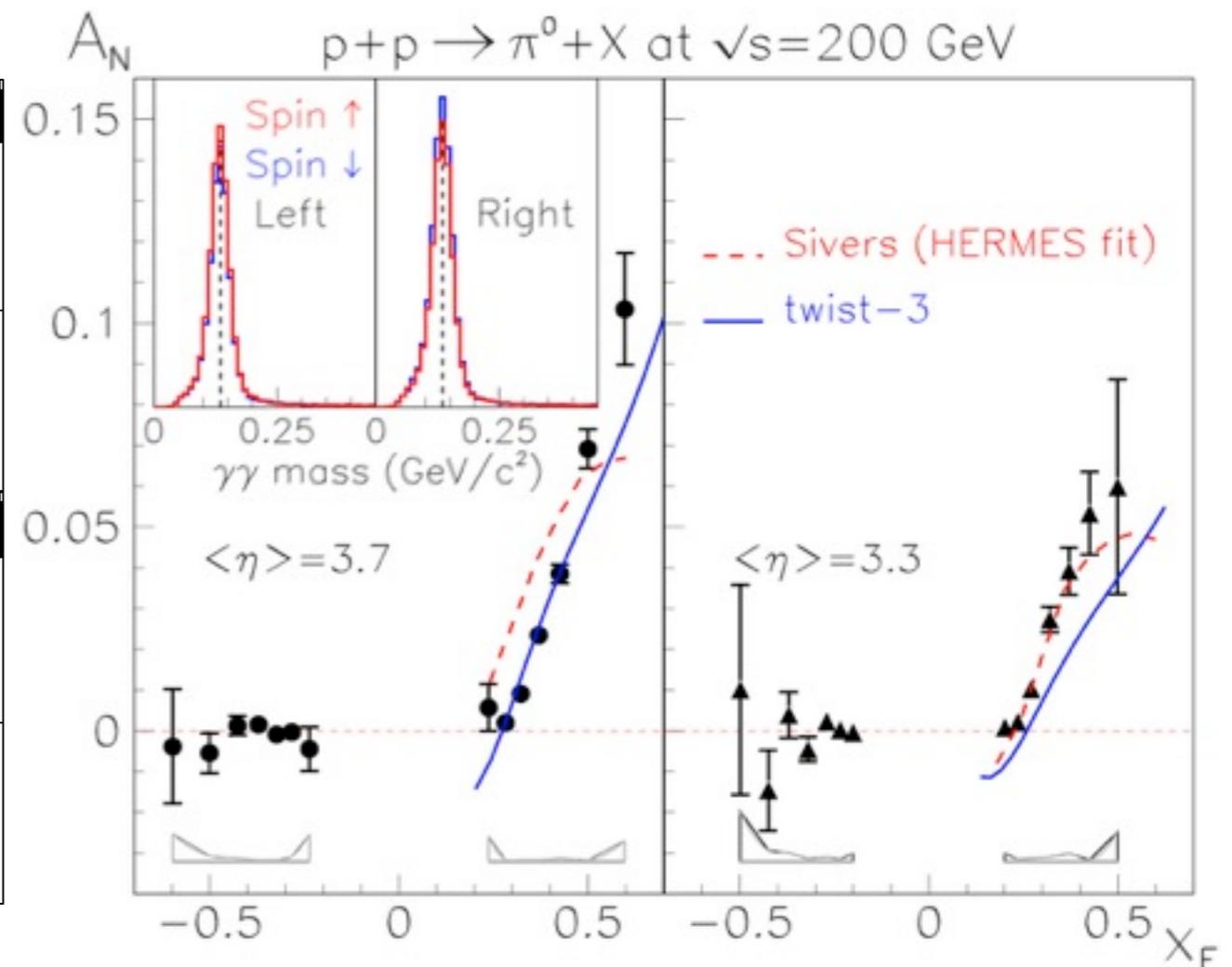
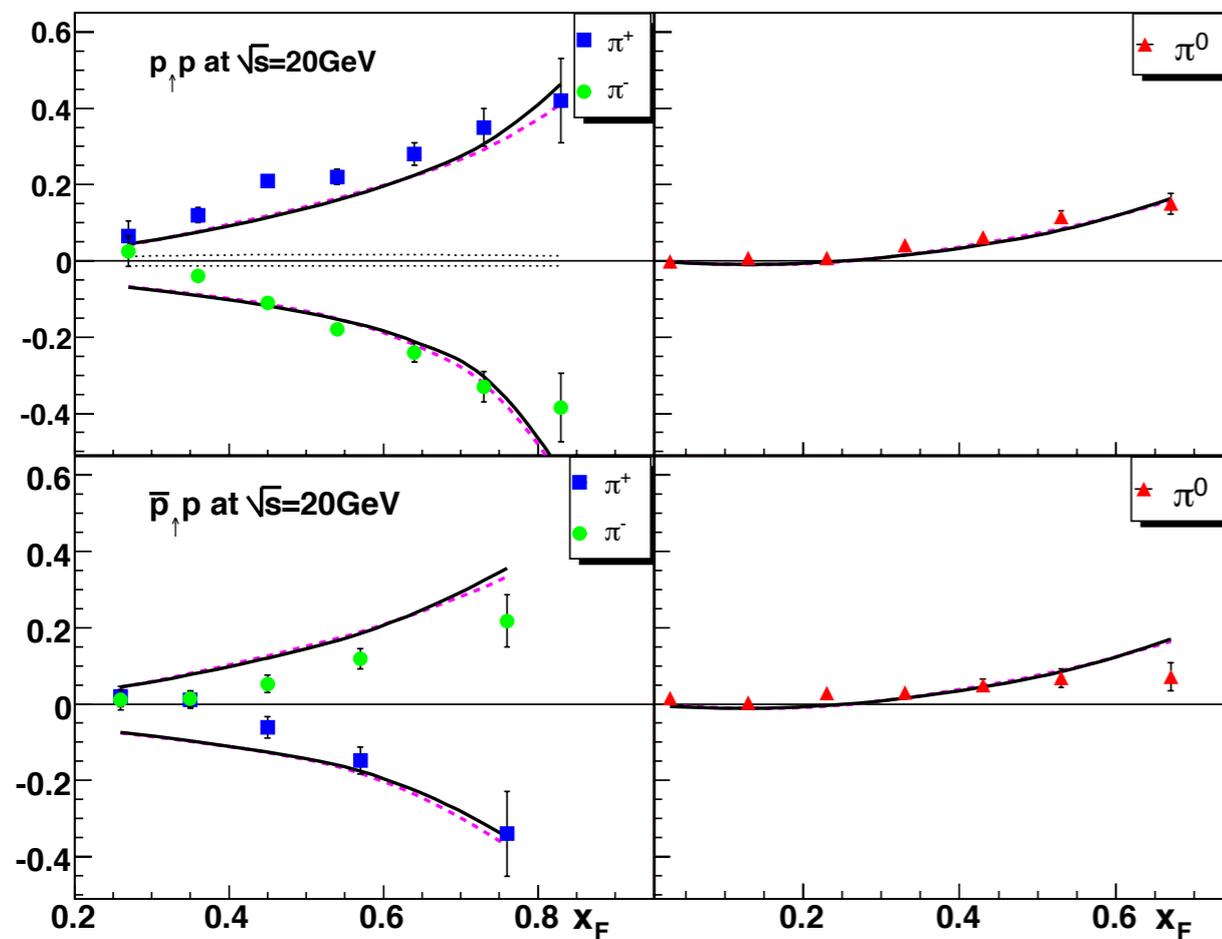
- The effects of initial- and final-state interaction are absorbed to  $H_{ab \rightarrow c}^{\text{Siv}}$
- ETQS function  $T_{q,F}(x, x)$  is universal
- Since TMD and collinear twist-3 approaches provide a unified picture for the SSAs, ETQS function and Sivers function are closely related to each other

# Initial success of twist-3 approach

- Describe both fixed-target and RHIC well: a fit

$$T_{q,F}(x, x) = N_q x^{\alpha_q} (1-x)^{\beta_q} \phi_q(x)$$

Kouvaris-Qiu-Vogelsang-Yuan, 2006



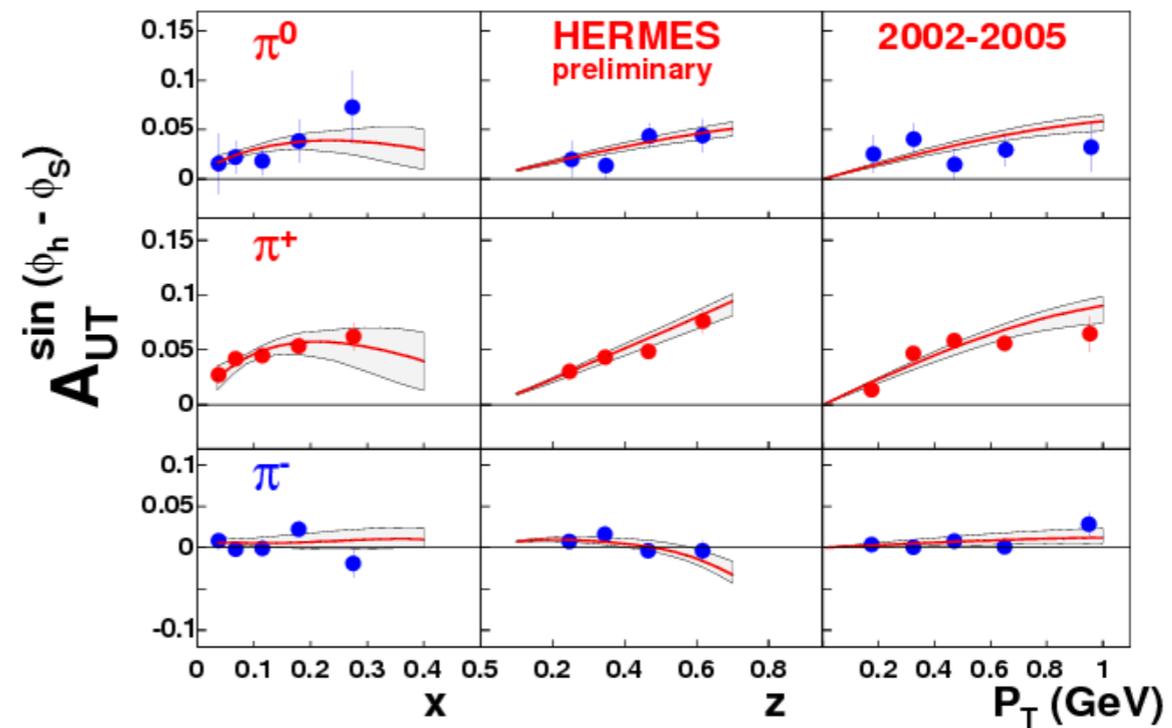
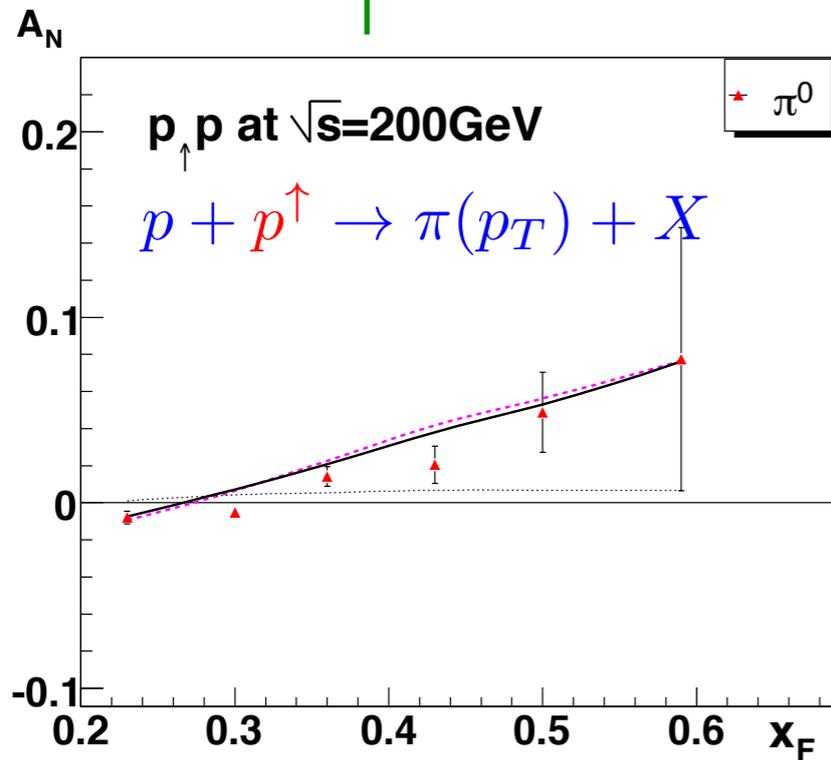
$$p^\uparrow p \rightarrow \pi + X$$

# What about the connections?

- Both seem to describe the data well (in their own kinematic region), but what about their connections?
  - At the operator level, ETQS function is related to the first kt-moment of the Sivers function

Boer, Mulders, Pijlman, 2003  
 Ji, Qiu, Vogelsang, Yuan, 2006

$$gT_{q,F}(x, x) = - \int d^2 k_{\perp} \frac{|k_{\perp}|^2}{M} f_{1T}^{\perp q}(x, k_{\perp}^2) |_{\text{SIDIS}}$$



# kt-dependence is a Gaussian in current parameterization

- To extract the Siverson function, the following parametrization is used

- unpolarized PDFs:  $f_1^q(x, k_\perp^2) = f_1^q(x)g(k_\perp)$

- Siverson function:  $\Delta^N f_{q/h^\uparrow}(x, k_\perp) = 2\mathcal{N}_q(x)f_1^q(x)h(k_\perp)g(k_\perp)$

$\mathcal{N}_q(x)$  is a fitted function

$$g(k_\perp) = \frac{1}{\pi\langle k_\perp^2 \rangle} e^{-k_\perp^2 / \langle k_\perp^2 \rangle}$$

old Siverson:  $h(k_\perp) = \frac{2k_\perp M_0}{k_\perp^2 + M_0^2}$  Anselmino, et.al, 2005

new Siverson:  $h(k_\perp) = \sqrt{2}e \frac{k_\perp}{M_1} e^{-k_\perp^2 / M_1^2}$  Anselmino, et.al, 2009

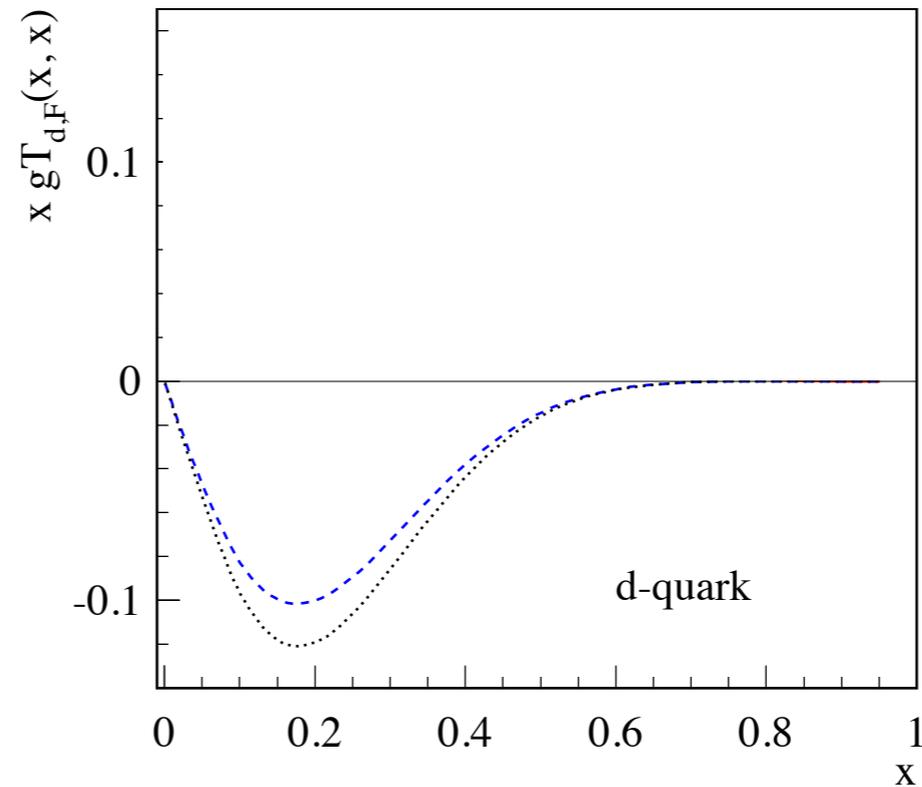
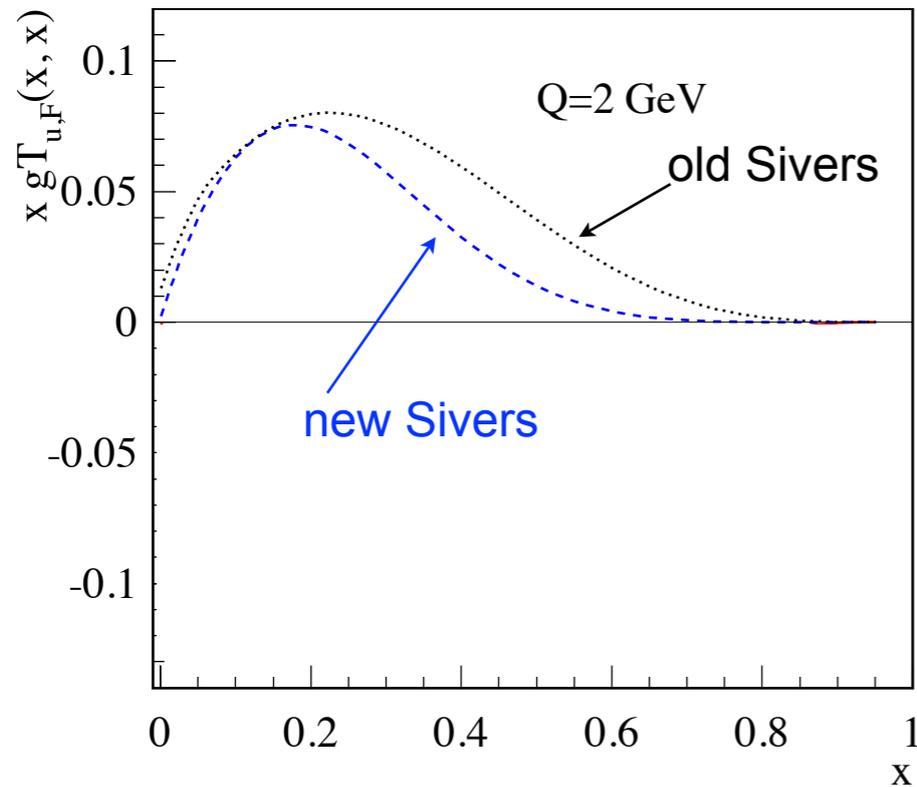
- Using  $\Delta^N f_{q/A^\uparrow}(x, k_\perp) = -\frac{2k_\perp}{M} f_{1T}^{\perp q}(x, k_\perp^2)$ , one can obtain

$$gT_{q,F}(x, x)|_{\text{old Siverson}} = 0.40 f_1^q(x)\mathcal{N}_q(x)|_{\text{old}}$$

$$gT_{q,F}(x, x)|_{\text{new Siverson}} = 0.33 f_1^q(x)\mathcal{N}_q(x)|_{\text{new}}$$

# Indirectly obtained ETQS function

- The plot of indirectly obtained ETQS function  $T_{q,F}(x, x)$

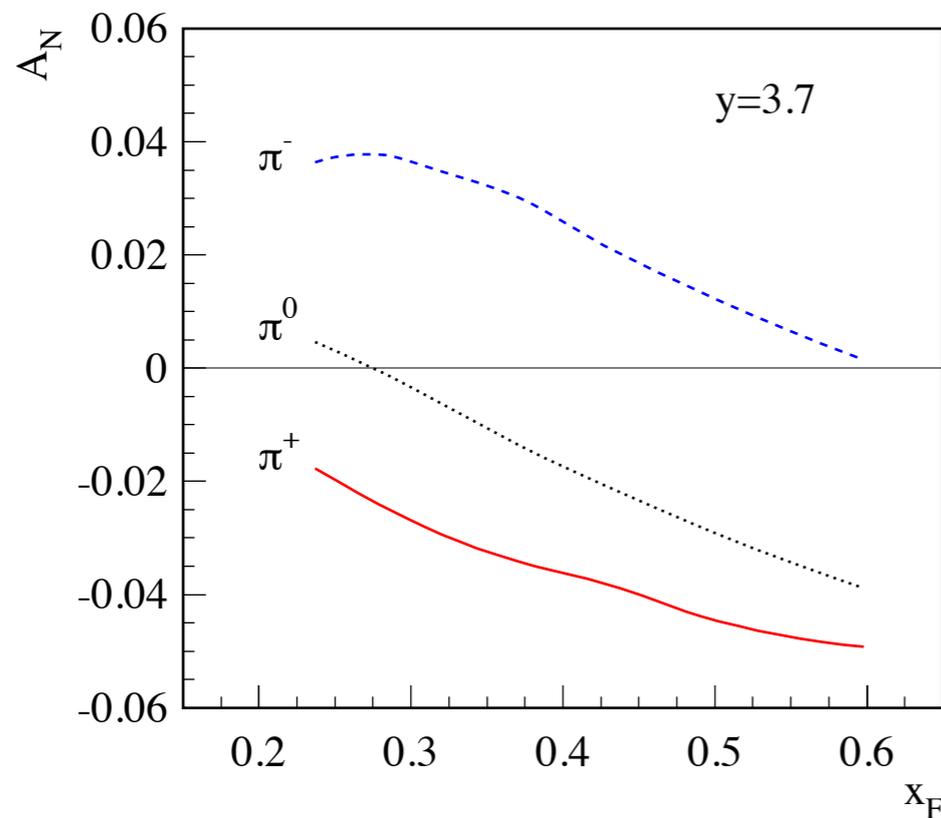


- ETQS function is positive for u-quark
- ETQS function is negative for d-quark

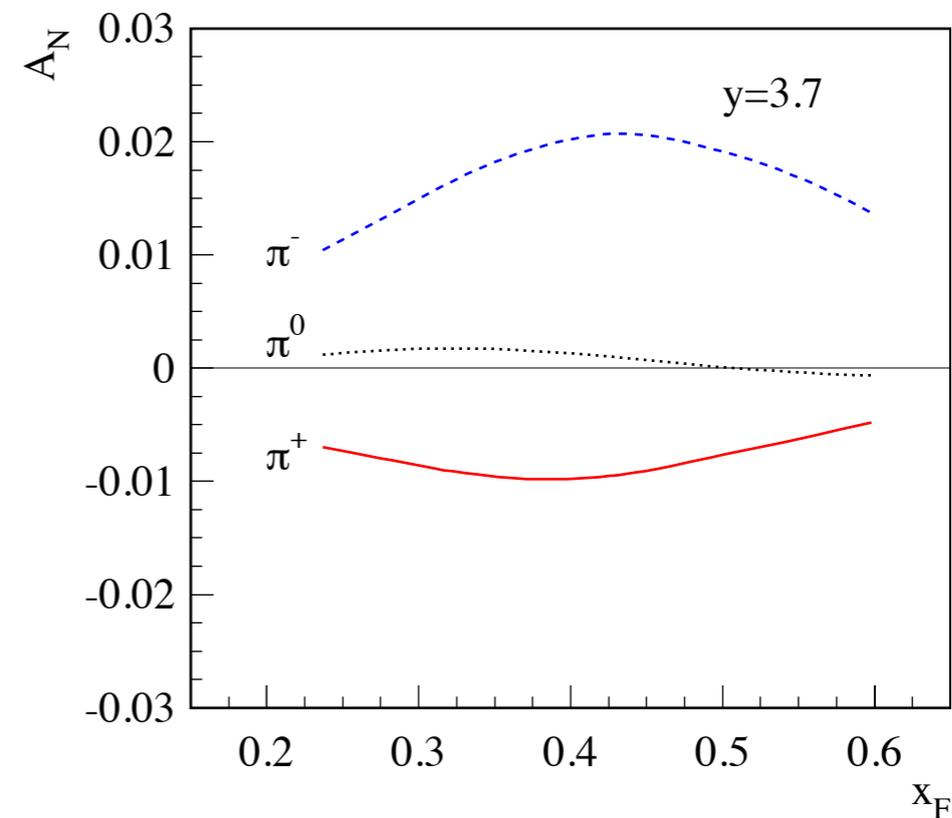
$$gT_{q,F}(x, x) = - \int d^2 k_{\perp} \frac{|k_{\perp}|^2}{M} f_{1T}^{\perp q}(x, k_{\perp}^2) |_{\text{SIDIS}}$$

# Apparent sign mismatch

- Use the ETQS function derived from the old Sivers and new Sivers functions, one could make predictions for the single inclusive hadron production. We find they are opposite to the experimental observations.



old Sivers

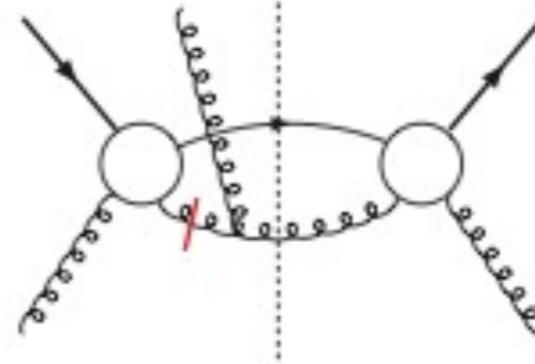
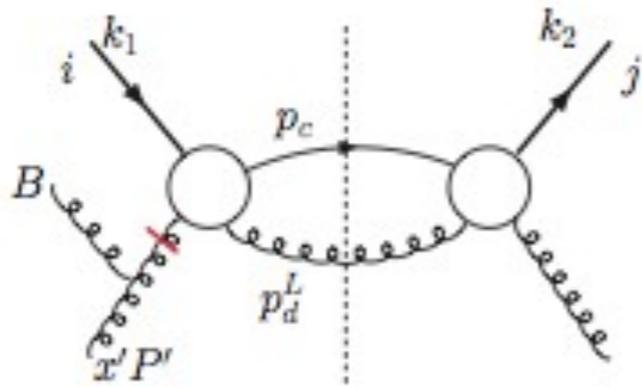


new Sivers



# Initial- and final-state interaction in pp collisions

- The dominant channel is  $qg \rightarrow qg$



$$H_{qg \rightarrow qg}^U = \frac{N_c^2 - 1}{2N_c^2} \begin{bmatrix} \hat{s} & \hat{u} \\ -\hat{u} & \hat{s} \end{bmatrix} \left[ 1 - \frac{2N_c^2}{N_c^2 - 1} \frac{\hat{s}\hat{u}}{\hat{t}^2} \right] \xrightarrow{|\hat{t}| \ll \hat{s} \sim |\hat{u}|} \begin{bmatrix} 2\hat{s}^2 \\ \hat{t}^2 \end{bmatrix}$$

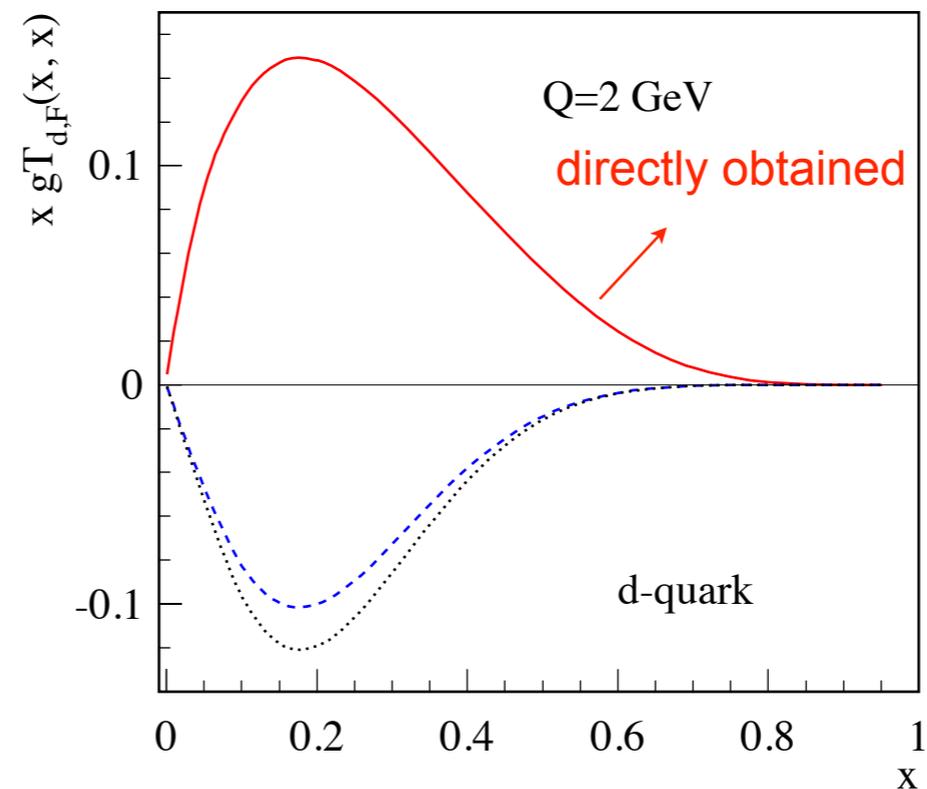
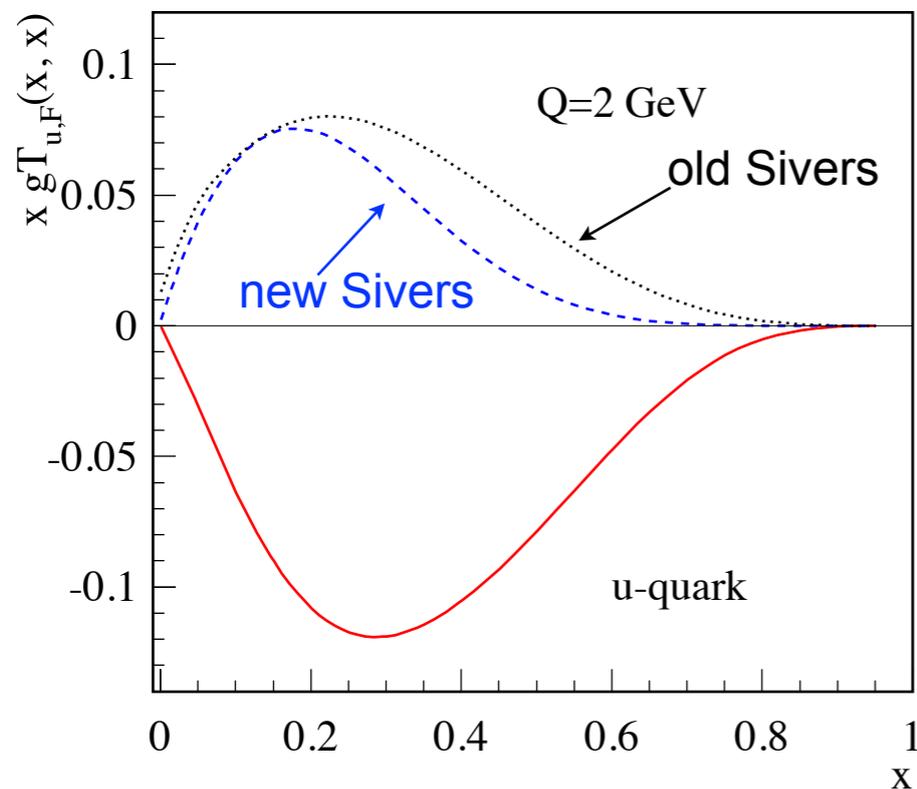
$$H_{qg \rightarrow qg}^I = \frac{1}{2(N_c^2 - 1)} \begin{bmatrix} \hat{s} & \hat{u} \\ -\hat{u} & \hat{s} \end{bmatrix} \left[ 1 - N_c^2 \frac{\hat{u}^2}{\hat{t}^2} \right] \xrightarrow{|\hat{t}| \ll \hat{s} \sim |\hat{u}|} \begin{bmatrix} N_c^2 \\ -2(N_c^2 - 1) \end{bmatrix} \begin{bmatrix} 2\hat{s}^2 \\ \hat{t}^2 \end{bmatrix}$$

$$H_{qg \rightarrow qg}^F = \frac{1}{2N_c^2(N_c^2 - 1)} \begin{bmatrix} \hat{s} & \hat{u} \\ -\hat{u} & \hat{s} \end{bmatrix} \left[ 1 + 2N_c^2 \frac{\hat{s}\hat{u}}{\hat{t}^2} \right] \xrightarrow{|\hat{t}| \ll \hat{s} \sim |\hat{u}|} \begin{bmatrix} 1 \\ -N_c^2 - 1 \end{bmatrix} \begin{bmatrix} 2\hat{s}^2 \\ \hat{t}^2 \end{bmatrix}$$

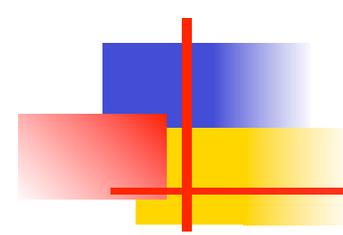
- Sivers effect in single hadron production is more similar to DY

## Directly obtained ETQS function

- ETQS function could be directly obtained from the global fitting of inclusive hadron production in hadronic collisions



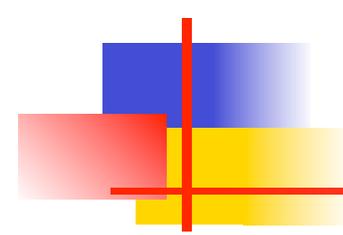
- directly obtained ETQS functions for both u and d quarks are opposite in sign to those indirectly obtained from the kt-moment of the quark Siverts function - "a sign mismatch"



## Question

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Does this apparent sign “mismatch” indicate an inconsistency in our current QCD formalism for describing the SSAs?



## Question

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Does this apparent sign “mismatch” indicate an inconsistency in our current QCD formalism for describing the SSAs?

The answer is possibly yes, but not necessarily.

## Scenario I

- Let us assume the directly obtained ETQS function from inclusive hadron production reflects the true sign of these functions.
- In such case, to make everything consistent, we need to explain how the sign of the kt-moment of the Sivers function is different from the sign of the Sivers function.

$$gT_{q,F}(x, x) = - \int d^2 k_{\perp} \frac{|k_{\perp}|^2}{M} f_{1T}^{\perp q}(x, k_{\perp}^2)|_{\text{SIDIS}}$$

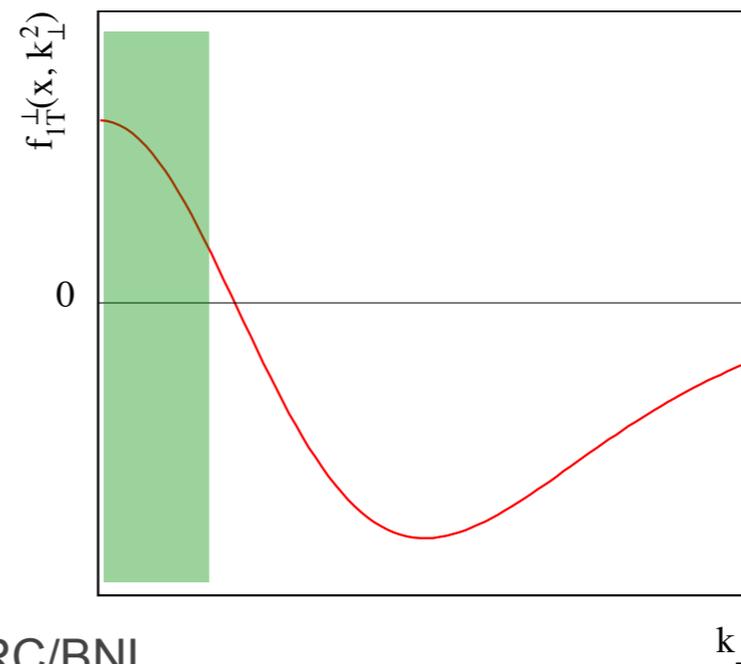
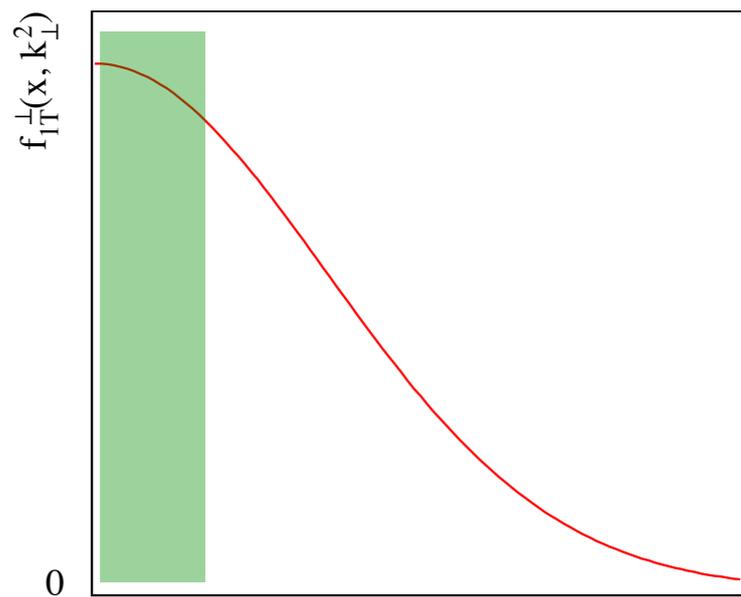
# What could go wrong - Scenario I

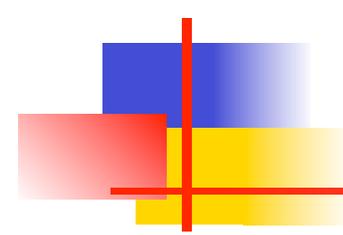
- To obtain ETQS function, one needs the full  $kt$ -dependence of the quark Sivers function

$$gT_{q,F}(x, x) = - \int d^2 k_{\perp} \frac{|k_{\perp}|^2}{M} f_{1T}^{\perp q}(x, k_{\perp}^2) |_{\text{SIDIS}}$$

- However, the Sivers functions are extracted mainly from HERMES data at rather low  $Q^2 \sim 2.4 \text{ GeV}^2$ , and TMD formalism is only valid for the kinematic region  $kt \ll Q$ .
  - HERMES data only constrain the behavior (or the sign) of the Sivers function at very low  $kt \sim \Lambda_{\text{QCD}}$ .

$$\Delta^N f_{q/h\uparrow}(x, k_{\perp}) \vec{S} \cdot \hat{p} \times \hat{\mathbf{k}}_{\perp} = f_{q/h\uparrow}(x, \mathbf{k}_{\perp}, \vec{S}) - f_{q/h\uparrow}(x, \mathbf{k}_{\perp}, -\vec{S})$$





## Measure $kt$ -dependence of Sivers function

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- To test whether we have a sign change in the  $kt$ -distribution (or have a node), we need to expand the reach of  $kt$  in the SIDIS
  - With a much broader  $Q$  and energy coverage
  - a Electron Ion Collider might be ideal
- A new global fitting including both SIDIS and pp data is underway:
  - Explore the possibility of a node in  $kt$  space or  $x$  space

Kang, Prokudin, in preparation

## Scenario II

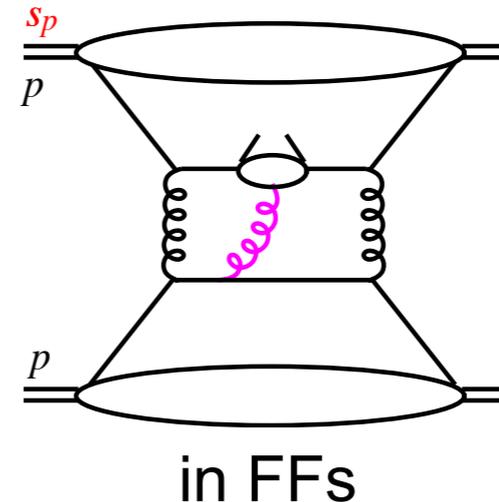
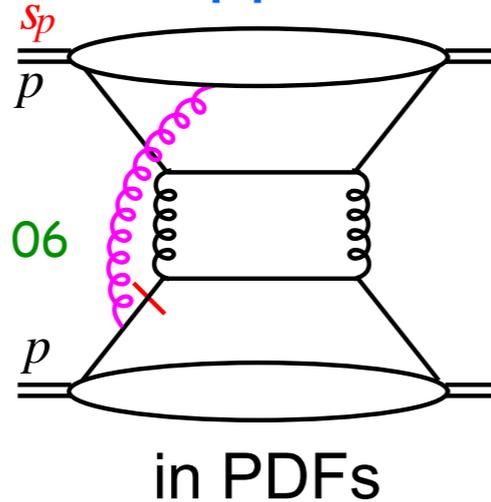
- Let assume the indirectly obtained (from the kt-moment of the Sivers function) ETQS function reflects the true sign of these functions
- In such case, to make everything consistent, we need to explain why we obtain a sign-mismatched ETQS function by analyzing the inclusive hadron data

$$gT_{q,F}(x, x) = - \int d^2 k_{\perp} \frac{|k_{\perp}|^2}{M} f_{1T}^{\perp q}(x, k_{\perp}^2) |_{\text{SIDIS}}$$

# Single inclusive hadron production is complicated

- There are two major contributions to the SSAs of the single inclusive hadron production in pp collisions

Efremov-Teryaev 82, 84,  
 Qiu-Sterman 91, 98,  
 Kouvaris-Qiu-Vogelsang-Yuan, 06  
 Kanazawa-Koike, 11



Kang-Yuan-Zhou 2010

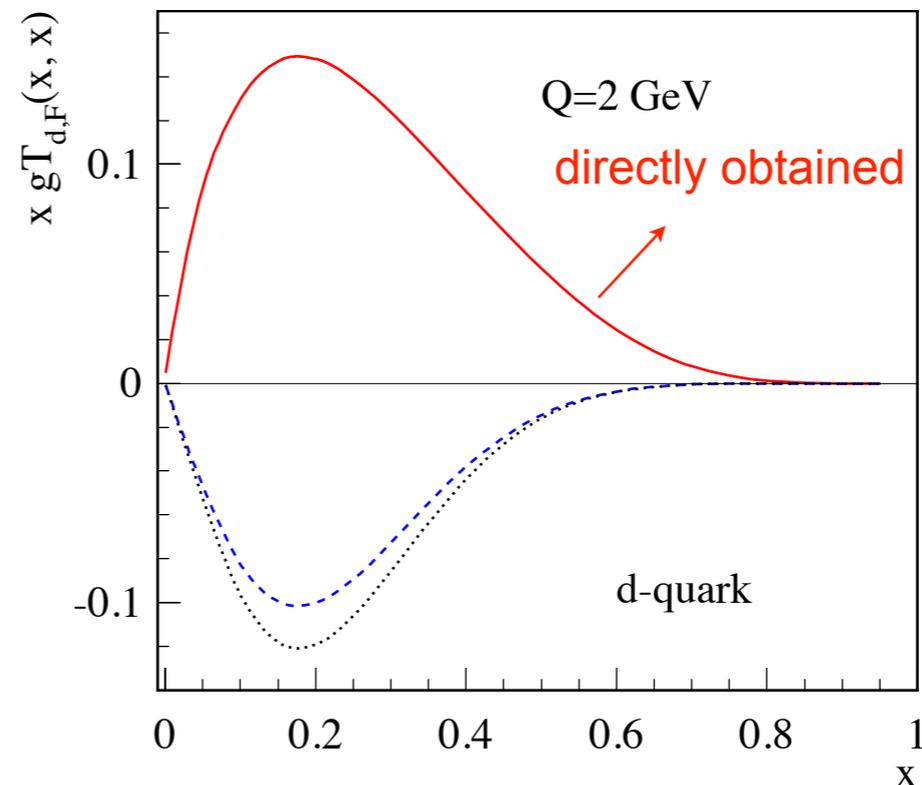
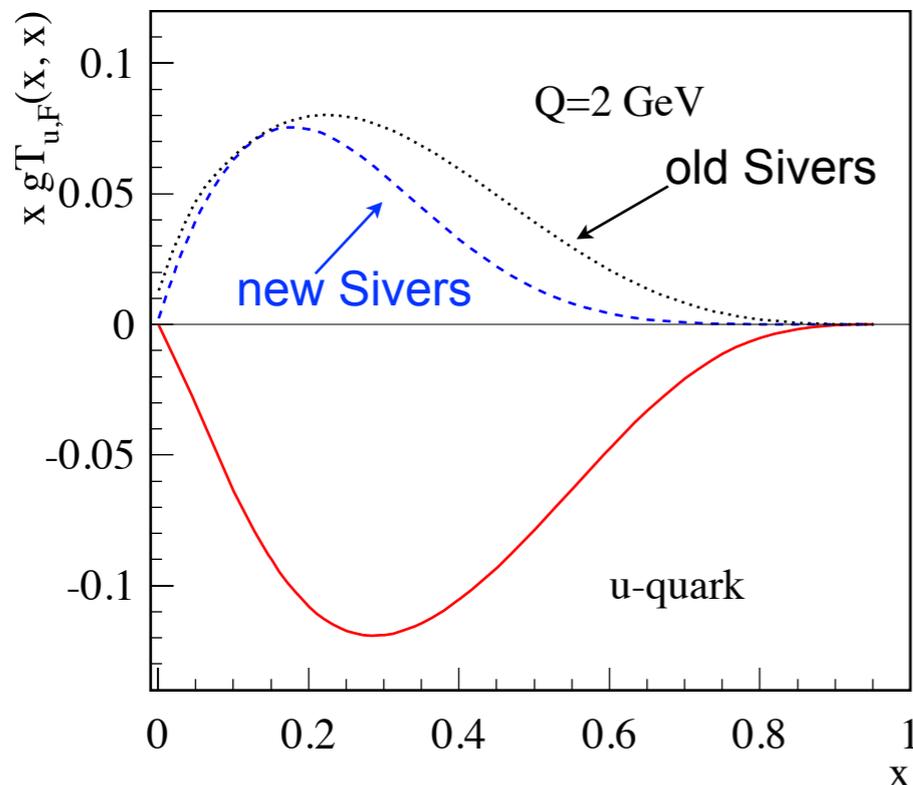
- So far the calculations related to three-parton correlation functions are more complete, while those related to the twist-3 fragmentation functions are available only very recently (not complete)
  - The current available global fittings are based on the assumptions that the SSAs mainly come from the twist-3 correlation functions, which might not be the case
  - If the contribution from the twist-3 fragmentation functions dominates, one might even reverse the sign of the ETQS function?

$$A_N = A_N|^{PDFs} + A_N|^{FFs}$$

If  $A_N|^{FFs} > A_N$ , sign of  $A_N|^{PDFs}$  is opposite to  $A_N$

# Distinguish scenario I and II

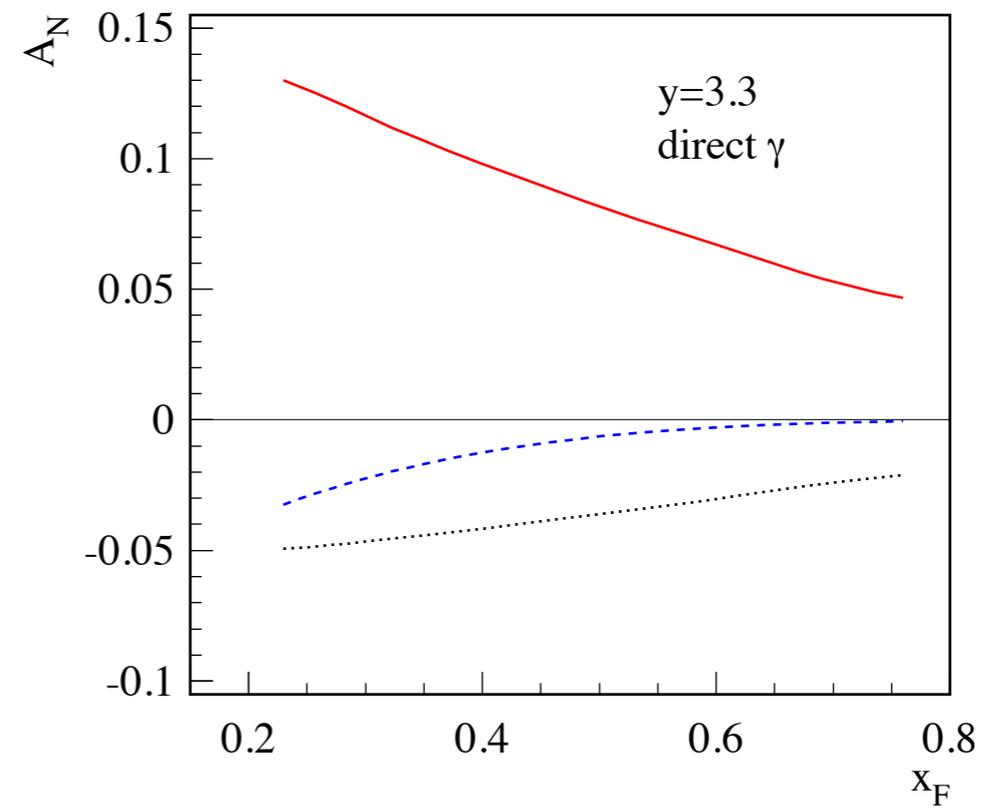
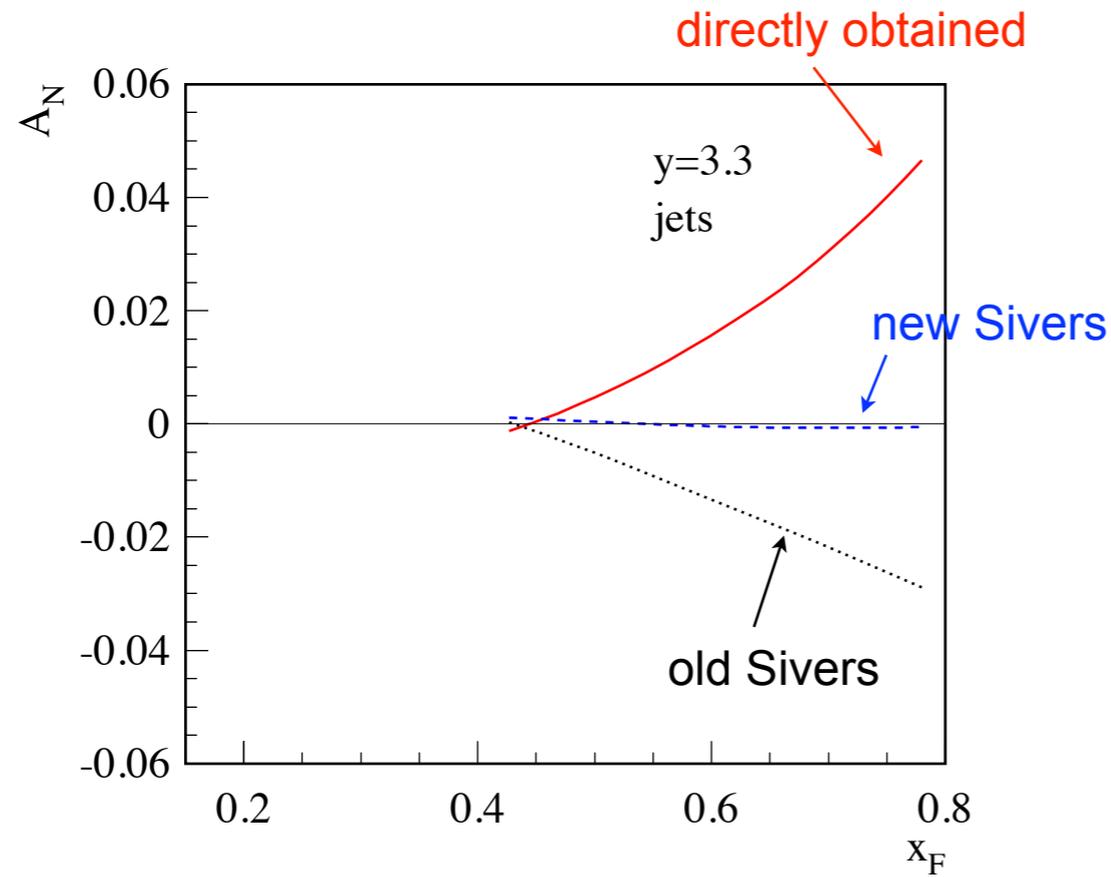
- Scenario I and II are completely different from each other

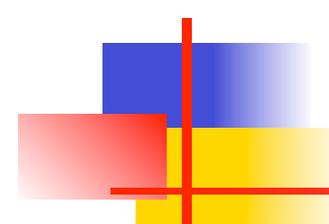


- To distinguish one from the other, in hadronic machine (like RHIC), one needs to find observables which are sensitive to twist-3 correlation function (not fragmentation function), such as single inclusive jet production, direct photon production

# Predictions for jet and direct photon

- at RHIC 200 GeV:

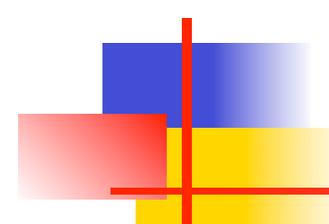




# Summary

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- The existence of Sivers function relies on the initial and final-state interactions
- Sivers effect is process dependent
  - Test process-dependence is very important to understand the SSAs: sign change between SIDIS and DY
  - Both TMD and collinear twist-3 approaches seem to be successful phenomenologically
- Their connection seems to have a puzzle
  - Directly obtained ETQS functions are opposite in sign to those indirectly obtained from the kt-moment of the quark Sivers function
  - This sign mismatch does not necessarily lead to any inconsistency in our current formalism for describing the SSAs
  - Future experiments could help resolve different scenarios, which will help understand the SSAs and hadron structure better

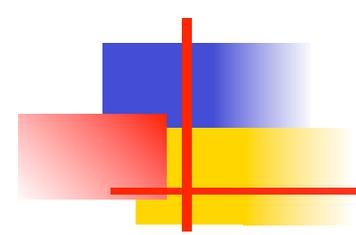


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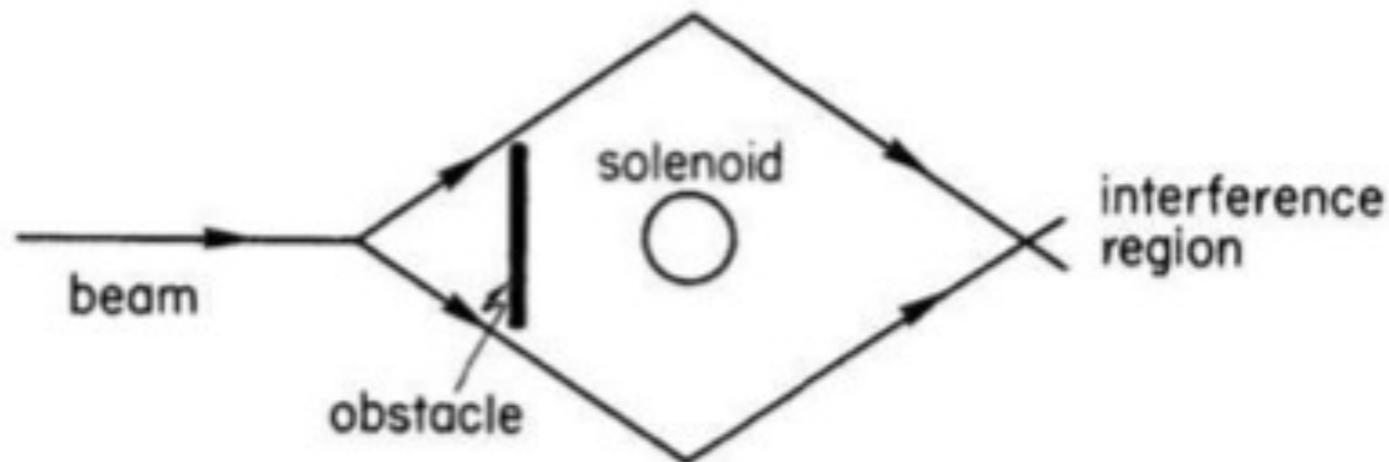
Thank you



# Backup

# QED: Aharonov-Bohm effect → non-abelian version

- In classical electrodynamics, gauge potential  $A^\mu = (V, \vec{A})$  is no more than an auxiliary mathematical quantity for defining  $E$  and  $B$  field, thus has no independent physical significance
- However, this is decidedly not the case in quantum theory, as the analysis of Aharonov and Bohm has first made clear
- In the following experiment, there is magnetic-**B**-field confined inside the solenoid. Outside it is magnetic-field-free region, but gauge potential  $A$  exists, which eventually leads to a phase for different paths and interference pattern when beams recombine



C. Quigg, Gauge theory of The Strong, Weak and Electromagnetic Interactions

$$\Psi = \Psi_1^0 e^{iS_1/\hbar} + \Psi_2^0 e^{iS_2/\hbar}$$

$$S_i = e \int_{\text{path } i} d\vec{x} \cdot \vec{A}$$

## Process-dependence: TMD vs collinear twist-3

- TMD approach: the process-dependence of the SSAs is completely absorbed into the process-dependence of the Sivers function

- Sivers function is process-dependent

$$\sigma \sim H^U \otimes f(x, k_\perp)$$

$$\Delta\sigma \sim \Delta H \otimes f_{1T}^\perp(x, k_\perp)$$

$$\Delta H = H^U$$

- Collinear twist-3 approach: the process-dependence of the SSAs is completely absorbed into the hard-part functions, thus the relevant collinear twist-3 correlation functions are universal

- twist-3 correlation function is universal

$$\sigma \sim H^U \otimes f(x)$$

$$\Delta\sigma \sim \Delta H \otimes T_F(x, x)$$

$$\Delta H = H^I + H^F$$

# Difference of distributions has a node is not new

- Current best fit for gluon helicity distribution function  $\Delta g(x)$  seems to favor a  $x$ -distribution with a node

