Dihadron fragmentation functions for large invariant mass

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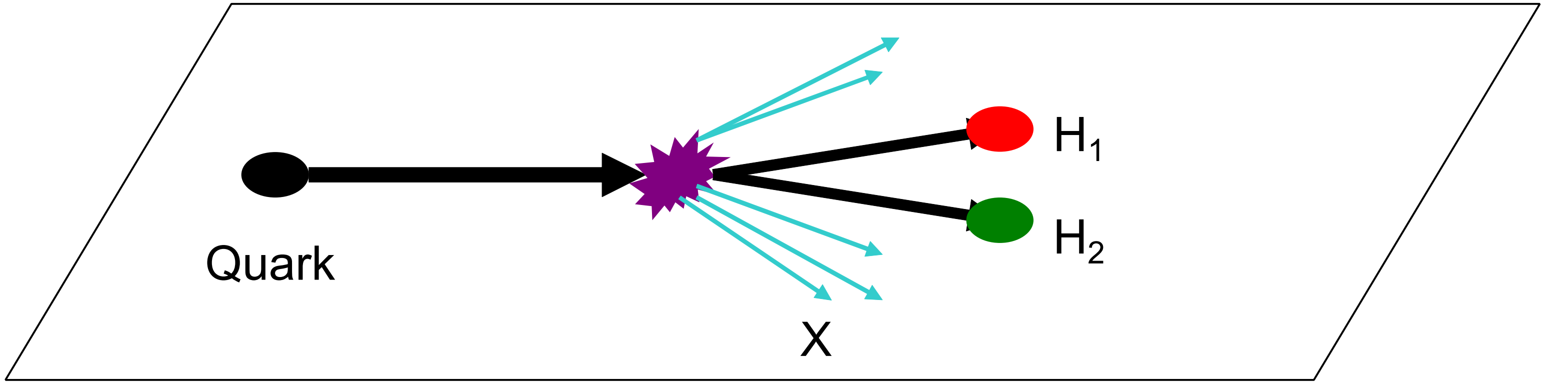
Outline:

- 1: Introduction
- 2: Interference fragmentation function for large invariant mass
- 3: Dihadron production in SIDIS
- 4: Summary

Dihadron fragmentation function

In late 1970's, dihadron fragmenation functions were already introduced.

Konishi, Ukawa and Veneziano 1978



It is also shown that DFFs are needed for absorbing collinear divergence for production of two hadron in e^-e^+ annihilation at NLO.

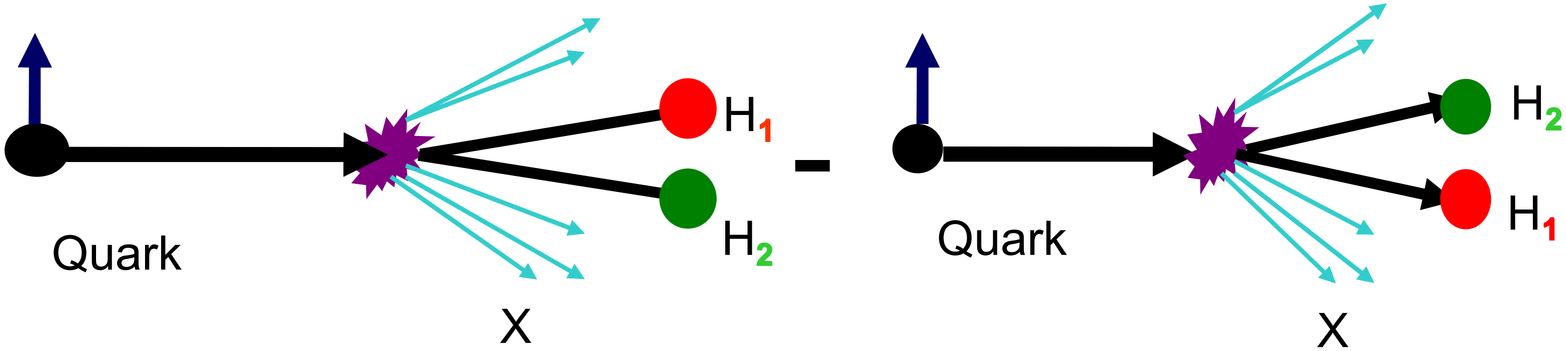
Florian and Vanni 2004

In the meantime, DiFFs also play a considerable role in heavy ion physics.

Majumder, E. Wang and X. N. Wang 2007

Interference Fragmentation Function(IFF) $H_1^{\sphericalangle}$

It describes the strength of the correlation $\mathbf{S}_T \times (\mathbf{P}_{1hT}-\mathbf{P}_{2hT})$.



It was proposed that $H_1^{\sphericalangle}$ can be used to address transversity h_1 of the nucleon.

Collins, Heppelmann and Landinsky 1994

Azimuthal orientation of hadron pair → spin analyzer of fragmenting quark

Advantage: (as compared to Collins mechanism)
One can integrate over the total transverse momentum of the two hadrons in the final state, leading to a collinear factorization formula.

State of the art on IFF $H_1^{\sphericalangle}$

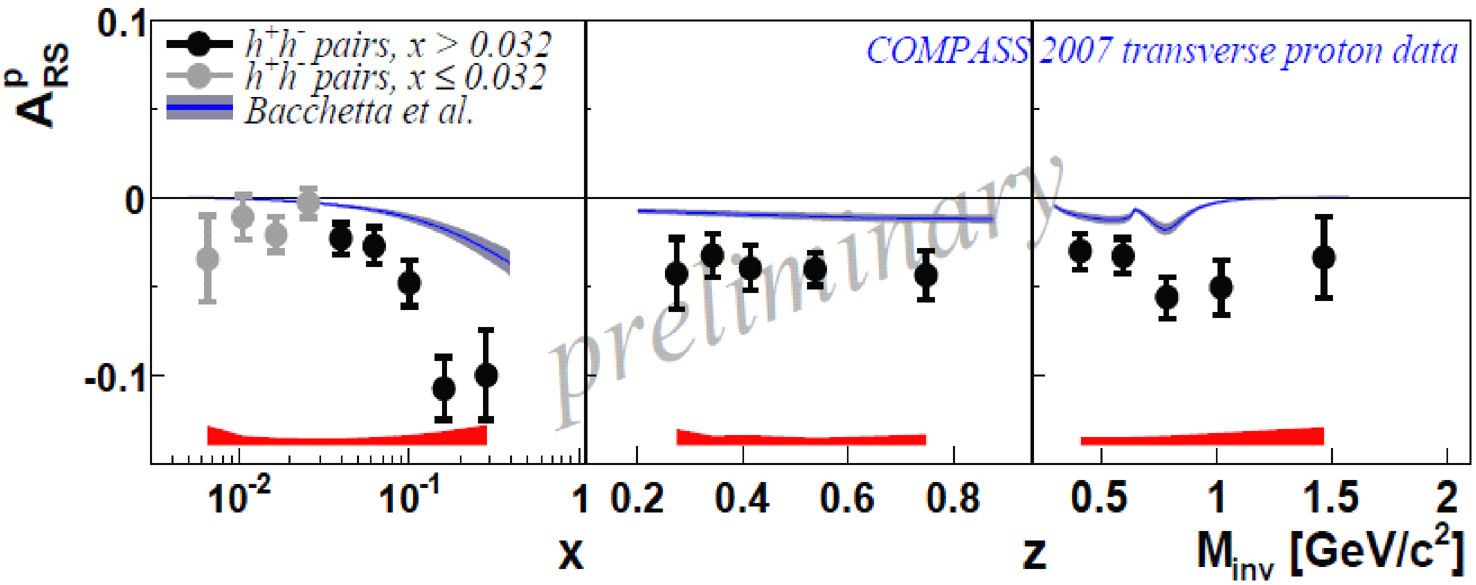
Model calculation: **Jaffe, Jing and Tang 1998**
Radici, Jakob and Bianconi 2002
Bacchetta and Radici 2006

Experimental status: $\sin(\phi_R + \phi_S)$ in SIDIS from HERMES and COMPASS

$$h_1^q(x_B) H_1^{\sphericalangle q}(z_{hh}, M_{hh}^2)$$

$\cos(\phi_R + \bar{\phi}_R)$ in e^-e^+ from Belle

$$H_1^{\sphericalangle q}(z_{hh}, M_{hh}^2)\; H_1^{\sphericalangle q}(z_{hh}, M_{hh}^2)$$

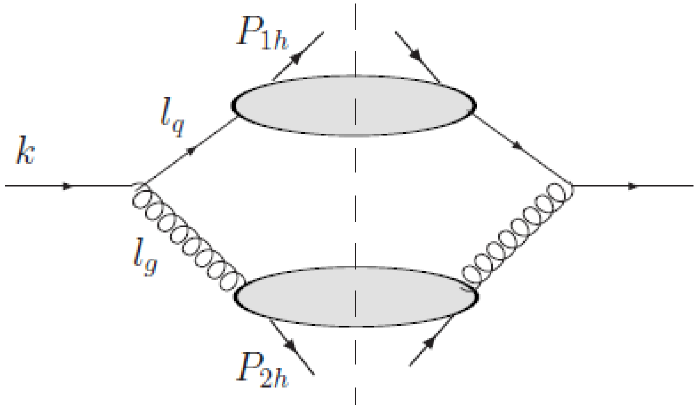


DFFs for large invariant mass

Matrix element definition(in the frame $P_{\mathbf{1hT}}$=0)

$$\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{d\hat{z}_{hh}}{32(2\pi)^3} \frac{z_{hh}^2}{z_{1h}^2} \sum_X \int \frac{dy^+}{2\pi} e^{ik^- y^+} \langle 0|\psi^q(y^+)|P_{1h}, P_{2h}, X\rangle\langle P_{1h}, P_{2h}, X|\bar{\psi}^q(0)|0\rangle$$

$$= \frac{\gamma^+}{2} D_1^q(z_{hh}, M_{hh}^2) + \frac{\sigma^{\alpha+} R_{T\alpha}}{2|\vec{R}_T|} H_1^{\sphericalangle q}(z_{hh}, M_{hh}^2)$$



When $M_{hh} >> \Lambda_{QCD}$,DFFs can be calcluated in collinear factorization and expressed as convolution of two single hadron FFs

$$D_1^q(z_{hh}, M_{hh}^2) = \frac{\alpha_s}{2\pi\, M_{hh}^2} \int_{-\frac{1}{2}}^{\frac{1}{2}} d\hat{z}_{hh} \int_{z_{1h}}^{1-z_{2h}} \frac{d\xi}{\xi(1-\xi)}\, C_F \frac{1+\xi^2}{1-\xi} D_1^{h_1/q}\left(\frac{z_{1h}}{\xi}\right) D_1^{h_2/g}\left(\frac{z_{2h}}{1-\xi}\right)$$

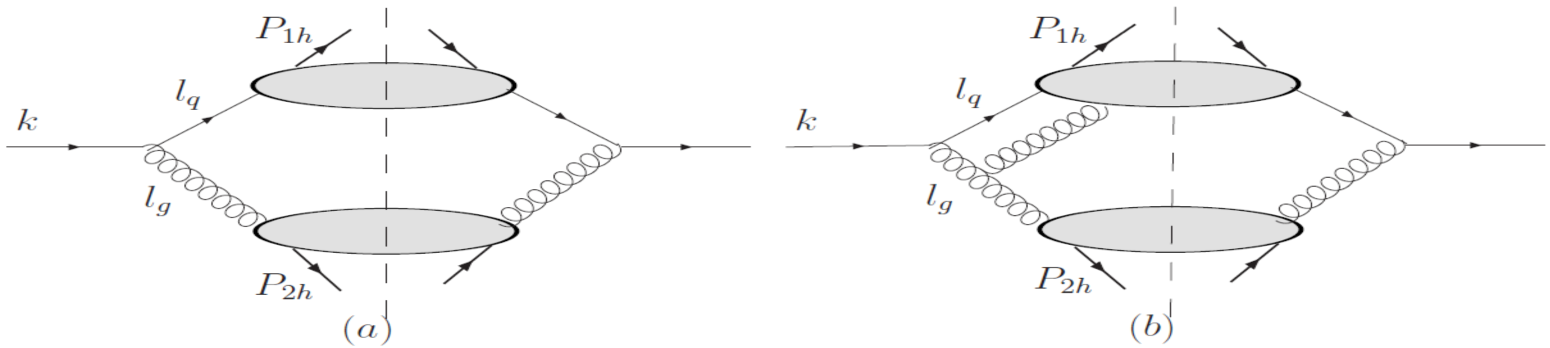
Konishi, Ukawa and Veneziano 1978
Florian and Vanni 2004

Final state interaction & T-odd distributions

If one switches off the QCD interaction, the T-odd distributions, such as , Sivers function, Collins function and IFF will entirely vanish

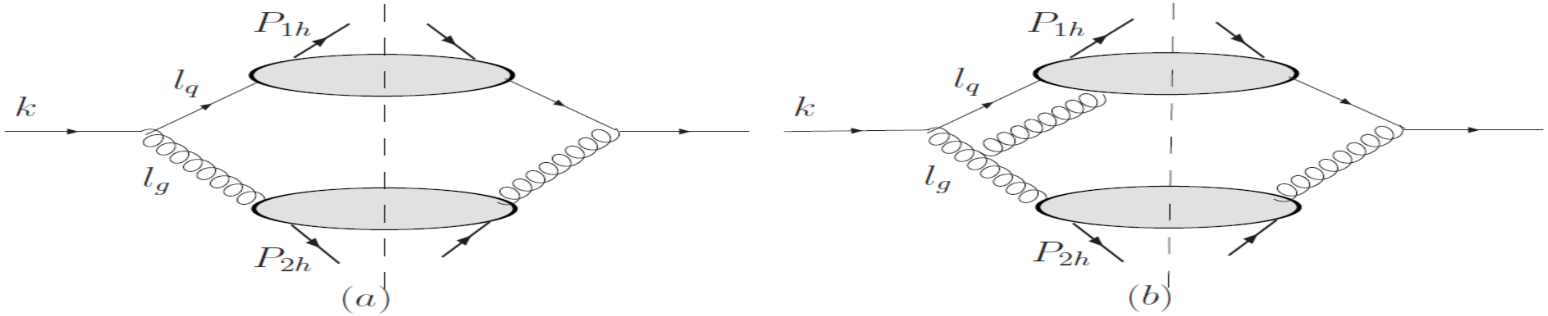
For having nonvanishing IFF, final state interaction between the active parton and the remanents is crucial

In the case of IFF,



When $M_{hh} >> \Lambda_{QCD}$, final state interaction is perturbatively calculable.

IFF for large invarint mass



In collinear twist-3 factorization: $\langle \bar{\psi} \partial_\perp \psi \rangle \; \langle \bar{\psi} A_\perp \psi \rangle$

A_T part is essentially identical to the corresponding part in the treatment of the Collins function at large transverse momentum.

$$l_q = \frac{P'_{1h}}{z'_1} + l_{q\perp}$$

$$z'_1 = z_1 + \delta z_1$$

$$P'_{1h} = P_{1h} + \delta P_{1h}$$

$$P'_{2h} = P_{2h} + \delta P_{2h}$$

$$\delta P_{2h\perp} = -z_2 l_{q\perp}$$

$$P_{1h} \cdot \delta P_{1h} = 0 \,, \quad P_{2h} \cdot \delta P_{2h} = 0 \,,$$

$$P_{1h} \cdot \delta P_{2h} + P_{2h} \cdot \delta P_{1h} = 0 \,,$$

$$\delta P_{1h}^- + \delta P_{2h}^- = 0 \,, \quad \delta P_{1h\perp} = 0 \,,$$

IFF for large invariant mass

Taylor expansion:
$$M(P'_{1h}, P'_{2h}, z'_1) = M(P_{1h}, P_{2h}, z_1) + \left.\frac{\partial M(P'_{1h}, P'_{2h}, z'_1)}{\partial l_{q\perp}}\right|_{l_{q\perp}=0} l_{q\perp} + \ldots$$

One obtains:

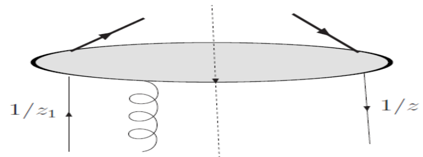
$$H_1^{\sphericalangle q}(z_{hh}, M_{hh}^2) = \frac{\alpha_s}{2\pi M_{hh}^3} \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{d\hat{z}_{hh}}{\sqrt{z_{1h} z_{2h}}} \int_{z_{1h}}^{1-z_{2h}} \frac{d\xi}{\xi} \; A^{h_1/q}(\hat{z}_{hh}, \xi) D_1^{h_2/g}\left(\frac{z_{2h}}{1-\xi}\right)$$

$$A = C_F\left[\left(z_1^3 \frac{\partial}{\partial z_1} \frac{\hat{H}(z_1)}{z_1^2}\right) 2\xi^2 \frac{z_1 - z_2}{z_{hh}} + \hat{H}(z_1) \frac{2\xi^2}{1-\xi}\right] + \int \frac{d\bar{z}_1}{\bar{z}_1^2} PV\left(\frac{1}{\frac{1}{z_1} - \frac{1}{\bar{z}_1}}\right) \hat{H}_F(z_1, \bar{z}_1)$$
$$\times \left[-C_F \frac{2z_{1h}}{z_1}\left(1 + \frac{z_{1h}}{\bar{z}_1} - \frac{z_{1h}}{z_1}\right) - \frac{C_A}{2} \frac{2z_{1h}}{z_1} \frac{z_1 \bar{z}_1 (z_1 + \bar{z}_1) - z_{1h}(z_1^2 + \bar{z}_1^2)}{z_1 (z_1 - \bar{z}_1)(\bar{z}_1 - z_{1h})}\right]$$

What can we learn from this calculation ?

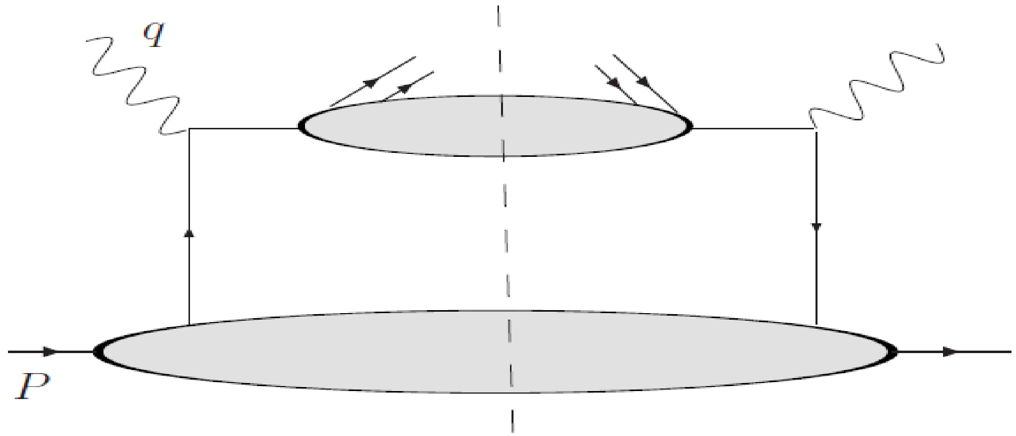
- Power behavior: $D_{hh} \sim 1/M_{hh}^2$; $H_1^{\sphericalangle} \sim 1/M_{hh}^3$; Asymmetry in SIDIS $\sim 1/M_{hh}$
- Final state dynamics:

$$\hat{H}(z) = \int d^2 p_\perp \frac{p_\perp^2}{2M} H_1^\perp(z, p_\perp) \qquad \hat{H}_F$$



Both contribute to large k_T collins function. **Yuan and ZJ 2009**

Two hadrons production in SIDIS

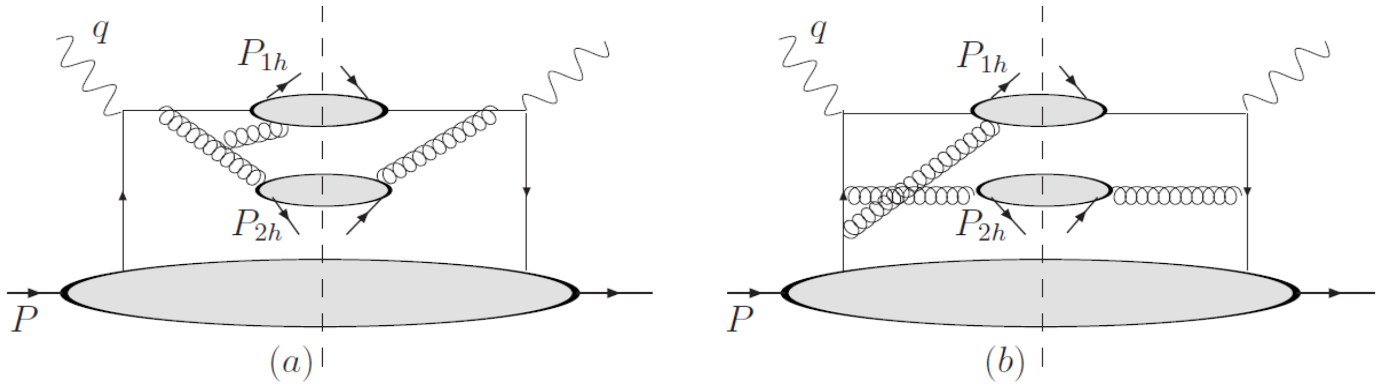


Cross section derived from the collinear factorization in terms of DFFs(M_{hh}<<Q).

$$\frac{d\sigma}{dx_B dy d\phi_S dz_{hh} dM_{hh}^2 d\phi_R} = \frac{2\alpha_{em}^2 s x_B}{Q^4} \sum_q e_q^2$$

$$\times \left[\left(1 - y + \frac{y^2}{2}\right) f_1^q(x_B) D_1^q(z_{hh}, M_{hh}^2) + (1-y)\sin(\phi_R + \phi_S) h_1^q(x_B) H_1^{\sphericalangle q}(z_{hh}, M_{hh}^2) \right]$$

Dihadrons production in SIDIS



When $\Lambda_{QCD} << M_{hh}$, collinear factorization in terms of single fragmentation functions applies

$$\frac{d\sigma}{dx_B dy d\phi_S dz_{hh} dM_{hh}^2 d\phi_R} = \frac{2\alpha_{em}^2 s}{Q^4} x_B (1-y)\sin(\phi_S+\phi_R) h_1(x_B) \frac{\alpha_s}{2\pi^2}\frac{1}{M_{hh}^3}\int \frac{d\hat{z}_{hh}}{\sqrt{z_{1h}z_{2h}}}\frac{d\xi}{\xi}$$
$$\times\left\{ C_F \left[\left(z_1^3 \frac{\partial}{\partial z_1}\frac{\hat{H}(z_1)}{z_1^2}\right) 2\xi^2 \frac{z_1 - z_2}{z_{hh}} + \hat{H}(z_1)\frac{2\xi^2}{1-\xi}\right] + \int \frac{dz_1'}{z_1'^2} PV\left(\frac{1}{\frac{1}{z_1}-\frac{1}{z_1'}}\right)\hat{H}_F(z_1,z_1')\right.$$
$$\left.\times\left[-C_F \frac{2z_{1h}}{z_1}\left(1+\frac{z_{1h}}{z_1'}-\frac{z_{1h}}{z_1}\right) - \frac{C_A}{2}\frac{2z_{1h}}{z_1}\frac{z_1 z_1'(z_1+z_1') - z_{1h}(z_1^2+z_1'^2)}{z_1(z_1-z_1')(z_1'-z_{1h})}\right]\right\}$$

Unified picture

$\Lambda_{QCD} << M_{hh} << Q$
Collinear factorization in terms of dihadron fragmentation functions &
Collinear factorization in terms of single hadron fragmentation functions

Similar to recent work:

$\Lambda_{QCD} << L_T << Q$,
Collinear factorization & TMD factorization, Sivers effect in SIDIS and DY

Ji, Qiu, Vogelsang, Yuan 2006

Summary

- We calculated the IFF $H_1^{\sphericalangle}$ at large invariant mass. The asymmery in SIDIS dorps like $1/M_{hh}$.

- Our calculation revealed that IFF $H_1^{\sphericalangle}$ and Collins function originate from the same type of FSI.

- Collinear factorization in terms of dihadron FFs and single hadron FFs provide same result at imtermediate invariant mass region.