# Running-mass definition for the DIS

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- FFN and VFN schemes
- Progress for the FFN scheme
  - threshold soft-gluon resummation
  - running mass definition
- Running mass in the VFN scheme

**sa, Moch [hep-ph 1011.5790]**

Newport News, 13 Apr 2011

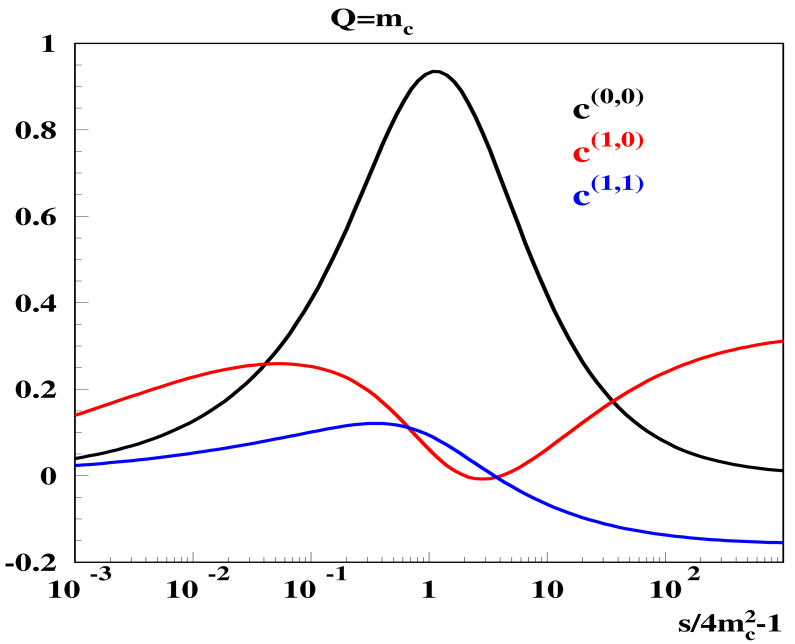
# The heavy-quark electro-production

The dominant mechanism is photon-gluon fusion, contributes up to 30% to the inclusive structure functions. The massive coefficient functions are known up to the NLO.

$$C_{2,g}^{LO} = c^{(0,0)}$$ **Witten NPB 104, 445 (1976)**

$$C_{2,g}^{NLO} = c^{(1,0)} + c^{(1,1)} \ln(\mu_F^2/m_c^2)$$

**Laenen, Riemersma, Smith, van Neerven NPB 392, 162 (1993)**




## FFNS

- Only 3 light flavors in the initial state are considered.
- Accurate at $Q \sim m_c$
- At large $Q$ the fixed-order results may be insufficient due to big logs $\sim \ln^n(Q/m_c)$ must be resummed
- Involved high-order calculations. The full NNLO corrections are missed → inconsistency in the NNLO PDF fits.

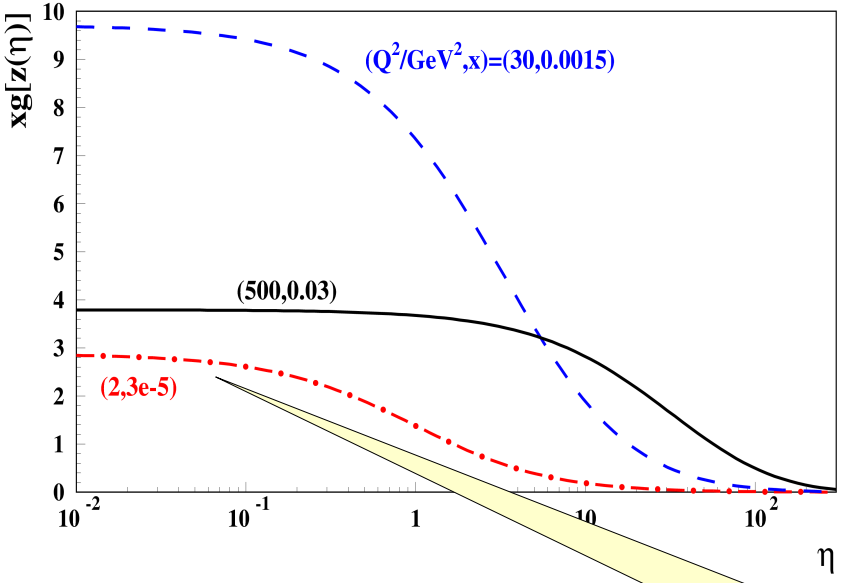
## ZMVFNS

**Collins, Tung NPB 278, 934 (1986)**

- At $Q >> m_c$ the heavy quarks are considered as massless → the NNLO evolution and the coefficient functions up to N$^3$LO are ready
- The big logs $\sim \ln^n(Q/m_c)$ are in a natural way resummed in the QCD evolution
- Matching conditions for the 3(4)-flavor and the 4(5)-flavor massless theories
- *A smooth matching with the FFNS in the limit of $Q \to m_c$ must be provided*

# Threshold soft-gluon resummation

$$x^{-1}F_2^c(x,Q^2,m^2) = \frac{\alpha_s\, e_q^2}{\pi^2} \sum_{i=q,\bar{q},g} \int_0^{\eta_{max}} d\eta\; f_i\left(z(\eta),\mu^2\right)\; c_{i,k}\left(\eta,\xi,\mu^2\right)$$




- At small *x* and small *Q* the main contribution comes from η<1 due to the gluon distribution shape (threshold production)
- The large logs ~ $\ln^{2n}(\beta)$ can be resummed in all orders, this gives a good approximation to the exact NNLO expression at small β with the tower of large logs

**Laenen, Moch PRD 59, 034027 (1999)**

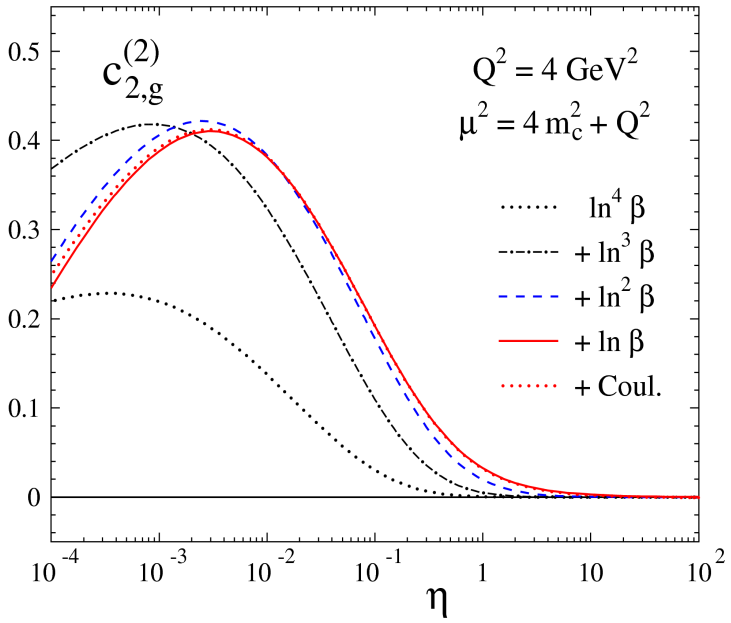
**sa, Moch PLB 672, 166 (2009)**

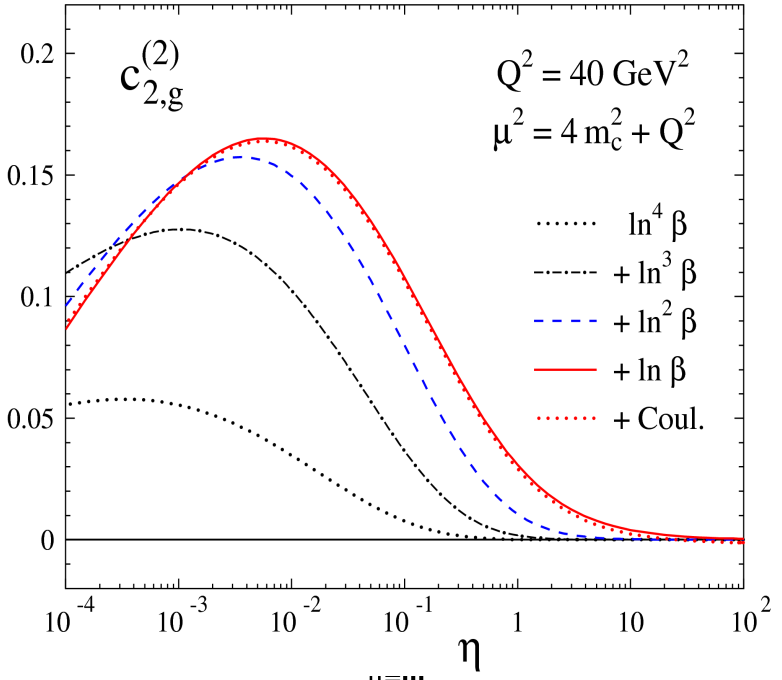
$\eta = s/4m^2 - 1$ – distance to the threshold

$\beta = \sqrt{1-4m^2/s}$ – quark velocity

*The threshold approximation works in a best way at small Q and x*

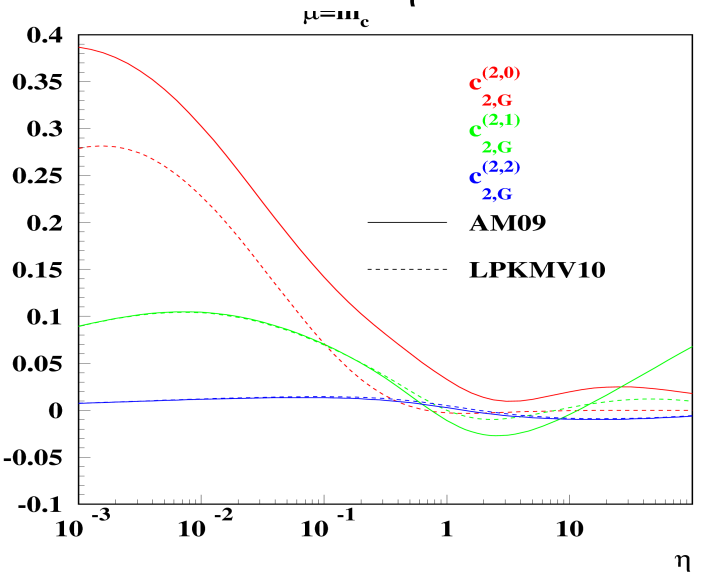
# Threshold soft-gluon resummation (cont'd)




**Lo Presti, Kawamura, Moch, Vogt [hep-ph 1008.0951]**

- The first log and Coulumb terms have been added → $F_2^c$ gets somewhat smaller at small Q and somewhat bigger at large Q



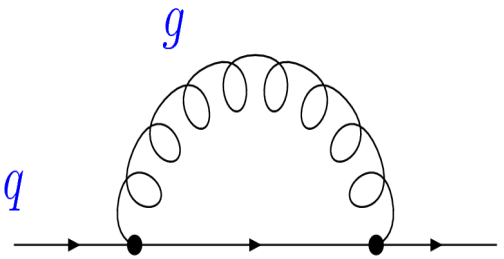
# Pole mass definition

The pole mass is defined as a the QCD Lagrangian parameter and is commonly used in the QCD calculations

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \sum_{\text{flavors}} \bar{q} \left( \mathrm{i} \not{D} - m_q \right) q$$

Pole mass is defined for the free (unobserved) quarks

$$\not{p} - m_q - \Sigma(p, m_q) \Big|_{p^2 = m_q^2}$$




The quantum corrections due to the self-energy loop integrals receive contribution down to scale of $O(\Lambda_{QCD})$ → sensitivity to the high order corrections, particularly at the production threshold

Example: the ttbar production in hadronic collisions

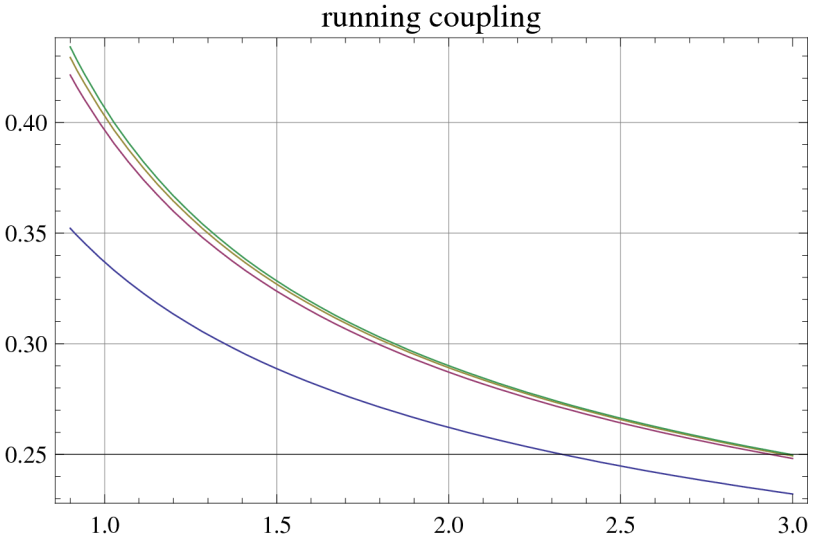
**Laengenfeld, Moch, Uwer PRD 80, 054009 (2009)**

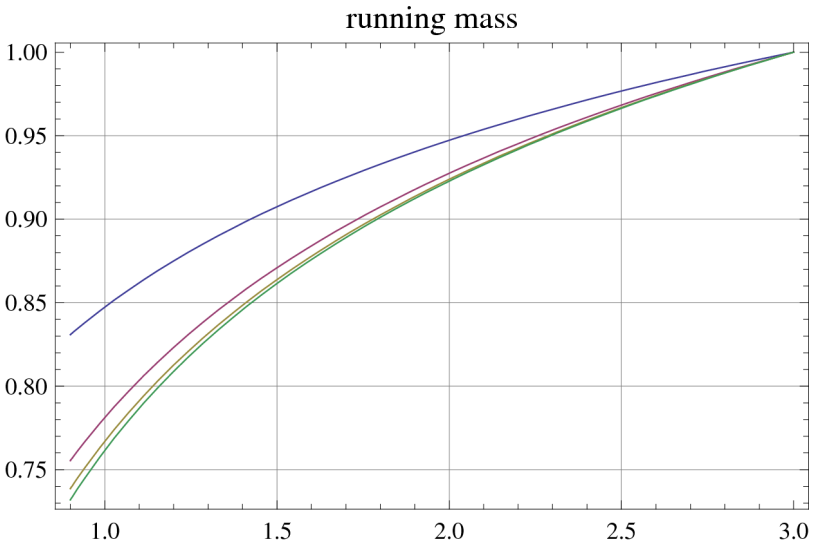
# Running quark mass

The renormgroup equation for mass is similar to one for the coupling constant

$$\mu^2 \frac{d}{d\mu^2}\alpha_s(\mu) = \beta(\alpha_s)$$

$$\mu^2 \frac{d}{d\mu^2} m(\mu) = \gamma(\alpha_s) m(\mu)$$




The corrections up to 4-loops are known

**van Ritbergen, Vermaseren, Larin PLB 400, 379 (1997)**

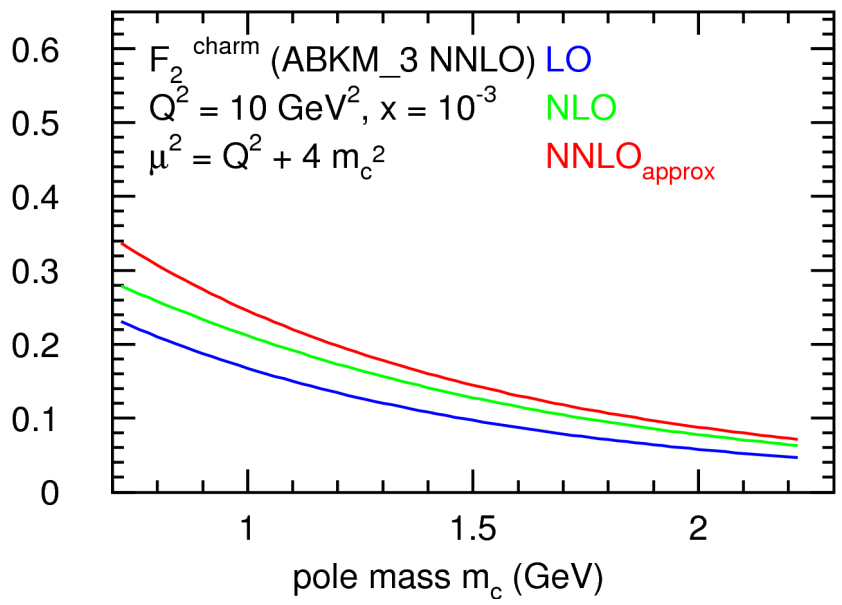
**Chetyrkin PLB 404, 161 (1997)**

**Vermaseren, Larin, van Ritbergen PLB 405, 327 (1997)**

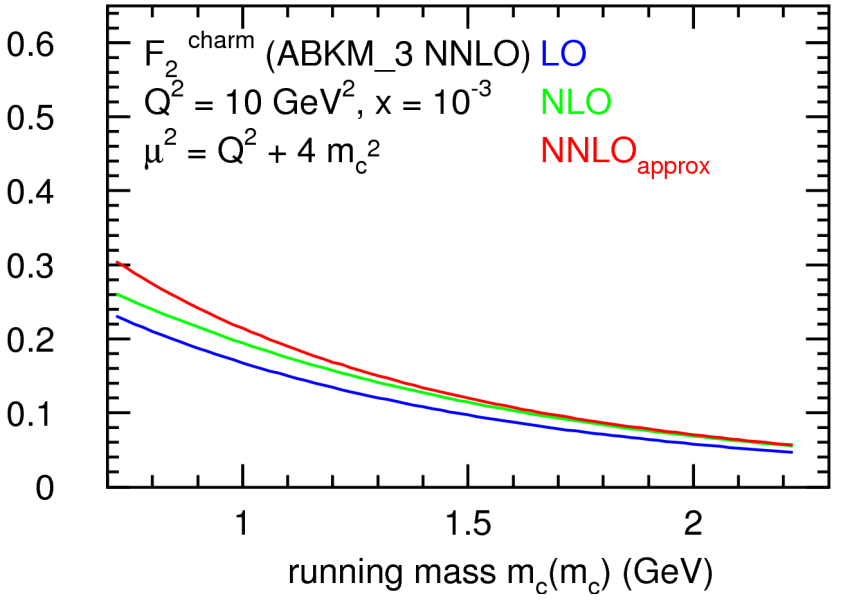
The choice of $\mu_R = m_c$ is close to the DIS data kinematic → better perturbative convergence and reduced scale dependence

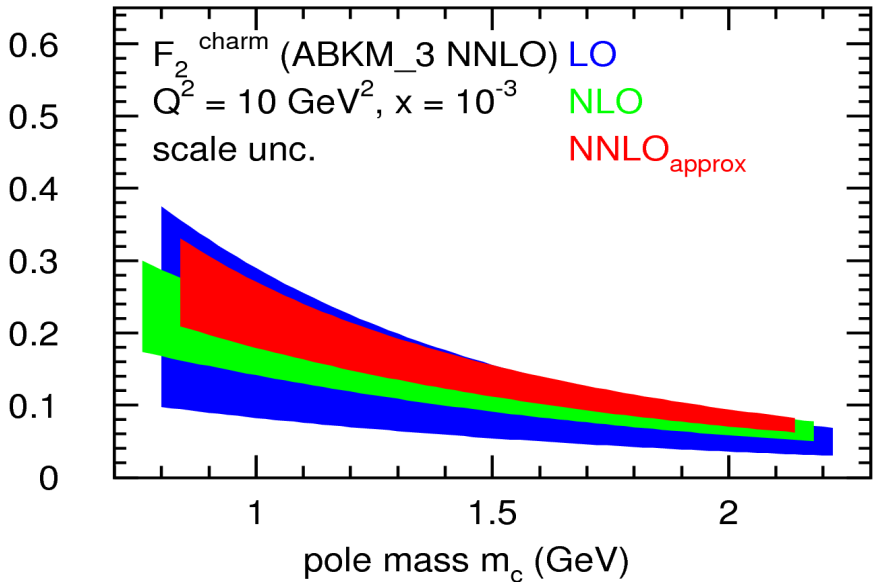
# Running mass definition for the DIS SFs

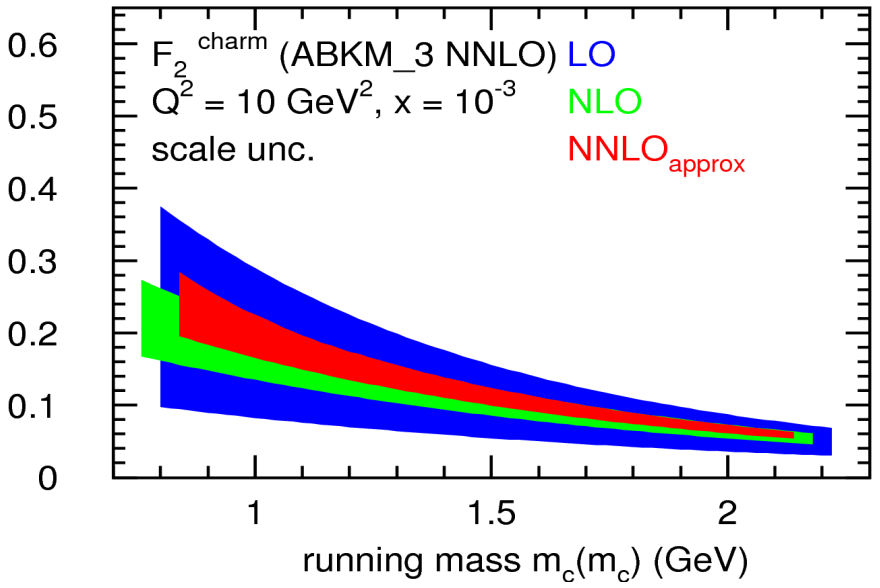
## Pole mass




## Running mass



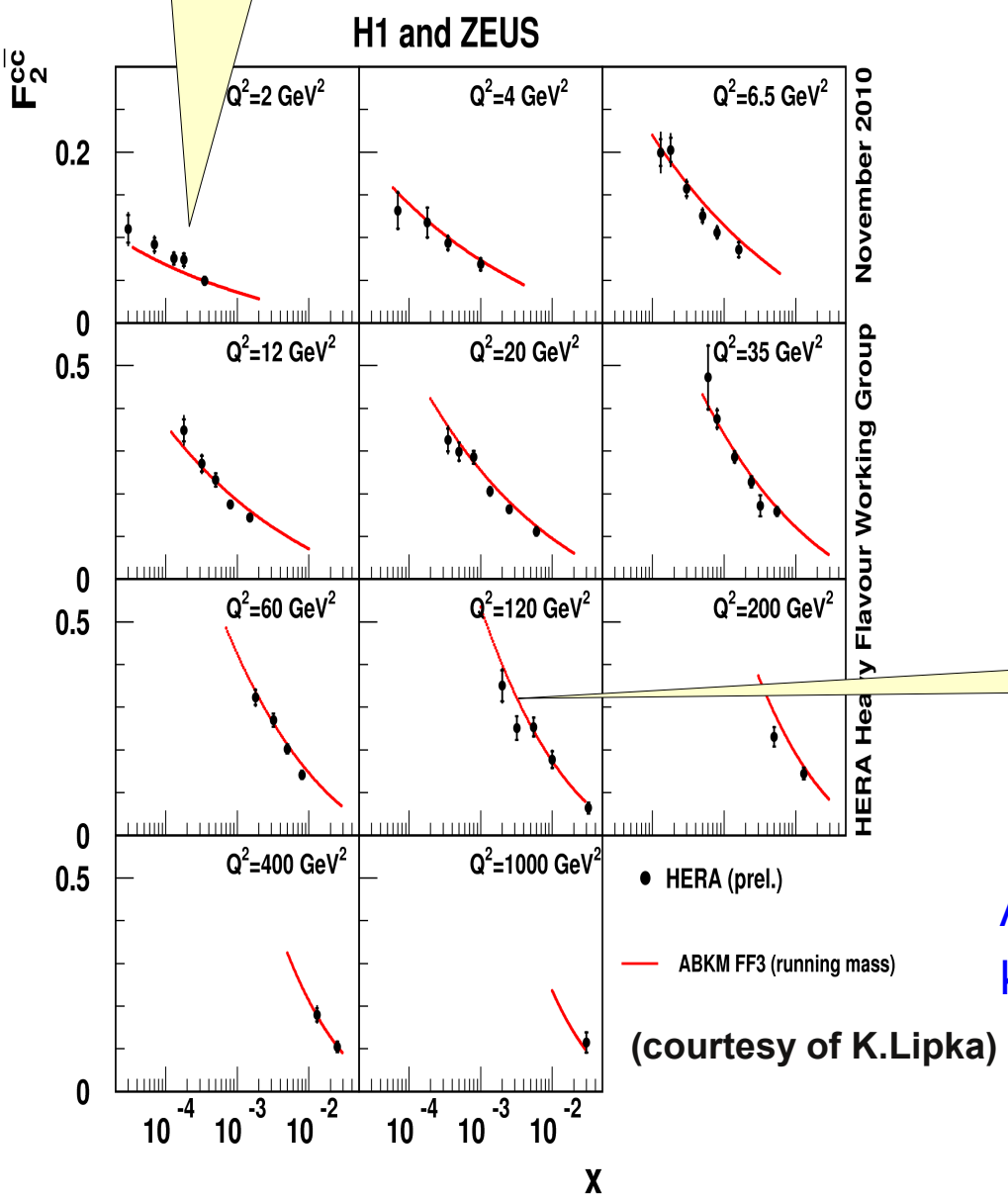



# c-quark production

The NNLO(approx.) FFNS ABM ***predictions*** based on the running mass definition are In nice agreement with the new HERA data

N$^3$LO corrections?




(courtesy of K.Lipka)

$m_c(m_c)$=1.27±0.08 GeV (PDG '10)

$m_c(m_c)$=1.18±0.06 GeV (incl.$F_2$ +PDG)

The HERA data prefer $m_c(m_c)$ close to the PDG value

*Improved accuracy due to correlation between quark and gluon PDFs must be reduced*

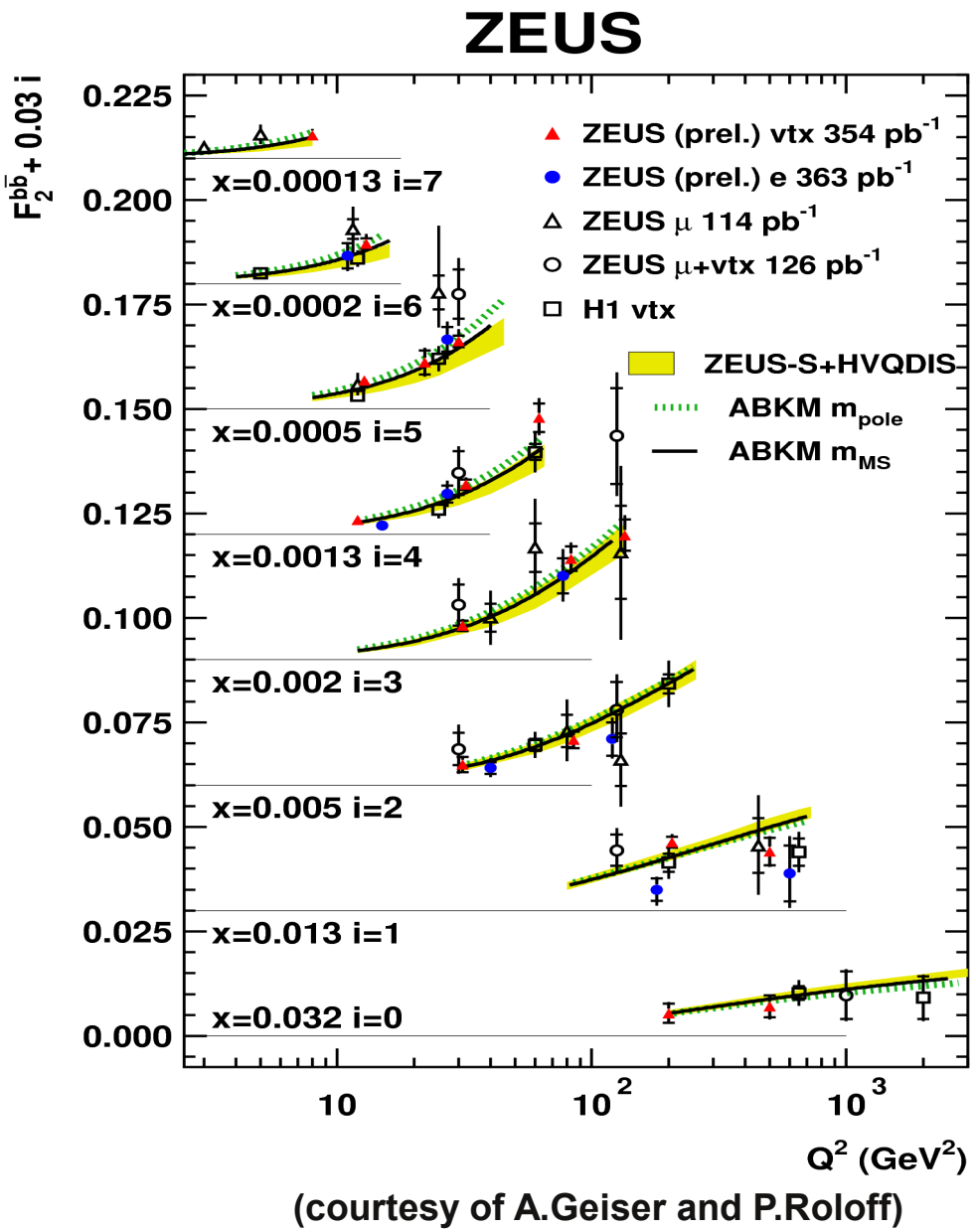
No need for the VFN scheme

At $Q >> m_c$ first Mellin NNLO moments are known

**Ablinger at al. NPB 844, 26 (2011)**

**Bierenbaum, Blümlein, Klein NPB 829, 417 (2009)**

# b-quark production

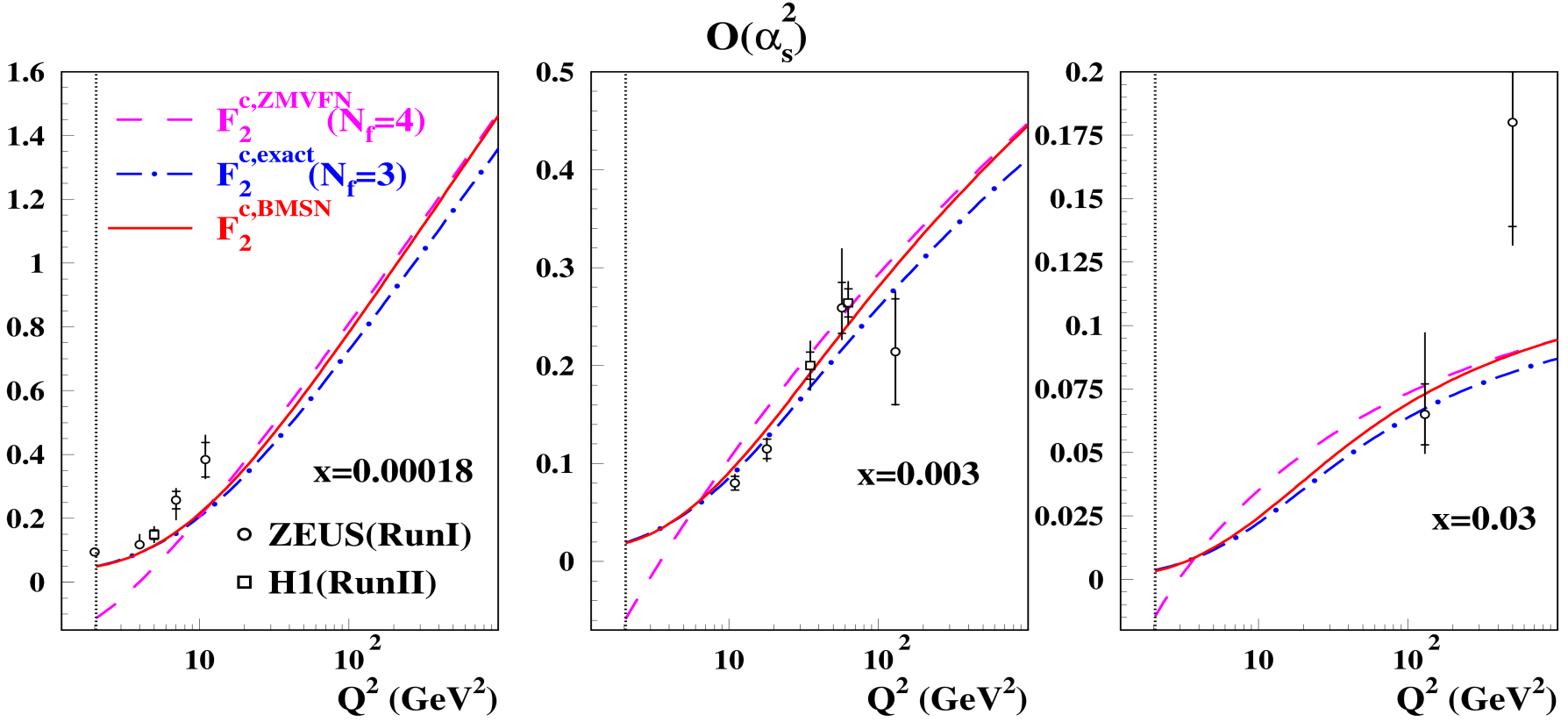



(courtesy of A.Geiser and P.Roloff)

For the b-quark production NNLO$_{approx}$ predictions work well → the threshold approximation is better justified

No sensitivity to $m_b$ → fixed at the PDG value $m_b(m_b)$=4.19±0.12 GeV

# FFN and VFN schemes



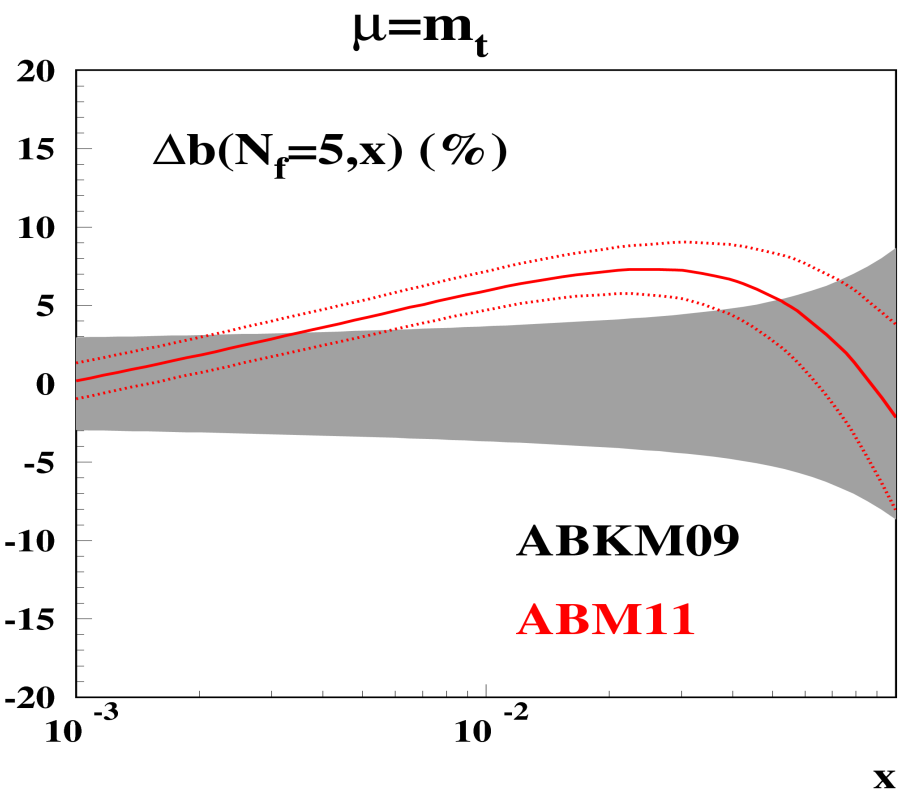

sa, Blümlein, Klein, Moch PRD 81, 014032 (2010)

*In the NLO the FFNS is sufficient for description of the realistic DIS data – the big logs appear in the high order corrections to the massive coefficient functions*

Glück, Reya, Stratmann NPB 422, 37 (1994)

- With the running mass definition the difference between VFN and FFN is even reduced?

# Running mass and the VFN scheme




The 4- and 5-flavour PDFs are generated from the ABM11 fit preformed with the running-mass definition; the massive OMEs with the running-mass definition are used

The change in the heavy-quark distribution id due to:

- change in the 3-flavor distributions from ABKM09 to ABM11
- change in the masses: $m_b$ = 4.5 → 4.19±0.12 GeV (PDG '10)
- modification of the massive OMEs

The b-quark distribution uncertainty is reduced → impact on the single-top production

# Summary and outlook

- The running mass definition is implemented for the DIS semi-inclusive structure functions
  - Improved perturbative stability and the scale variation uncertainty
  - Consistent treatment of the mass in DIS and other processes, like e+e- initiated
  - First determination of running mass from the DIS data
- Better determination on the heavy-quark PDFs
- Improved uncertainty foreseen with inclusion of the HERA combined charm data
  - Resolving correlation between gluon and sea distribution
  - Good agreement with the PDG value of $m_c$ expected