TMD Factorization, Factorization Breaking and Evolution

Ted C. Rogers

Vrije Universiteit Amsterdam

Based on:

S.M. Aybat, TCR (2011)

TCR, P.J. Mulders (2010)

See also, ***Foundations of Perturbative QCD***, J.C. Collins.

(available May 2011)

DIS 2011 April 13, 2011

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- Complicated issues involved in defining TMDs.
  - Divergences.
  - Wilson lines / gauge links.
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- Related: Factorization Theorems:

  Definitions dictated by requirements for factorization!

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Watch out for sign flips!

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TMD-Factorization

- TMD Parton model intuition (Drell-Yan):

$$W^{\mu\nu} = \sum_f \left|\mathcal{H}_f(Q)^2\right|^{\mu\nu} \int d^2\mathbf{k}_{1T}\, d^2\mathbf{k}_{2T}\, F_{f/P_2}(x_1, \mathbf{k}_{1T})\, F_{\bar{f}/P_2}(x_2, \mathbf{k}_{2T}) \times \times\, \delta^{(2)}(\mathbf{k}_{1T} + \mathbf{k}_{2T} - \mathbf{q}_T)$$

Leading order hard part

No evolution

Generalized Parton Model

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Evolved Cross Section:

- Contrast: Typical appearance of Collins-Soper-Sterman formalism:

(1985)

$$d\sigma \sim \int d^2\mathbf{b}\, e^{-i\mathbf{b}\cdot\mathbf{q}_T}$$

$$\int_{x_1}^{1} \frac{d\hat{x}_1}{\hat{x}_1} \tilde{C}_{f/j}(x_1/\hat{x}_1, b_*; \mu_b^2, \mu_b, g(\mu_b)) f_{j/P_1}(\hat{x}_1, \mu_b)$$

$$\int_{x_2}^{1} \frac{d\hat{x}_2}{\hat{x}_2} \tilde{C}_{f/j}(x_2/\hat{x}_2, b_*; \mu_b^2, \mu_b, g(\mu_b)) f_{j/P_2}(\hat{x}_2, \mu_b)$$

$$\exp\left[\int_{1/b^2}^{Q^2} \frac{d\mu'^{\,2}}{\mu'^{\,2}} \left\{\mathcal{A}(\alpha_s(\mu')) \ln\frac{Q^2}{\mu'^2} + \mathcal{B}(\alpha_s(\mu'))\right\}\right]$$

$$\exp\left[-g_k(b)\ln\frac{Q^2}{Q_0^2} - g_1(x_1, b) - g_2(x_2, b)\right]$$

+ Large q_T term

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- How are these related? How to connect to phenomenology?

What is needed?

- TMD Parton model intuition (Drell-Yan):

$$W^{\mu\nu} = \sum_f \left|\mathcal{H}_f(Q)^2\right|^{\mu\nu} \int d^2\mathbf{k}_{1T}\, d^2\mathbf{k}_{2T}\, F_{f/P_2}(x_1, \mathbf{k}_{1T})\, F_{\bar{f}/P_2}(x_2, \mathbf{k}_{2T}) \times$$
$$\times\, \delta^{(2)}(\mathbf{k}_{1T} + \mathbf{k}_{2T} - \mathbf{q}_T)$$

- Using newest definitions:

$$W^{\mu\nu} = \sum_f |\mathcal{H}_f(Q;\mu)^2|^{\mu\nu}$$
$$\times \int d^2\mathbf{k}_{1T}\, d^2\mathbf{k}_{2T}\, F_{f/P_1}(x_1, \mathbf{k}_{1T}; \mu; \zeta_1)\, F_{\bar{f}/P_2}(x_2, \mathbf{k}_{2T}; \mu; \zeta_2)$$
$$\times\, \delta^{(2)}(\mathbf{k}_{1T} + \mathbf{k}_{2T} - \mathbf{q}_T)$$
$$+\, Y(Q, q_T) + \mathcal{O}((\Lambda/Q)^a).$$

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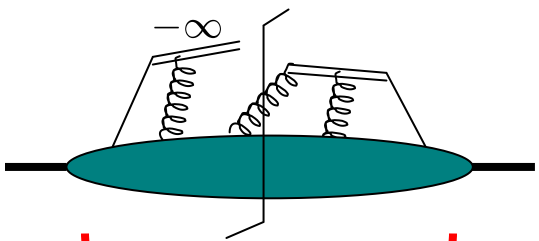
Process dependence in hard part

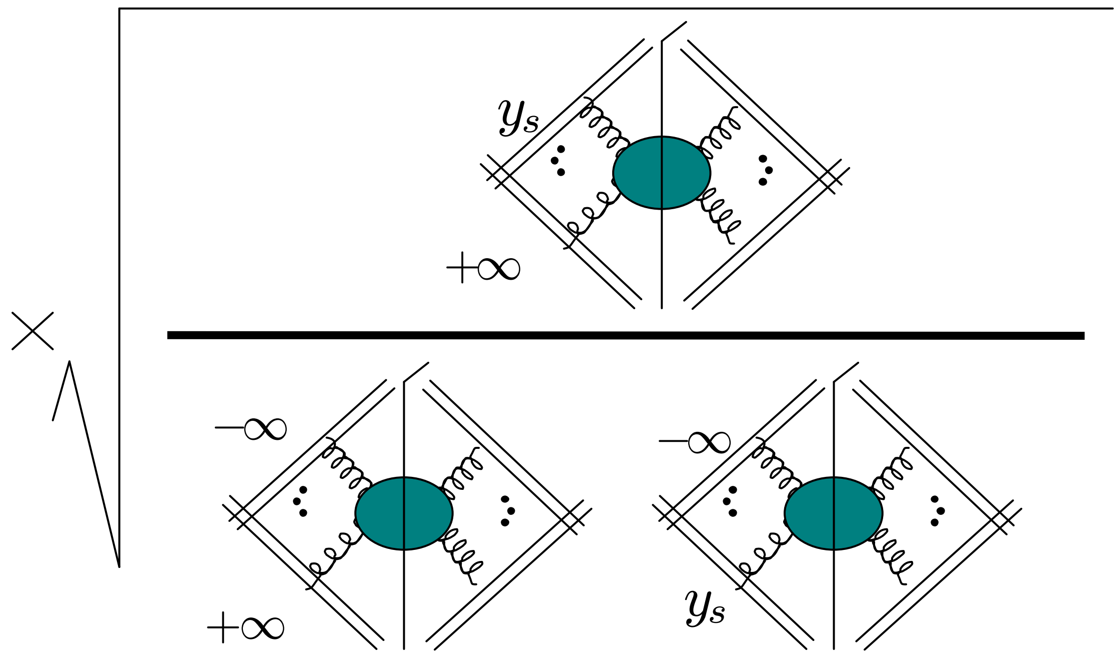
Universal PDFs with evolution

TMD PDF, Complete Definition:

$$F_{f/P}(x, b; \mu; \zeta_F) =$$



"Unsubtracted"



Implements Subtractions/Cancellations

From *Foundations of Perturbative QCD*, J.C. Collins,
See also, Collins, TMD 2010 Trento Workshop

Our Strategy:

- Use evolution to extrapolate between existing fits, to build unified fits that include evolution.

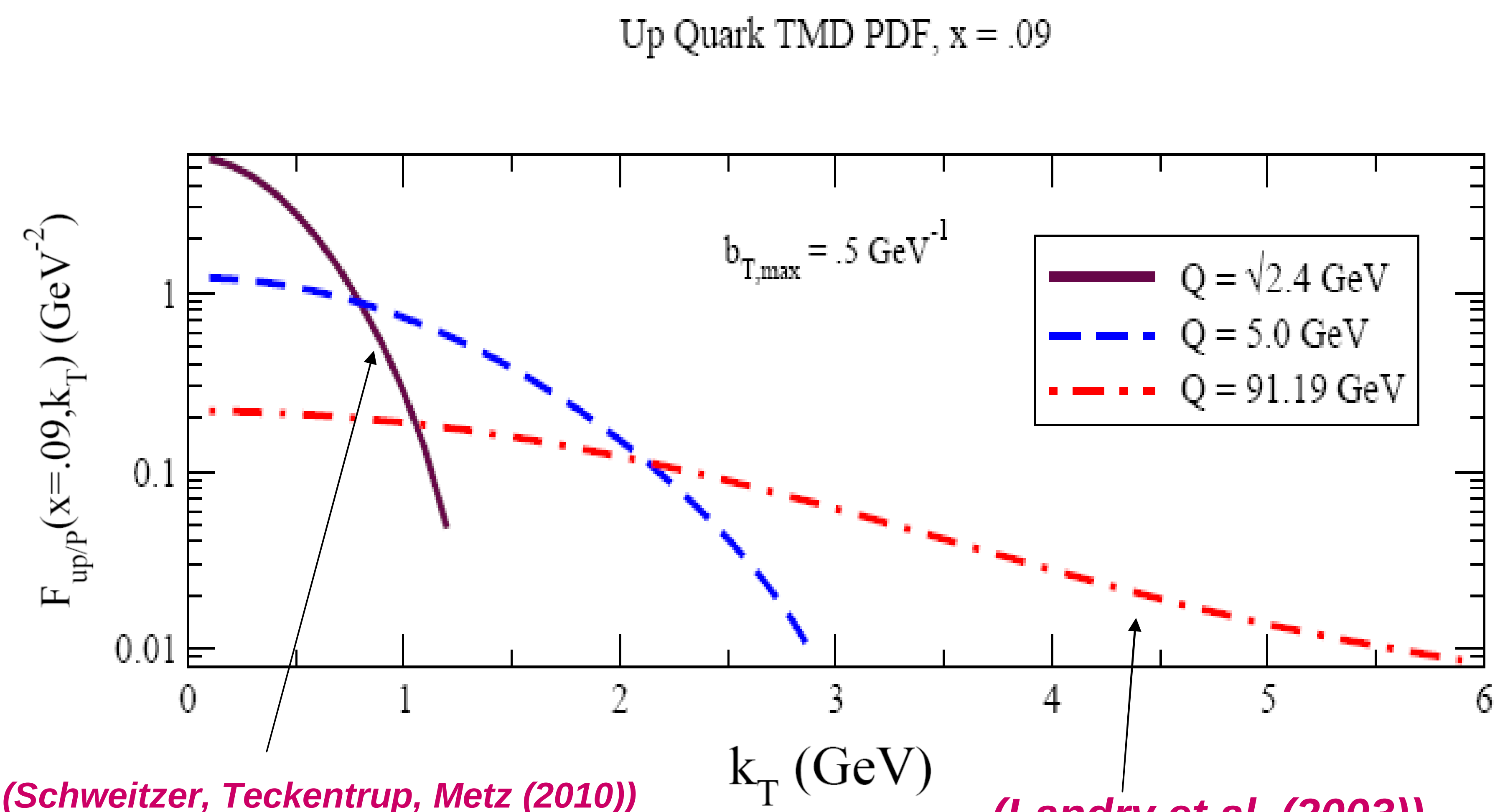
(S.M. Aybat, TCR (2011))

  - PDFs:
    - Start with DY:
      (Landry et al, (2003); Konychev, Nadolsky (2006)) ***(BLNY)***
    - Modify to match to SIDIS:
      (Schweitzer, Teckentrup, Metz (2010)) ***(STM)***

(For details, see Aybat talk, pre-DIS meeting.)

- Can supply explicit, evolved TMD PDF fit.

Evolving TMD PDFs



(Schweitzer, Teckentrup, Metz (2010))

(SIDIS)

(Landry et al, (2003))

(Drell-Yan)

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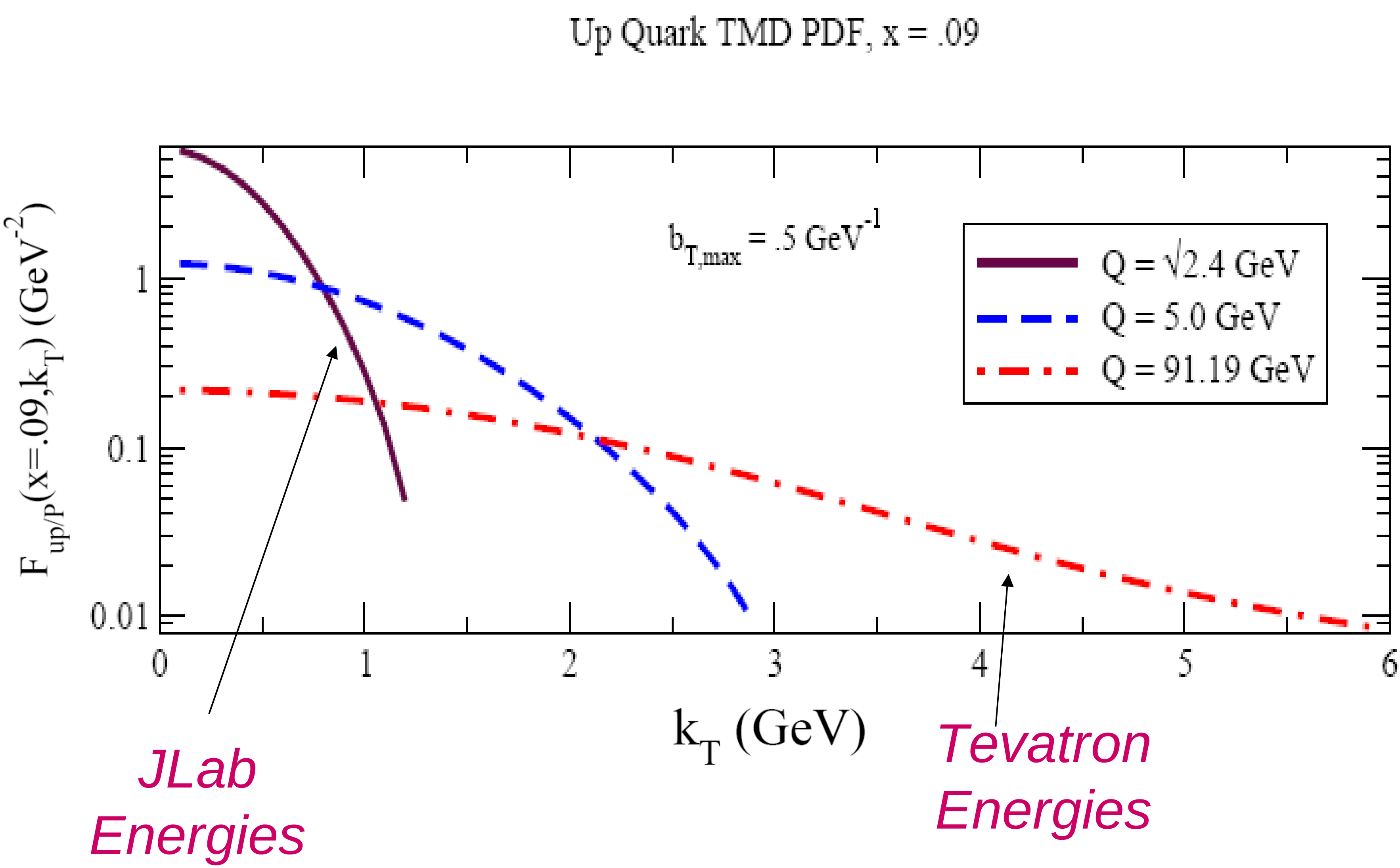

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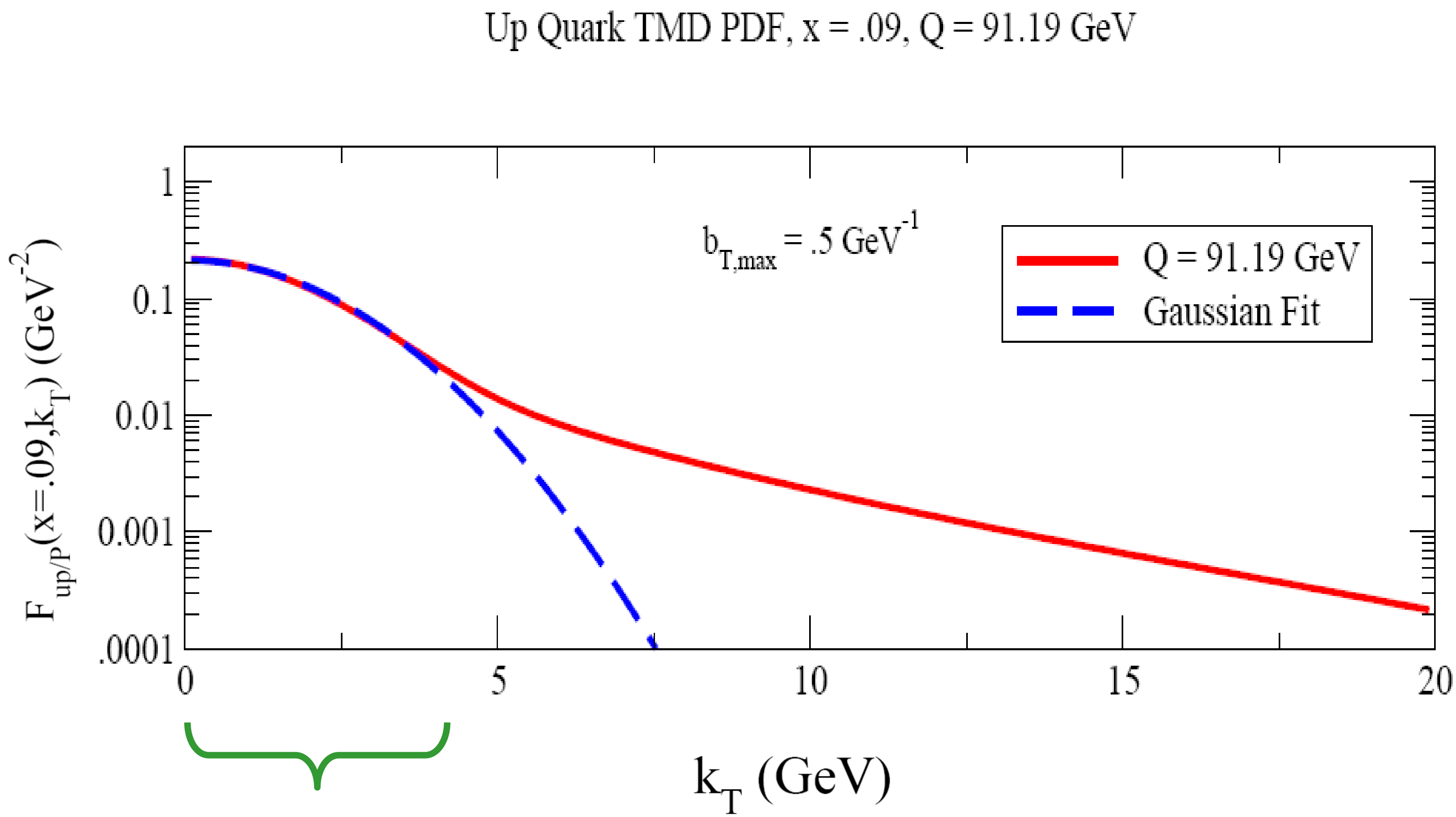

Gaussian fit good at small k_T.

Unambiguous Hard Part

- Higher orders follow systematically from definitions:

$$W^{\mu\nu} = |\mathcal{H}_f(Q;\mu/Q)^2|^{\mu\nu} \; F_{f/P_1} \otimes F_{f/P_2}$$

$$\Rightarrow \quad |\mathcal{H}_f(Q;\mu/Q)^2|^{\mu\nu} = \frac{W^{\mu\nu}}{F_{f/P_1} \otimes F_{f/P_2}}$$

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- Definition:

$$|\mathcal{H}_f(Q;\mu/Q)^2|^{\mu\nu} = \frac{W^{\mu\nu}}{F_{f/P_1} \otimes F_{f/P_2}}$$

- Drell-Yan: $(\overline{\text{MS}})$

$$|\mathcal{H}_f(Q;\mu/Q)^2|^{\mu\nu} =$$

$$e_f^2|H_0^2|^{\mu\nu}\left(1+\frac{C_F\alpha_s}{\pi}\left[\frac{3}{2}\ln\left(Q^2/\mu^2\right)-\frac{1}{2}\ln^2\left(Q^2/\mu^2\right)-4+\frac{\pi^2}{2}\right]\right)+\mathcal{O}(\alpha_s^2)$$

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- SIDIS

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Space-like photon!

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Long-Term Goal:

- Repository of new TMD fits with evolution.

- Based on well-understood operator definitions.
  - *Take Collins definitions*.

Agenda:

- Improve fits. Combine SIDIS, DY, e^+e^- in global fit. Extend to higher orders. Gaussian fits.
- Extend to polarization dependent functions (Sivers, Boer-Mulders, etc…).
- TMD gluon distribution.
- Factorization breaking??
- Updates to appear at:

https://projects.hepforge.org/tmd/

Thanks!

Backup Slides

Understanding the Definition:

- Start with only the hard part factorized:

Naïve Factorization:

$$d\sigma = |\mathcal{H}|^2 \, \tilde{F}_1^{\rm unsub}(y_1 - (-\infty)) \times \tilde{F}_2^{\rm unsub}(+\infty - y_2).$$

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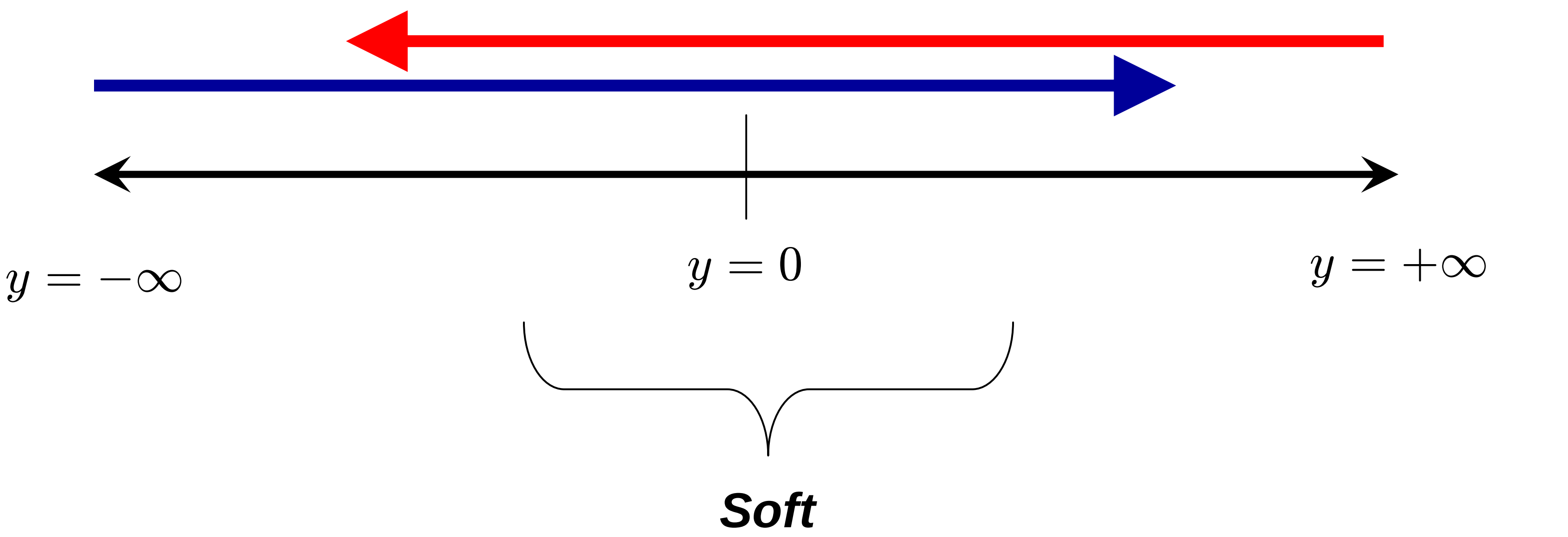

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- Separate soft part:

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- Multiply by: $\dfrac{\sqrt{\tilde{S}(+\infty, y_s)\,\tilde{S}(y_s, -\infty)}}{\sqrt{\tilde{S}(+\infty, y_s)\,\tilde{S}(y_s, -\infty)}}$

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- Rearrange factors: $d\sigma = |\mathcal{H}|^2 \left\{ \tilde{F}_1^{\rm unsub}(y_1 - (-\infty)) \sqrt{\dfrac{\tilde{S}(+\infty, y_s)}{\tilde{S}(+\infty, -\infty)\tilde{S}(y_s, -\infty)}} \right\}$

$$\times \left\{ \tilde{F}_2^{\rm unsub}(+\infty - y_2) \sqrt{\frac{\tilde{S}(y_s, -\infty)}{\tilde{S}(+\infty, -\infty)\tilde{S}(+\infty, y_s)}} \right\}$$

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Separately Well-defined

Evolution

- Collins-Soper Equation:
  - $$\frac{\partial \ln \tilde{F}(x, b_T, \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_T; \mu)$$

Perturbatively calculable from definition at small b.

$$\tilde{K}(b_T; \mu) = \frac{1}{2} \frac{\partial}{\partial y_n} \ln \frac{\tilde{S}(b_T; y_n, -\infty)}{\tilde{S}(b_T; +\infty, y_n)}$$

- RG:
  - $$\frac{d\tilde{K}}{d \ln \mu} = -\gamma_K(g(\mu))$$
  - $$\frac{d \ln \tilde{F}(x, b_T; \mu, \zeta)}{d \ln \mu} = -\gamma_F(g(\mu); \zeta/\mu^2)$$

Perturbatively calculable, from definitions

TMD-Factorization

- TMD-factorization with consistent definitions:

$$
\begin{aligned}
W^{\mu\nu} = \sum_f |\mathcal{H}_f(Q;\mu)^2|^{\mu\nu} \\
\times \int d^2\mathbf{k}_{1T}\, d^2\mathbf{k}_{2T}\, F_{f/P_1}(x_1, \mathbf{k}_{1T}; \mu; \zeta_1)\, F_{\bar{f}/P_2}(x_2, \mathbf{k}_{2T}; \mu; \zeta_2) \\
\times \delta^{(2)}(\mathbf{k}_{1T} + \mathbf{k}_{2T} - \mathbf{q}_T) \\
+ Y(Q, q_T) + \mathcal{O}((\Lambda/Q)^a).
\end{aligned}
$$

Implementing Evolution

- After evolution:

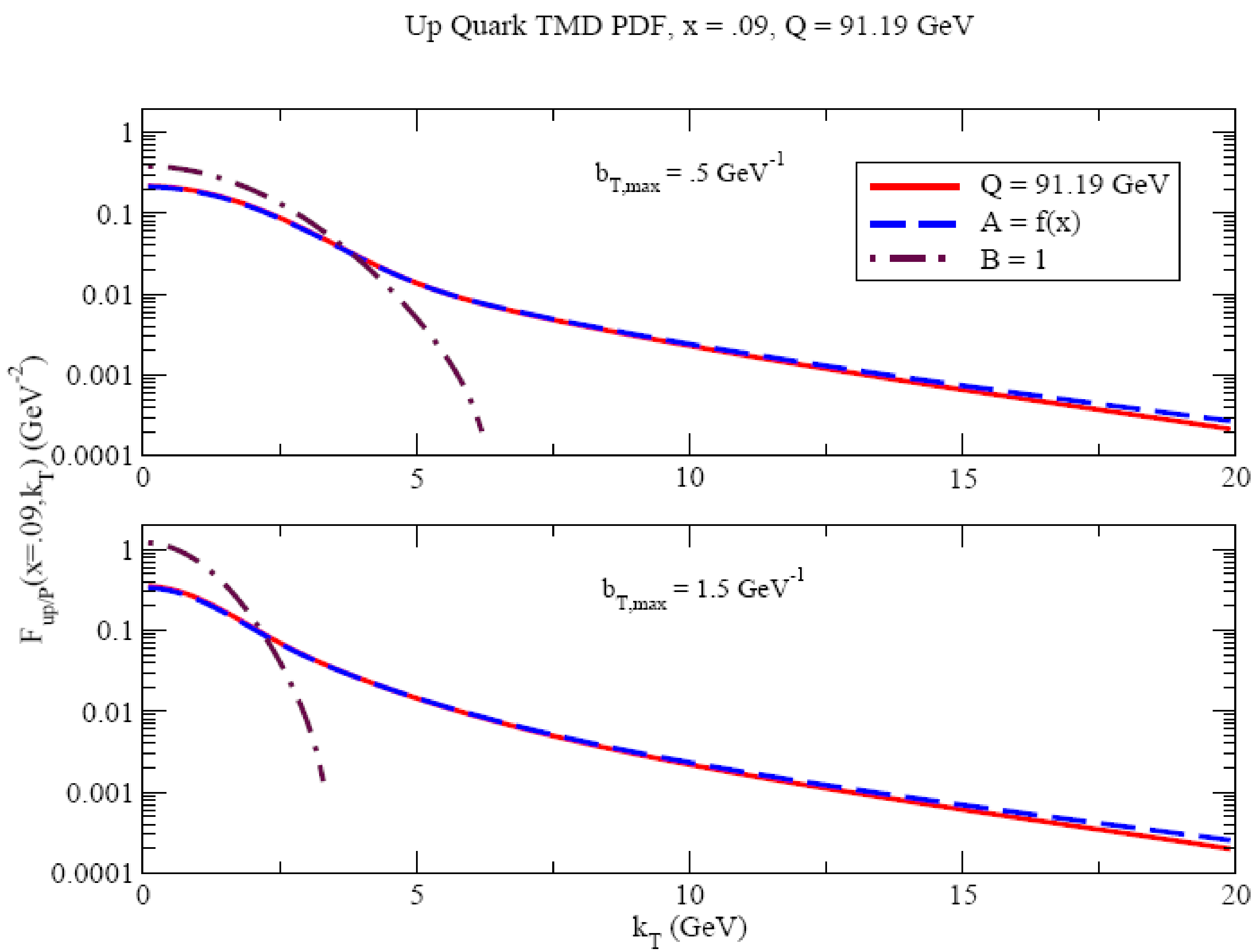
$$\tilde{F}_{f/H}(x, b_T, \mu, \zeta) = \sum_j \int_x^1 \frac{d\hat{x}}{\hat{x}} \tilde{C}_{f/j}(x/\hat{x}, b_*; \mu_b, g(\mu_b)) f_{j/H}(x, \mu_b) \times$$ A

$$\times \exp\left\{ \ln \frac{\sqrt{\zeta}}{\mu_b} \tilde{K}(b_*; \mu_b) + \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_F(g(\mu'); 1) - \ln \frac{\sqrt{\zeta}}{\mu'} \gamma_K(g(\mu')) \right] \right\} \times$$ B

$$\times \exp\left\{ g_{j/H}(x, b_T) + g_K(b_T) \ln \frac{\sqrt{\zeta}}{Q_0} \right\}$$ C

$$b_*(b_T) \equiv \frac{b_T}{\sqrt{1 + b_T^2/b_{max}^2}} \qquad \mu_b(b_T) \sim 1/b_*$$

CSS matching procedure



Up Quark TMD PDF, x = .09, Q = 91.19 GeV
$b_{T,max}$ = .5 GeV^{-1}
Q = 91.19 GeV
A = f(x)
B = 1
$b_{T,max}$ = 1.5 GeV^{-1}
$F_{up/P}(x=.09,k_T)$ (GeV^{-2})
k_T (GeV)