Bessel Weighted Asymmetries

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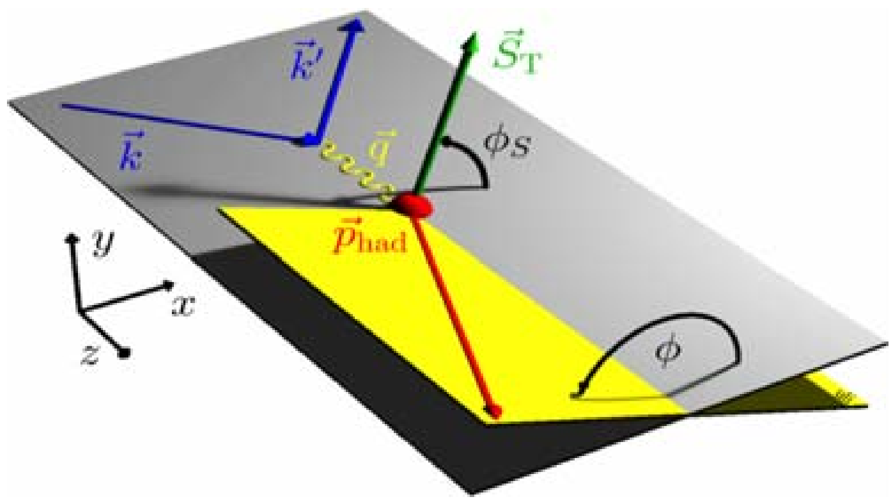
D. Boer, B. Musch and A. Prokudin in preparation

Outline

- On the merit of Bessel Weighted asymmetries
- **Elements of factorization theorem in SIDIS**
  - **Role of Soft Factor**
- Fourier Transformed SIDIS cross section
- Propose general Bessel Weighting procedure
- Cancellation of the Soft Factor from WA
- Cancellation of soft factor from av. trans. shift see talk of B. Musch (PRE-DIS wkshp http://conferences.jlab.org/QCDEvolution/index.html)

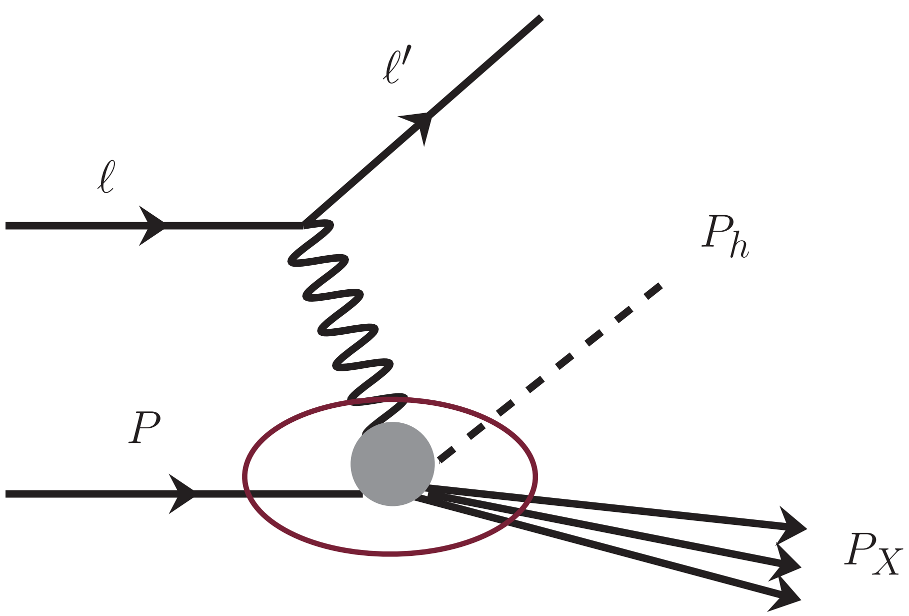
Summarize factorization in SIDIS to discuss weighted asymmetries

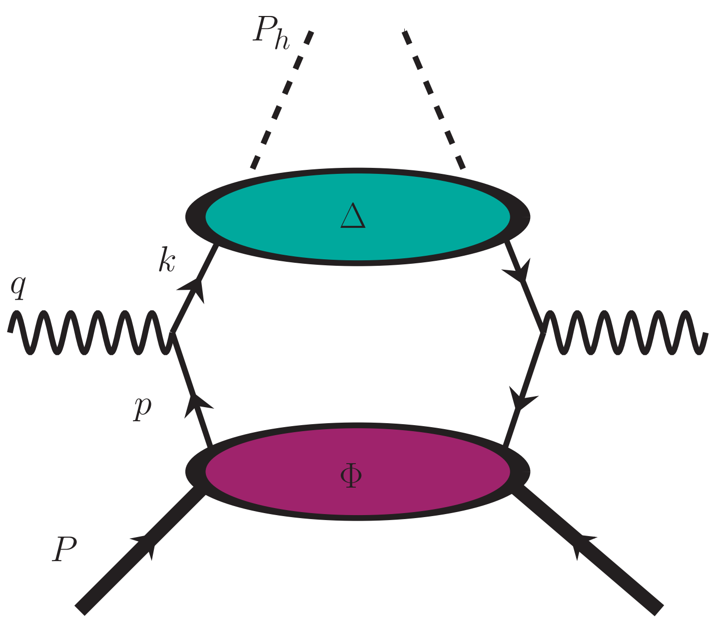


$$x_B = \frac{Q^2}{2P\cdot q}$$

$$z_h = \frac{P\cdot P_h}{P\cdot q} \approx \frac{P_h^-}{q^-}$$

Parton model & DIS kinematics



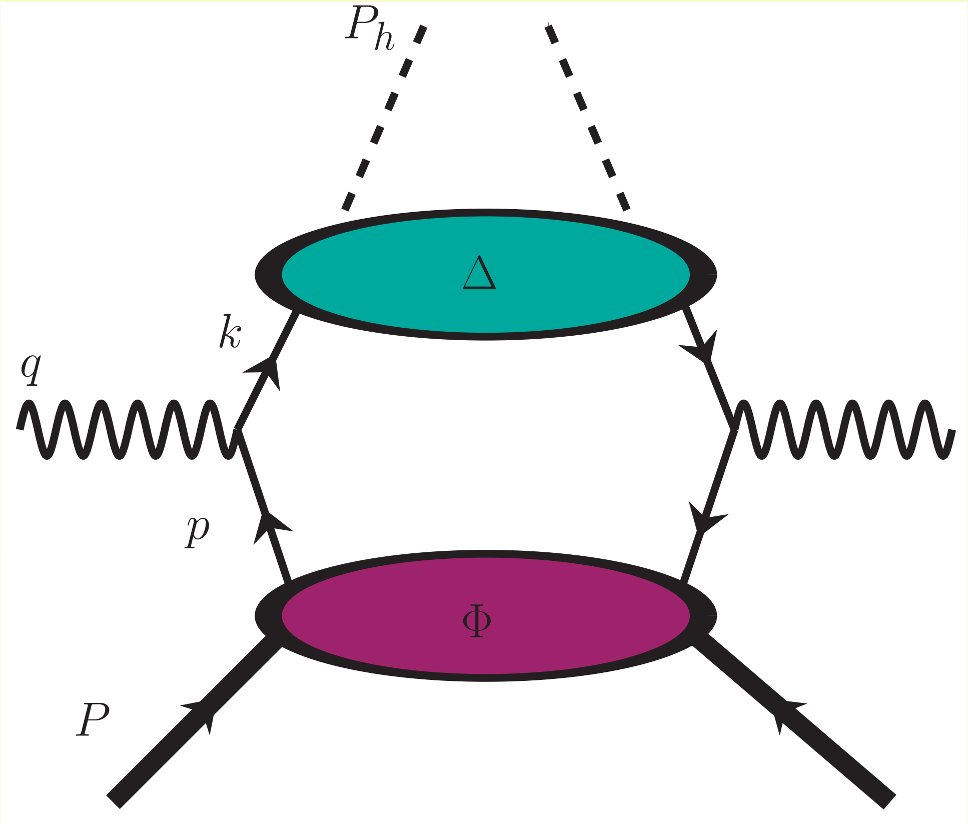


Factorization & Sensitivity to $P_T \sim k_\perp \longrightarrow$ TMDs

Based on QCD factorization at tree level

$$d^6\sigma = \hat{\sigma}_{\rm hard}\, \mathcal{C}[w\, f\, D]$$

Ralson & Soper NPB 1979-Drell Yan, Mulders Tangerman NPB 1996-SIDIS



Structure functions are convolutions in momentum-space

$$\mathcal{C}\left[w\, f\, D\right] \equiv x_B \sum_a e_a^2 \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{K}_T\, \delta^{(2)}(z\boldsymbol{p}_T + \boldsymbol{K}_T - \boldsymbol{P}_{h\perp})\, w\left(\boldsymbol{p}_T, -\frac{\boldsymbol{K}_T}{z}\right)\, f^a(x, p_T^2)\, D^a(z, K_T^2)$$

$$F_{UT,T}^{\sin(\phi_h-\phi_S)} = \mathcal{C}\left[-(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T/M)\, f_{1T}^{\perp}\, D_1\right] \times (1 + \mathcal{O}(\alpha_s))$$

where $\boldsymbol{K}_T \equiv -\boldsymbol{k}_T z$ $\qquad$ with $\hat{\boldsymbol{h}} \equiv \boldsymbol{P}_{h\perp}/|\boldsymbol{P}_{h\perp}|$

SIDIS cross section model indpen. thru structure functions

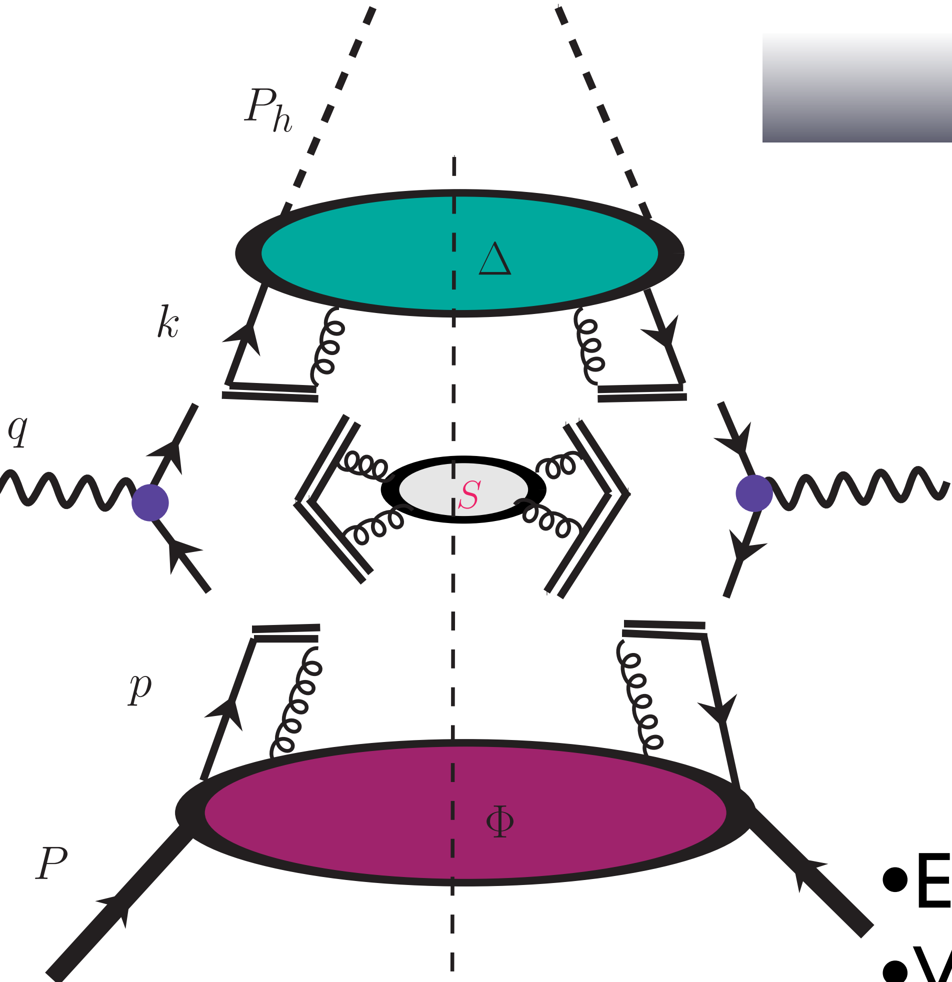
Kotzinian NPB 95, Mulders Tangermann NPB 96, Bacchetta et al JHEP 08

$$\frac{d^6\sigma}{dx\,dy\,d\psi\,dz_h\,d\phi_h\,dP_{h\perp}^2} \propto \frac{\alpha^2}{xyQ^2}\,\frac{y^2}{2}\left\{ \begin{aligned} & F_{UU,T} \\ & + \quad |S_\perp|\,\sin(\phi_h-\phi_S)\,F_{UT,T}^{\sin(\phi_h-\phi_S)} \\ & + \quad \ldots 16 \quad \text{more} \quad \text{structures} \quad \ldots \end{aligned} \right.$$

Projected from cross section

$$A_{XY}^{\mathcal{F}} \equiv 2\frac{\int d\phi_h\,d\phi_S\,\mathcal{F}(\phi_h,\phi_S)\,\left(d\sigma^{\uparrow}-d\sigma^{\downarrow}\right)}{\int d\phi_h d\phi_S\,(d\sigma^{\uparrow}+d\sigma^{\downarrow})}\,,$$

$X\,Y$-polarization e.g. $\mathcal{F}(\phi_h,\phi_s)=\sin(\phi_h-\phi_s)$



Beyond "tree level" factorization

CS NPB 81, Collins Hautman PLB 00, Ji Ma Yuan PRD 05,
Cherednikov Karanikas Stefanis NPB 10, Collins Oxford Press 2011,
Abyat & Rogers arXiv: 2011

- Extra divergences at one loop and higher
- Various strategies to address them
- Extra variables needed to regulate divergences
- Modifies convolution integral by introduction soft factor
- Will show cancels in certain weighted asymmetries

Hard

$$\mathcal{C}\left[H; w\, f\, S\, D\right] \equiv x_B H(Q^2,\mu^2,\rho) \sum_a e_a^2 \int d^2p_T\, d^2K_T\, d^2\ell_T\, \delta^{(2)}\left(zp_T + K_T + \ell_T - P_{h\perp}\right) w\left(p_T, -\frac{K_T}{z}\right)$$
$$\times f^a(x, p_T^2, \mu^2, x\zeta, \rho)\, \underline{S(\ell_T^2, \mu^2, \rho)}\, D^a(z, K_T^2, \mu^2, \hat{\zeta}/z, \rho)$$

TMD

Soft

FF

Collins Soper NPB 1981, Collins Metz PRL 2004, Ji, Ma, Yuan PRD 2005, also Bacchetta Boer Diehl Mulders JHEP 2008

Structure Function

$$F_{UT,T}^{\sin(\phi_h-\phi_S)} = \mathcal{C}\big[-(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T/M)\, f_{1T}^{\perp} D_1\big] \times (1+\mathcal{O}(\alpha_s))$$

Momentum conv. becomes

$$F_{UT,T}^{\sin(\phi_h-\phi_S)} = \mathcal{C}\big[H_{UT,T}^{\sin(\phi_h-\phi_S)}; -(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T/M)\; f_{1T}^{\perp}\; S^{+}\, D_1\big]$$

or,

$$F_{UT,T}^{\sin(\phi_h-\phi_S)} = x_B\, H_{UT,T}^{\sin(\phi_h-\phi_S)} \sum_a e_a^2 \times \int d^2p_T\, d^2K_T\, d^2l_T \delta^{(2)}\big(zp_T + K_T + l_T - P_{h\perp}\big) \times \frac{p_T \cos(\phi_h-\phi_s)}{M}\, f_{1T}^{\perp}{}^{a}(x, p_T^2)\, S(l_T^2)\, D^a(z, K_T^2)$$

Comments on Soft factor

- Collective effect of soft gluons not associated with distribution or fragmentation function-factorizes
- Considered to be universal in hard processes
  (Collins & Metz PRL 04, Ji, Ma, Yuan, PRD 05)
- At tree level (zeroth order α_s) unity
- Absent tree level pheno analyses of experimental data
  (e.g. Anselmino et al PRD 05 & 07, Efremov et al PRD 07)
- Potentially, results of analyses can be difficult to compare at different energies **issue for EIC**
- Correct description of energy scale dependence of cross section and asymmetries in TMD picture soft factor must be included (see Collins Oxford Press 2011, & Abyat & Rogers arXive: 1101.5057)
- However, possible to consider observables where it cancels e.g. weighted asymmetries

Weighted asymmetries

Disentangle in model independent way cross section in terms of moments of TMDs

Kotzinian, Mulders PLB 97, Boer, Mulders PRD 98

$$A_{XY}^{\mathcal{W}} = 2\frac{\int d|\boldsymbol{P}_{h\perp}|\,|\boldsymbol{P}_{h\perp}|\,d\phi_h\,d\phi_S\,\mathcal{W}(|\boldsymbol{P}_{h\perp}|,\phi_h,\phi_S)\,\left(d\sigma^{\uparrow}-d\sigma^{\downarrow}\right)}{\int d|\boldsymbol{P}_{h\perp}|\,|\boldsymbol{P}_{h\perp}|\,d\phi_h\,d\phi_S\,\left(d\sigma^{\uparrow}+d\sigma^{\downarrow}\right)}$$

e.g. $$\mathcal{W}_{\text{Sivers}} = \frac{|\boldsymbol{P}_{h\perp}|}{M}\sin(\phi_h-\phi_S)$$

$$A_{UT}^{\frac{|\boldsymbol{P}_{h\perp}|}{z_h M}\sin(\phi_h-\phi_s)} = -2\frac{\sum_a e_a^2\; f_{1T}^{\perp(1)}(x)\;D_1^{a(0)}(z)}{\sum_a e_a^2\; f_1^{a(0)}(x)\;D_1^{a(0)}(z)}$$

Undefined w/o regularization to subtract infinite contribution at large transverse momentum

Bacchetta et al. JHEP 08

Comments

- Propose generalize Bessel Weights-"BW"
- BW procedure has advantages
- Introduces a free parameter $\mathcal{B}_T\ [\mathrm{GeV}^{-1}]$ that is Fourier conjugate to $\boldsymbol{P}_{h\perp}$
- Provides a regularization of infinite contributions at lg. transverse momentum when $\mathcal{B}_T^2$ is non-zero for moments
- Addtnl. bonus soft factor eliminated from weighted asymmetries
- Possible to compare observables at different scales.... could be useful for an EIC

Advantages of Bessel Weighting

1. "Deconvolution"-SIDIS struct fnct simple products "$\mathcal{P}$"
2. Soft Factor Cancels
3. Circumvents the problem of ill-defined $\boldsymbol{p}_T$ moments
4. Bessel Weight asymmetris sensitive to low $\boldsymbol{P}_{h\perp}$ region

★ $w_1 = 2J_1(|\boldsymbol{P}_{h\perp}|\mathcal{B}_T)/zM\mathcal{B}_T$

$$A_{UT}^{\frac{2J_1(|\boldsymbol{P}_{h\perp}|\mathcal{B}_T)}{zM\mathcal{B}_T}\sin(\phi_h-\phi_s)} = -2\frac{\sum_a e_a^2\, \tilde{f}_{1T}^{\perp(1)a}(x, z^2\mathcal{B}_T^2)\, \tilde{D}_1^a(z, \mathcal{B}_T^2)}{\sum_a e_a^2\, \tilde{f}_1^a(x, z^2\mathcal{B}_T^2)\, \tilde{D}_1^a(z, \mathcal{B}_T^2)},$$

Where $\tilde{f}_1,\ \tilde{f}_{1T}^{\perp(1)}$, and $\tilde{D}_1$ are Fourier Transf. of TMDs/FFs and finite

1. “Deconvolution”-SIDIS structure functions simple products

1. “Deconvolution”-SIDIS structure functions simple products

a) F.T. SIDIS cross section w/ following defintions

$$\begin{aligned}
\tilde{f}(x,\boldsymbol{b}_T^2) &\equiv \int d^2\boldsymbol{p}_T\, e^{i\boldsymbol{b}_T\cdot\boldsymbol{p}_T}\; f(x,\boldsymbol{p}_T^2) \\
&= 2\pi \int d|\boldsymbol{p}_T||\boldsymbol{p}_T|\; J_0(|\boldsymbol{b}_T||\boldsymbol{p}_T|)\; f^a(x,\boldsymbol{p}_T^2)\ , \\
\tilde{f}^{(n)}(x,\boldsymbol{b}_T^2) &\equiv n!\left(-\frac{2}{M^2}\partial_{\boldsymbol{b}_T^2}\right)^n \tilde{f}(x,\boldsymbol{b}_T^2) \\
&= \frac{2\pi\ n!}{(M^2)^n}\int d|\boldsymbol{p}_T||\boldsymbol{p}_T|\left(\frac{|\boldsymbol{p}_T|}{|\boldsymbol{b}_T|}\right)^n J_n(|\boldsymbol{b}_T||\boldsymbol{p}_T|)\; f(x,\boldsymbol{p}_T^2)\ ,
\end{aligned}$$

b) n.b. connection to $\boldsymbol{p}_T$ moments

$$\tilde{f}^{(n)}(x,0) = \int d^2\boldsymbol{p}_T\left(\frac{\boldsymbol{p}_T^2}{2M^2}\right)^n f(x,\boldsymbol{p}_T^2) \equiv f^{(n)}(x)$$

Structure functions are "products" $\mathcal{P}$ vs. "convolutions" $\mathcal{C}$

$$\frac{d\sigma}{dx_B\, dy\, d\phi_S\, dz_h\, d\phi_h\, d|\boldsymbol{P}_{h\perp}|^2} \propto \frac{\alpha^2}{x_B Q^2} \int \frac{d|\boldsymbol{b}_T|}{(2\pi)} |\boldsymbol{b}_T|\, \tilde{\mathcal{S}}(\boldsymbol{b}_T^2) \Big\{ \quad \ldots$$

$$+ J_0(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{P}[\tilde{f}_1\ \tilde{D}_1]$$

$$+ |\boldsymbol{S}_\perp|\, \sin(\phi_h - \phi_S)\, J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{P}[\tilde{f}_{1T}^{\perp(1)}\ \tilde{D}_1]$$

$$+\varepsilon \cos(2\phi_h)\, J_2(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{P}[\tilde{h}_1^{\perp(1)}\ \tilde{H}_1^{\perp(1)}]$$

$$+ \quad \ldots 15 \quad \text{more} \quad \text{structure} \quad \text{functions}$$

Soft factor is

- spin blind
- flavor blind
- factors in $\mathcal{P}$

$\Big\}$ Ibildi,Ji,Ma,Yuan PRD 05

Products in terms of "$\boldsymbol{b}_T$ moments"

$$\mathcal{P}[\tilde{f}_{1T}^{\perp(1)}\, \tilde{D}^{(0)}] \quad \equiv \quad x_B\, (zM|\boldsymbol{b}_T|)$$

$$\times \sum_a e_a^2\, \tilde{f}_{1T}^{\perp a(1)}(x, z^2\boldsymbol{b}_T^2)\, \tilde{D}^a(z, \boldsymbol{b}_T^2)$$

Full Cross Section expansion in Bessel functions

$$
\begin{aligned}
\frac{d\sigma}{dx_B\, dy\, d\phi_S\, dz_h\, d\phi_h\, |\boldsymbol{P}_{h\perp}| d|\boldsymbol{P}_{h\perp}|} &= \frac{\alpha^2}{x_B y Q^2} \frac{y^2}{(1-\varepsilon)} \left(1 + \frac{\gamma^2}{2x_B}\right) \int \frac{d|\boldsymbol{b}_T|}{(2\pi)} |\boldsymbol{b}_T| \Bigg\{ \\
& + J_0(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{P}_{UU,T} + \varepsilon\, J_0(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{P}_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\, \cos\phi_h\, J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{P}_{UU}^{\cos\phi_h} \\
& + \varepsilon \cos(2\phi_h)\, J_2(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{P}_{UU}^{\cos(2\phi_h)} + \lambda_e \sqrt{2\,\varepsilon(1-\varepsilon)}\, \sin\phi_h\, J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{P}_{LU}^{\sin\phi_h} \\
& + S_\parallel \left[\sqrt{2\,\varepsilon(1+\varepsilon)}\, \sin\phi_h\, J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{P}_{UL}^{\sin\phi_h} + \varepsilon \sin(2\phi_h)\, J_2(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{P}_{UL}^{\sin 2\phi_h} \right] \\
& + S_\parallel \lambda_e \left[\sqrt{1-\varepsilon^2}\, J_0(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{P}_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\, \cos\phi_h\, J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{P}_{LL}^{\cos\phi_h} \right] \\
& + |\boldsymbol{S}_\perp| \Bigg[\sin(\phi_h - \phi_S)\, J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|) \left(\mathcal{P}_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\, \mathcal{P}_{UT,L}^{\sin(\phi_h-\phi_S)} \right) \\
& \quad + \varepsilon\, \sin(\phi_h + \phi_S)\, J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{P}_{UT}^{\sin(\phi_h+\phi_S)} + \varepsilon\, \sin(3\phi_h - \phi_S)\, J_3(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{P}_{UT}^{\sin(3\phi_h-\phi_S)} \\
& \quad + \sqrt{2\,\varepsilon(1+\varepsilon)}\, \sin\phi_S\, J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{P}_{UT}^{\sin\phi_S} + \sqrt{2\,\varepsilon(1+\varepsilon)}\, \sin(2\phi_h - \phi_S)\, J_2(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{P}_{UT}^{\sin(2\phi_h-\phi_S)} \Bigg] \\
& + |\boldsymbol{S}_\perp| \lambda_e \Bigg[\sqrt{1-\varepsilon^2}\, \cos(\phi_h - \phi_S)\, J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{P}_{LT}^{\cos(\phi_h-\phi_S)} + \sqrt{2\,\varepsilon(1-\varepsilon)}\, \cos\phi_S\, J_0(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{P}_{LT}^{\cos\phi_S} \\
& \quad + \sqrt{2\,\varepsilon(1-\varepsilon)}\, \cos(2\phi_h - \phi_S)\, J_2(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{P}_{LT}^{\cos(2\phi_h-\phi_S)} \Bigg] \Bigg\}
\end{aligned}
$$

truncates at J_3

2. Bessel Weighting & cancellation of soft factor

- Various strategies developed to take into account extra divergences that appear at 1 loop and beyond
- Requires introduction of variables that act as regularization scales--TMD evolution (PRE-DIS wkshp http://conferences.jlab.org/QCDEvolution/index.html
- Soft factor coming from gluon radiation can be absorbed in definition of TMDs or can appear in structure functions

Ji, Ma, Yuan PRD 05 , Collins 2011 Oxford Press, Abyet , Rogers

- With both definitions we show it cancels in weighted asymmetries Boer, LG, Musch, Prokudin (in preparation)

Hard

$$\mathcal{C}\left[H; w f S D\right] \equiv x_B H(Q^2, \mu^2, \rho) \sum_a e_a^2 \int d^2 p_T \, d^2 K_T \, d^2 \ell_T \, \delta^{(2)}(z p_T + K_T + \ell_T - P_{h\perp}) \, w\left(p_T, -\frac{K_T}{z}\right)$$

$$\times f^a(x, p_T^2, \mu^2, x\zeta, \rho) \, \underbrace{S(\ell_T^2, \mu^2, \rho)}_{\text{Soft}} \, D^a(z, K_T^2, \mu^2, \hat{\zeta}/z, \rho)$$

TMD

FF

2. Bessel Weighting & cancellation of soft factor

Bessel weighting-projecting out Sivers using orthogonality of Bessel Fncts.

$$\frac{\mathcal{J}_1^{\mathcal{B}_T}(|\boldsymbol{P}_{hT}|)}{zM} = \frac{2\,J_1(|\boldsymbol{P}_{hT}|\mathcal{B}_T)}{zM\mathcal{B}_T}$$

$$A_{UT}^{\frac{\mathcal{J}_1^{\mathcal{B}_T}(|\boldsymbol{P}_{hT}|)}{zM}\sin(\phi_h-\phi_S)}(\mathcal{B}_T) \quad =$$

$$2\,\frac{\int d|\boldsymbol{P}_{h\perp}|\,|\boldsymbol{P}_{h\perp}|\,d\phi_h\,d\phi_S\,\frac{\mathcal{J}_1^{\mathcal{B}_T}(|\boldsymbol{P}_{hT}|)}{zM}\sin(\phi_h-\phi_S)\,(d\sigma^{\uparrow}-d\sigma^{\downarrow})}{\int d|\boldsymbol{P}_{h\perp}|\,|\boldsymbol{P}_{h\perp}|\,d\phi_h\,d\phi_S\;\mathcal{J}_0^{\mathcal{B}_T}(|\boldsymbol{P}_{hT}|)\;(d\sigma^{\uparrow}+d\sigma^{\downarrow})}$$

$$A_{UT}^{\frac{\mathcal{J}_1^{\mathcal{B}_T}(|\boldsymbol{P}_{hT}|)}{zM}\sin(\phi_h-\phi_s)}(\mathcal{B}_T) \quad =$$

$$-2\,\frac{\cancel{\tilde{S}(\mathcal{B}_T^2)}\,H_{UT,T}^{\sin(\phi_h-\phi_S)}(Q^2)\,\sum_a e_a^2\,\tilde{f}_{1T}^{\perp(1)a}(x,z^2\mathcal{B}_T^2)\,\tilde{D}_1^a(z,\mathcal{B}_T^2)}{\cancel{\tilde{S}(\mathcal{B}_T^2)}\,H_{UU,T}(Q^2)\sum_a e_a^2\;\tilde{f}_1^a(x,z^2\mathcal{B}_T^2)\,\tilde{D}_1^a(z,\mathcal{B}_T^2)}$$

Sivers asymmetry with full dependences

$$A_{UT}^{\frac{\mathcal{J}_1^{\mathcal{B}_T}(|\boldsymbol{P}_{hT}|)}{zM}\sin(\phi_h-\phi_s)}(\mathcal{B}_T) \quad =$$

$$-2\,\frac{\tilde{S}(\mathcal{B}_T^2,\mu^2,\rho^2)\,H_{UT,T}^{\sin(\phi_h-\phi_S)}(Q^2,\mu^2,\rho)\,\sum_a e_a^2\,\tilde{f}_{1T}^{\perp(1)a}(x,z^2\mathcal{B}_T^2;\mu^2,\zeta,\rho)\,\tilde{D}_1^a(z,\mathcal{B}_T^2;\mu^2,\hat{\zeta},\rho)}{\tilde{S}(\mathcal{B}_T^2,\mu^2,\rho^2)\,H_{UU,T}(Q^2,\mu^2,\rho)\,\sum_a e_a^2\,\tilde{f}_1^a(x,z^2\mathcal{B}_T^2;\mu^2,\zeta,\rho)\,\tilde{D}_1^a(z,\mathcal{B}_T^2;\mu^2,\hat{\zeta},\rho)}$$

3. Circumvents the problem of ill-defined p_T moments

$$A_{UT}^{\frac{\mathcal{J}_1^{\mathcal{B}_T}(|\boldsymbol{P}_{hT}|)}{zM}\sin(\phi_h-\phi_s)}(\mathcal{B}_T) =$$

$$-2\frac{\tilde{S}(\mathcal{B}_T^2,\mu^2,\rho^2)H_{UT,T}^{\sin(\phi_h-\phi_S)}(Q^2,\mu^2,\rho)\sum_a e_a^2\,\tilde{f}_{1T}^{\perp(1)a}(x,z^2\mathcal{B}_T^2;\mu^2,\zeta,\rho)\,\tilde{D}_1^a(z,\mathcal{B}_T^2;\mu^2,\hat{\zeta},\rho)}{\tilde{S}(\mathcal{B}_T^2,\mu^2,\rho^2)H_{UU,T}(Q^2,\mu^2,\rho)\sum_a e_a^2\,\tilde{f}_1^a(x,z^2\mathcal{B}_T^2;\mu^2,\zeta,\rho)\,\tilde{D}_1^a(z,\mathcal{B}_T^2;\mu^2,\hat{\zeta},\rho)}$$

Traditional weighted asymmetry recovered but UV divergent

$$\lim_{\mathcal{B}_T\to 0} w_1 = 2J_1(|\boldsymbol{P}_{h\perp}|\mathcal{B}_T)/zM\mathcal{B}_T \longrightarrow |\boldsymbol{P}_{h\perp}|/zM$$

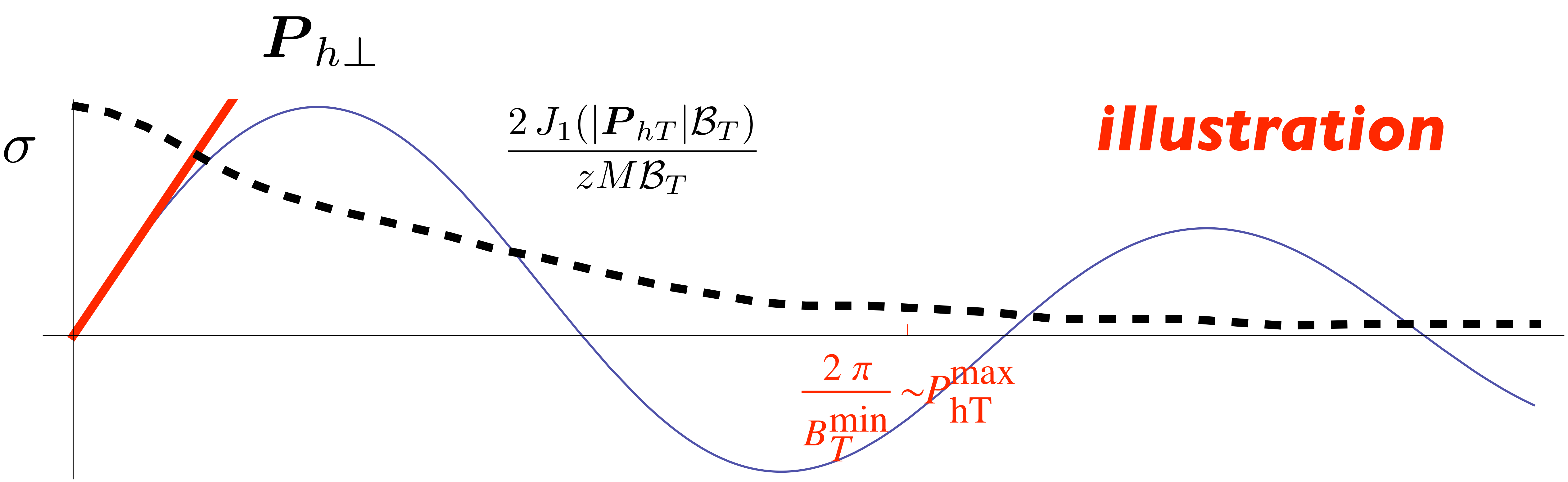
$$A_{UT}^{\frac{|\boldsymbol{P}_{h\perp}|}{z_h M}\sin(\phi_h-\phi_s)} = -2\frac{\sum_a e_a^2 f_{1T}^{\perp(1)}(x)\;D_1^{a(0)}(z)}{\sum_a e_a^2 f_1^{a(0)}(x)\;D_1^{a(0)}(z)}$$

undefined w/o regularization

Bacchetta et al. JHEP 08

4. More sensitive to low $\boldsymbol{P}_{h\perp}$ region

$\mathcal{B}_T$ can serve as a lever arm to enhance the low $\boldsymbol{P}_{h\perp}$ description and possibly dampen lg. momentum tail of cross section. We can use it to scan the cross section



Generalized av. quark trans. momentum shift
Soft Factor cancels

From talk of Berni Musch Pre-DIS wkshp.

$\langle \boldsymbol{p}_y \rangle_{TU}$:= average quark momentum in transverse y-direction measured in a proton polarized in transverse x-direction.

"dipole moment", "shift"

attention divergences from high-$\boldsymbol{p}_T$-tails!

$$\langle \boldsymbol{p}_y \rangle_{TU}(\mathcal{B}_T) \equiv M \frac{\int dx\, \tilde{f}_{1T}^{\perp(1)}(x, \mathcal{B}_T^2)}{\int dx\, \tilde{f}_1^{(0)}(x, \mathcal{B}_T^2)} = \frac{\cancel{\tilde{S}(\mathcal{B}_T^2, \ldots)}\, \tilde{A}_{12B}(\mathcal{B}_T^2, 0, 0, \tilde{\zeta}, \mu)}{\cancel{\tilde{S}(\mathcal{B}_T^2, \ldots)}\, \tilde{A}_{2B}(\mathcal{B}_T^2, 0, 0, \tilde{\zeta}, \mu)}$$

Conclusions

- Propose generalize Bessel Weights
- New theoretical weighting procedure w/ advantages
- Introduces a free parameter $\mathcal{B}_T \, [\mathrm{GeV}^{-1}]$ that is Fourier conjugate to $\boldsymbol{P}_{h\perp}$
- Provides a regularization of infinite contributions at lg. transverse momentum when $\mathcal{B}_T^2$ is non-zero
- Addtnl. bonus soft factor eliminated from weighted asymmetries
- Possible to compare observables at different scales.... could be useful for an EIC

EXTRA SLIDE

General Bessel Weighting

$$A_{XY}^{\mathcal{W}}(\mathcal{B}_T) = 2\,\frac{\int d|\boldsymbol{P}_{h\perp}|\,|\boldsymbol{P}_{h\perp}|\,d\phi_h\,d\phi_S\,\mathcal{W}(|\boldsymbol{P}_{h\perp}|,\phi_h,\phi_S,\mathcal{B}_T)\,\left(d\sigma^{\uparrow}-d\sigma^{\downarrow}\right)}{\int d|\boldsymbol{P}_{h\perp}|\,|\boldsymbol{P}_{h\perp}|\,d\phi_h\,d\phi_S\,J_0(|\boldsymbol{P}_{h\perp}|\mathcal{B}_T)\,(d\sigma^{\uparrow}+d\sigma^{\downarrow})}\,,$$

Old weights are asymptotic form of Bessel

$$|\boldsymbol{P}_{h\perp}|^n \to J_n(|\boldsymbol{P}_{h\perp}|\mathcal{B}_T)\,n!\left(\frac{2}{\mathcal{B}_T}\right)^n \equiv\ \mathcal{J}_n^{\mathcal{B}_T}(|\boldsymbol{P}_{h\perp}|)$$