Example calculation of fill-to-fill polarization uncertainties

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The fill-to-fill relative uncertainties account for the statistical fluctuations and the systematic fill-to-fill uncertainties in the beam polarization for the subset of the data that were used in this analysis. The beam polarizations and their statistical uncertainties are given fill-by-fill in the form of $P_0$, $\frac{dP}{dt}$, and the start time of the fill $t_0$. The relative fill-to-fill systematic uncertainty for a fill is normally negligible or small compared to the statistical fluctuation, and in this analysis it is negligible for the blue beam and 3.1% for the yellow beam.

In normal STAR analyses, beam polarizations are calculated for each run in a given fill where the luminosity is assumed to remain constant within a run. With the start and end times of the run, $t_{\text{start}}$ and $t_{\text{end}}$, the beam polarizations are taken as the average $P_{\text{run}} = P_0 + \frac{dP}{dt}(t_{\text{run}} - t_0)$, where $t_{\text{run}} = \frac{t_{\text{start}} + t_{\text{end}}}{2}$ and $P_0$, $\frac{dP}{dt}$, and $t_0$ are fill polarization parameters from the polarimeter group. The beam luminosity normally decreases noticeably during a fill for both beams. Therefore a luminosity averaged fill polarization is calculated:

$$P_{\text{fill}} = \frac{\sum_{\text{run}} L_{\text{run}} P_{\text{run}}}{L_{\text{fill}}}$$

$$= P_0 + \frac{dP}{dt} \cdot \left( \frac{\sum_{\text{run}} t_{\text{run}} L_{\text{run}}}{L_{\text{fill}}} - t_0 \right),$$

where $P_{\text{fill}}$ is the fill polarization, $L_{\text{run}}$ is the run luminosity, and $L_{\text{fill}} = \sum_{\text{run}} L_{\text{run}}$ is the total luminosity of the fill. The statistical uncertainty can be propagated through the statistical uncertainties on $P_0$ and $\frac{dP}{dt}$. A relative fill-to-fill systematic $\frac{\sigma_{(\text{fill-to-fill})}}{P}$ is then added in quadrature to provide the

\[1\text{The analysis discussed here is for STAR Run12 pp 500 GeV data.}\]
total uncertainty on the fill polarization:

\[
\sigma^2(P_{\text{fill}}) = \sigma^2(P_0) + \sigma^2\left(\frac{dP}{dt}\right) \cdot \left(\frac{\sum_{\text{run}} t_{\text{run}} L_{\text{run}}}{L_{\text{fill}}} - t_0\right)^2 + \left(\frac{\sigma(\text{fill-to-fill})}{P}\right)^2 \cdot P_{\text{fill}}^2.
\]  

(3)

The mean polarization for the data set \(P_{\text{set}}\) is the fill polarizations averaged with the weights of the fill luminosities:

\[
P_{\text{set}} = \frac{\sum_{\text{fill}} L_{\text{fill}} P_{\text{fill}}}{\sum_{\text{fill}} L_{\text{fill}}},
\]

(4)

and the fill systematic uncertainties are added in quadrature for the total fill-to-fill uncertainty in the analyzed data \(\sigma(P_{\text{set}})\):

\[
\sigma^2(P_{\text{set}}) = \frac{1}{(\sum_{\text{fill}} L_{\text{fill}})^2} \times \sum_{\text{fill}} L_{\text{fill}}^2 \sigma^2(P_{\text{fill}}).
\]

(5)

Since a fill-to-fill systematic uncertainty is already considered in the relative overall scale uncertainty, a correction factor of \(\sqrt{1 - \frac{M}{N}}\) for over-counting is applied to \(\sigma(P)\), where \(M\) is the number of fills analyzed and \(N\) is the number of fills that used in the overall scale uncertainty study. In this analysis\(^1\), \(M = 43\). For the entire 2012 \(pp\) 510 GeV run, \(N\) is 49 for both blue and yellow beams. For the double-spin measurement, the fill-to-fill uncertainty is calculated for both blue and yellow beam and final uncertainty is the square root of the quadrature sum. The blue and yellow beam relative fill-to-fill uncertainties are 0.4% and 0.4% respectively, and their total relative uncertainty is 0.6%.