Example calculation of fill-to-fill polarization uncertainties

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The fill-to-fill relative uncertainties account for the statistical fluctuations and the systematic fill-to-fill uncertainties in the beam polarization for the subset of the data that were used in this analysis[1]. The beam polarizations and their statistical uncertainties are given fill-by-fill in the form of P_0, $\frac{dP}{dt}$ and the start time of the fill t_0. The relative fill-to-fill systematic uncertainty for a fill is normally negligible or small compared to the statistical fluctuation, and in this analysis[1] it is negligible for the blue beam and 3.1% for the yellow beam.

In normal STAR analyses, beam polarizations are calculated for each run in a given fill where the luminosity is assumed to remain constant within a run. With the start and end times of the run, t_{start} and t_{end}, the beam polarizations are taken as the average $P_{run} = P_0 + \frac{dP}{dt}(t_{run} - t_0)$, where $t_{run} = \frac{t_{start}+t_{end}}{2}$ and P_0, $\frac{dP}{dt}$ and t_0 are fill polarization parameters from the polarimeter group. The beam luminosity normally decreases noticeably during a fill for both beams. Therefore a luminosity averaged fill polarization is calculated:

$$P_{fill} = \frac{\sum_{run} L_{run} P_{run}}{L_{fill}} \tag{1}$$

$$= P_0 + \frac{dP}{dt} \cdot \left(\frac{\sum_{run} t_{run} L_{run}}{L_{fill}} - t_0 \right) , \tag{2}$$

where P_{fill} is the fill polarization, L_{run} is the run luminosity, and $L_{fill} = \sum_{run} L_{run}$ is the total luminosity of the fill. The statistical uncertainty can be propagated through the statistical uncertainties on P_0 and $\frac{dP}{dt}$. A relative fill-to-fill systematic $\frac{\sigma(\text{fill-to-fill})}{P}$ is then added in quadrature to provide the

[1] The analysis discussed here is for STAR Run12 pp 500 GeV data.

total uncertainty on the fill polarization:

$$\sigma^2(P_{fill}) = \sigma^2(P_0) + \sigma^2(\frac{dP}{dt}) \cdot \left(\frac{\sum_{run} t_{run} L_{run}}{L_{fill}} - t_0\right)^2 + \left(\frac{\sigma(\text{fill-to-fill})}{P}\right)^2 \cdot P_{fill}^2 \,. \tag{3}$$

The mean polarization for the data set P_{set} is the fill polarizations averaged with the weights of the fill luminosities:

$$P_{set} = \frac{\sum_{fill} L_{fill} P_{fill}}{\sum_{fill} L_{fill}} \,, \tag{4}$$

and the fill systematic uncertainties are added in quadrature for the total fill-to-fill uncertainty in the analyzed data $\sigma(P_{set})$:

$$\sigma^2(P_{set}) = \frac{1}{(\sum_{fill} L_{fill})^2} \times \sum_{fill} L_{fill}^2 \sigma^2(P_{fill}) \,. \tag{5}$$

Since a fill-to-fill systematic uncertainty is already considered in the relative overall scale uncertainty, a correction factor of $\sqrt{1 - \frac{M}{N}}$ for over-counting is applied to $\sigma(P)$, where M is the number of fills analyzed and N is the number of fills that used in the overall scale uncertainty study. In this analysis[1], $M = 43$. For the entire 2012 pp 510 GeV run, N is 49 for both blue and yellow beams. For the double-spin measurement, the fill-to-fill uncertainty is calculated for both blue and yellow beam and final uncertainty is the the square root of the quadrature sum. The blue and yellow beam relative fill-to-fill uncertainties are 0.4% and 0.4% respectively, and their total relative uncertainty is 0.6%.