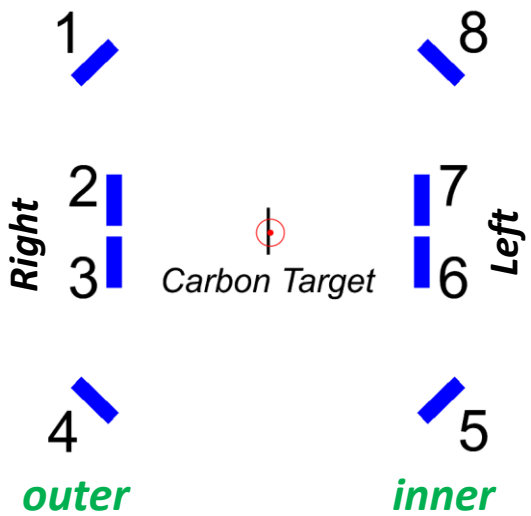
A. Poblaguev

Review of the Square Root Formula

AGS CNI Polarimeter 2012



4 different detector types:

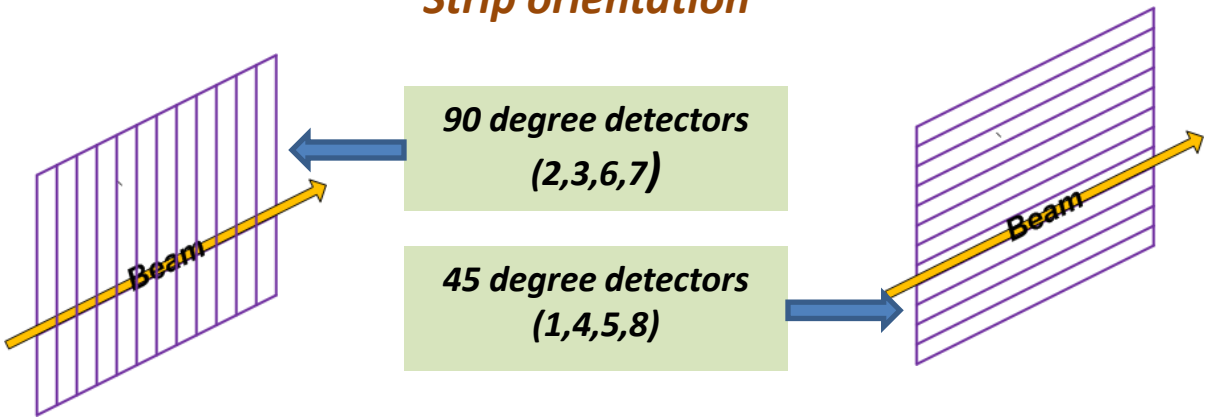
1,8 - Hamamatsu, slow preamplifiers, L = 51 cm

2,7 - BNL 2mm, fast preamplifiers, L = 30 cm

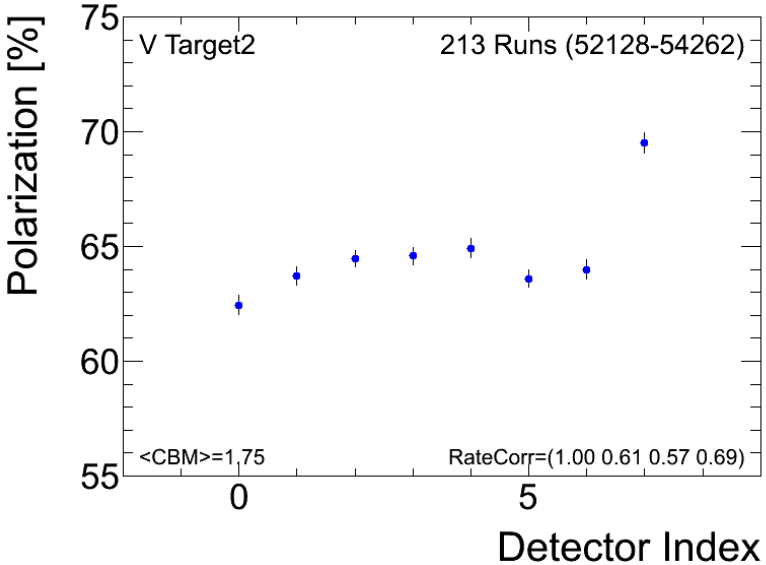
3,6 - BNL 1 mm, fast preamplifiers, L = 30 cm

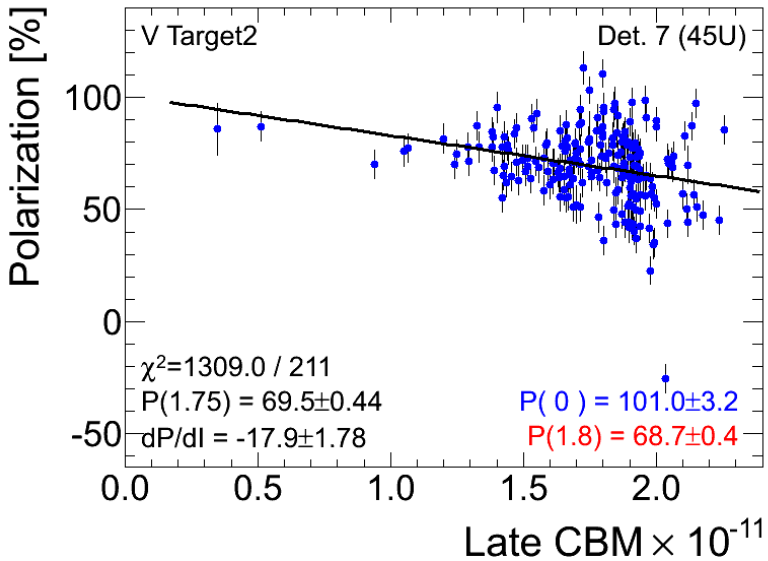
4,5 - Hamamatsu, fast preamplifiers, L = 51 cm

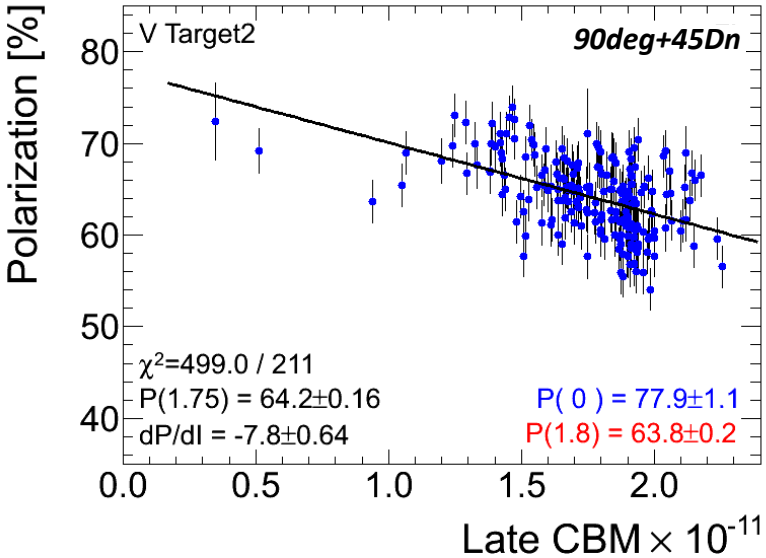
Strip orientation



2012 RHIC Reference Runs. Det. 7 puzzle.



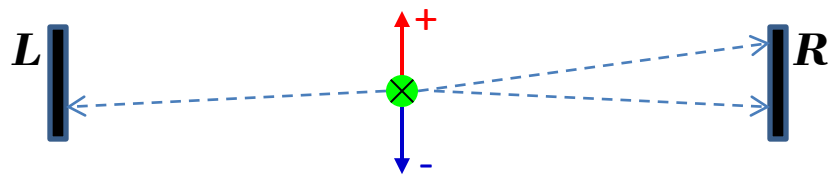




- ***Polarization measured in Det. 7 (inner, slow Hamamatsu) differs from the result in other detectors***
- ***Why can we trust to this result ? (square root formula is not applicable for a single detector measurement)***
- ***To understand this result we have to review what actually square root formula does.***

Square Root Formula

Systematic errors of the polarization measurement may be strongly suppressed if 2 left-right symmetric detectors are used and proton spin is flipping during the mesurements



$$\begin{cases} N_R^+ = N_0(1+a)(1+\epsilon)(1+\lambda) \\ N_R^- = N_0(1-a)(1+\epsilon)(1-\lambda) \\ N_L^+ = N_0(1-a)(1-\epsilon)(1+\lambda) \\ N_L^- = N_0(1+a)(1-\epsilon)(1-\lambda) \end{cases}$$

Exact solution

$$a = \frac{\sqrt{N_R^+ N_L^-} - \sqrt{N_L^+ N_R^-}}{\sqrt{N_R^+ N_L^-} + \sqrt{N_L^+ N_R^-}}$$

(Plus similar equations for the ε and λ)

a – polarization asymmetry
ε – acceptance asymmetry
λ – luminosity asymmetry

Square root formula is <u>absolutely</u> valid as long there is no correlation between ***a, ε,*** and ***λ.***
(and if we neglect statistical errors in measured number of events)

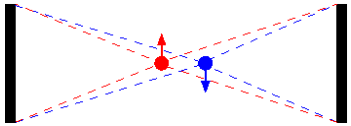
Possible correlation between asymmetries

$$\begin{cases} N_R^+ \propto (1+a+\delta_{LR}+\delta_\pm)(1+\epsilon+\delta_\epsilon)(1+\lambda) \\ N_R^- \propto (1-a-\delta_{LR}+\delta_\pm)(1+\epsilon-\delta_\epsilon)(1-\lambda) \\ N_L^+ \propto (1-a+\delta_{LR}-\delta_\pm)(1-\epsilon-\delta_\epsilon)(1+\lambda) \\ N_L^- \propto (1+a-\delta_{LR}-\delta_\pm)(1-\epsilon+\delta_\epsilon)(1-\lambda) \end{cases}$$

$$\begin{aligned} a_{\text{meas.}} &= a + \delta_\epsilon \\ \epsilon_{\text{meas.}} &= \epsilon + \delta_\pm - \cancel{a\delta_{LR}} \\ \lambda_{\text{meas.}} &= \lambda + \delta_{LR} - \cancel{a\delta_\pm} - \cancel{\epsilon\delta_\epsilon} \end{aligned}$$

Actually δ_ε, $\delta_\pm$, δ_{LR} are systematic errors in measurements of the *a*, *ε*, and *λ*, respectively

δ_ε - acceptance asymmetry dependence on beam polarity.
Possible source: a shift between averaged beam/target positions for positive and negative polarities.



δ_{LR} – left right asymmetry of the physics asymmetry (analyzing power)
due to inaccurate calibration etc.

$\delta_\pm$ - physics asymmetry (analyzing power) dependence on beam polarity.
(here added for completeness)

Since λ must be the same for all detectors, the monitoring of λ allows us to estimate possible variations of δ_{LR}

Calculation of the Analyzing Power

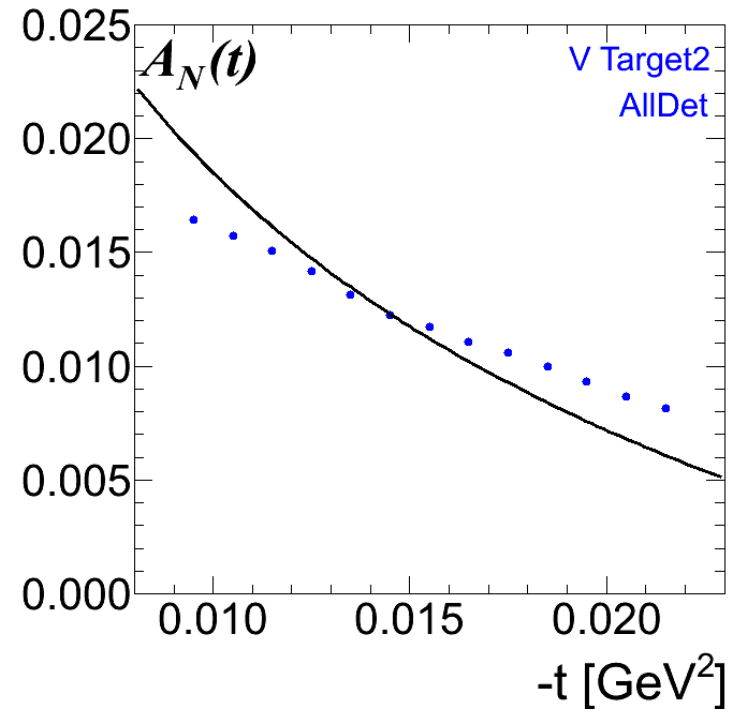
$$a_{\rm phy} = \langle A_N(t)\rangle P \sin\theta, \qquad t = -2M_c E_{\rm kin}$$

$$a_N = \langle A_N(t)\rangle = \frac{\int A_N(t)N(t)dt}{\int N(t)dt}$$

Since $N_\pm(t) = N_0(t)\times(1\pm A_N(t))$

The value of a_N is biased:

$$\frac{a_N^+ - a_N^-}{a_N^+ + a_N^-} = P\frac{\langle A_N^2\rangle - \langle A_N\rangle^2}{\langle A_N\rangle} \neq 0$$



$$\langle A_N(t)\rangle_{\rm unbias} = \frac{(1-\lambda)\int A_N(t)N_+(t)dt + (1+\lambda)\int A_N(t)N_-(t)dt}{(1-\lambda)\int N_+(t)dt + (1+\lambda)\int N_-(t)dt}$$

$$\approx \frac{\int A_N(t)N_+(t)dt + \int A_N(t)N_-(t)dt}{\int N_+(t)dt + \int N_-(t)dt}$$

We should use the average (over the beam polarity) value of the analyzing power .

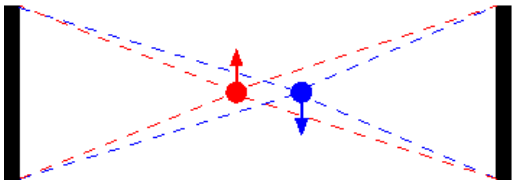
- There is a clear disagreement between theoretical *$A_N(t)$* and experimental results
- It may be caused both by wrong theory or by wrong measurements (e.g. wrong energy calibration, energy losses in target, etc)
- Substitution of the theoretical *$A_N(t)$* by the experimental one does not change, <u>qualitatively</u>, results of the polarization measurements.

δ_ε - acceptance asymmetry dependence on beam polarity

Vertical target:

x-coordinate is defined by the target position.

We may expect $\delta_\varepsilon \approx 0$.



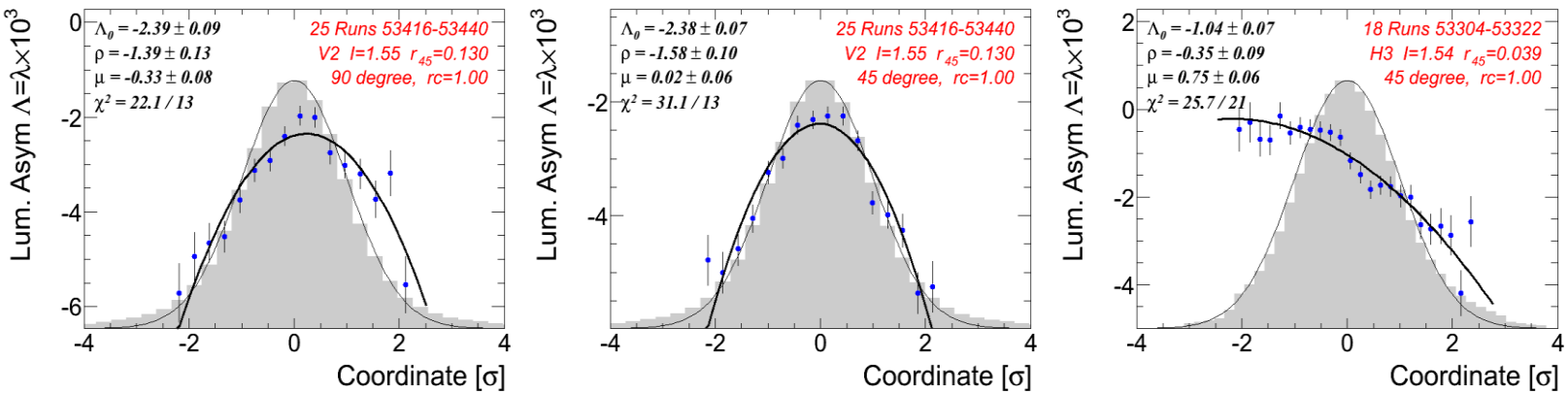
Horizontal Target:

x-coordinate is defined by beam position.

If $|x_+ - x_-| > 50\ \mu m$, a percent correction to the measured polarization is possible.

Horizontal target δ_ε might give an explanation of the discrepancy between vertical and horizontal target results.
This is not confirmed (?)

Luminosity asymmetry profile measurements allows one to evaluate δ_ε:



$$\langle x_+ \rangle - \langle x_- \rangle \approx 2\mu\sigma \lesssim 1\ \mu\mathrm{m}$$

$\delta_\pm$ - analyzing power dependence on beam polarity.

For a pair of Si detectors we can measure (determine) 4 averaged analyzing powers:

$$\begin{cases} a_R^+ = a + \delta_{LR} + \delta_\pm + \delta_n \\ a_R^- = a + \delta_{LR} - \delta_\pm - \delta_n \\ a_L^+ = a - \delta_{LR} + \delta_\pm - \delta_n \\ a_L^- = a - \delta_{LR} - \delta_\pm + \delta_n \end{cases}$$

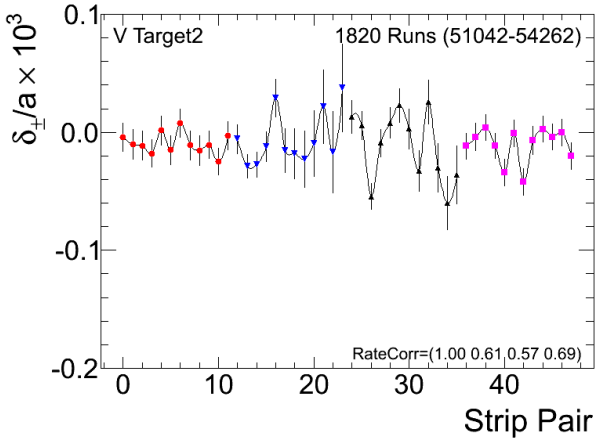
$$\begin{aligned} 4\delta_\pm &= a_R^+ - a_R^- + a_L^+ - a_L^- \\ 4\delta_n &= a_R^+ - a_R^- - a_L^+ + a_L^- \end{aligned}$$

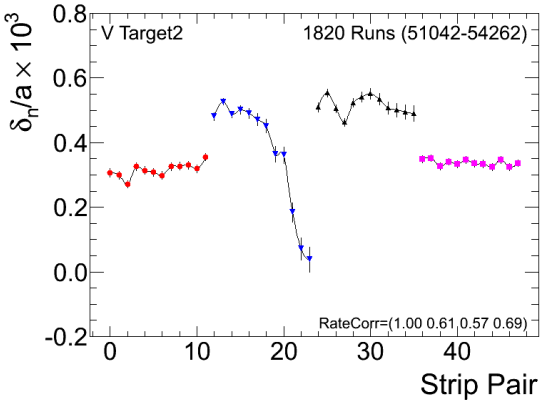
An estimate of the $\delta_\pm + \delta_n$:

$$a^\pm = \frac{\int A_N(t) N_\pm(t) dt}{\int N_\pm(t) dt}$$

$$N_\pm(t) = N_0(t) \times (1 \pm A_N(t) P)$$

$$\delta_\pm + \delta_n = \frac{a^+ - a^-}{a^+ + a^-} = P \frac{\langle A_N^2 \rangle - \langle A_N \rangle^2}{\langle A_N \rangle}$$

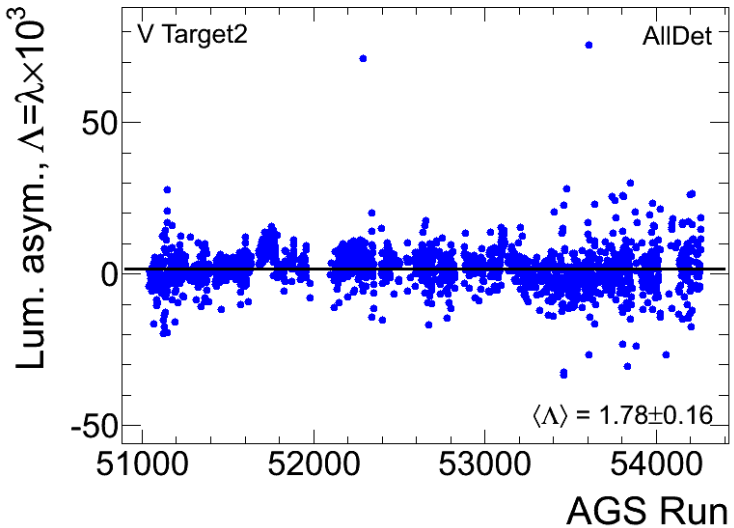


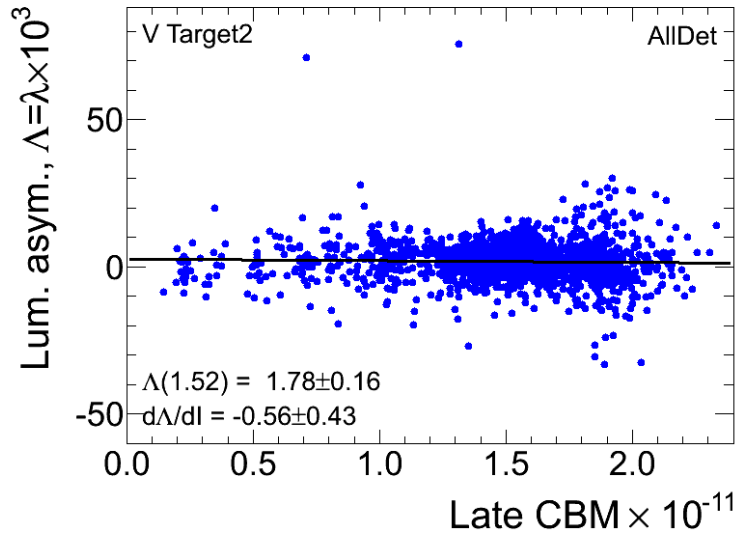


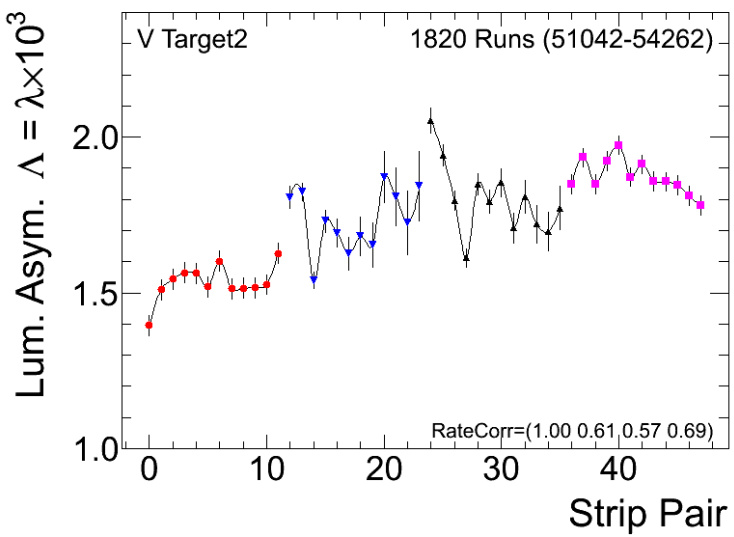
$|\delta_\pm/a| < 10^{-4}$ is very small and contributes only to the acceptance asymmetry. We may neglect this term.

$|\delta_n| < 10^{-5}$ is correction to normalization, e.g. $\delta_{LR} \rightarrow \delta_{LR}/(1+\delta_n)$ and, thus, may be neglected.

δ_{LR} – left right asymmetry of the analyzing power







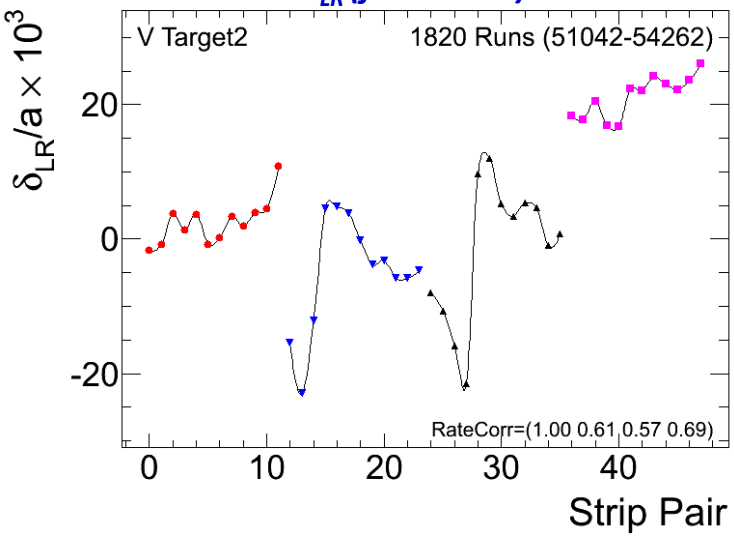
$$\lambda_{\mathrm{meas}} = \lambda + \delta_{\mathrm{LR}}$$

λ is the same for any strip pair.
We can evaluate δ_{LR}.

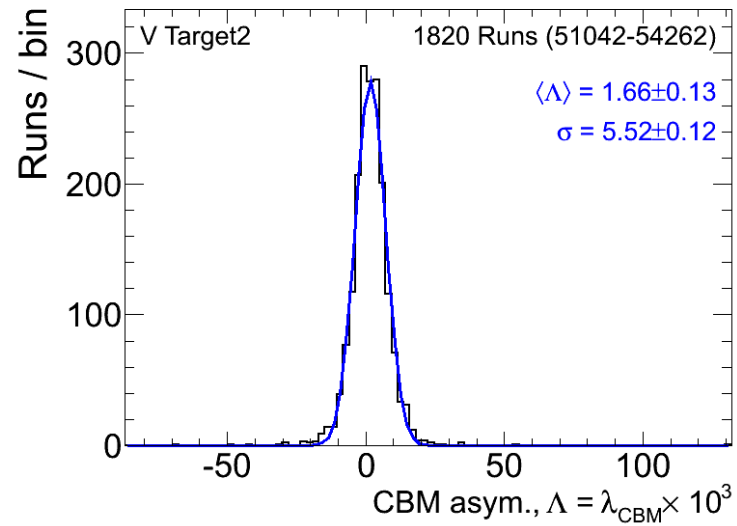
δ_{LR} should be compared with *a≈ 10^{-2}*

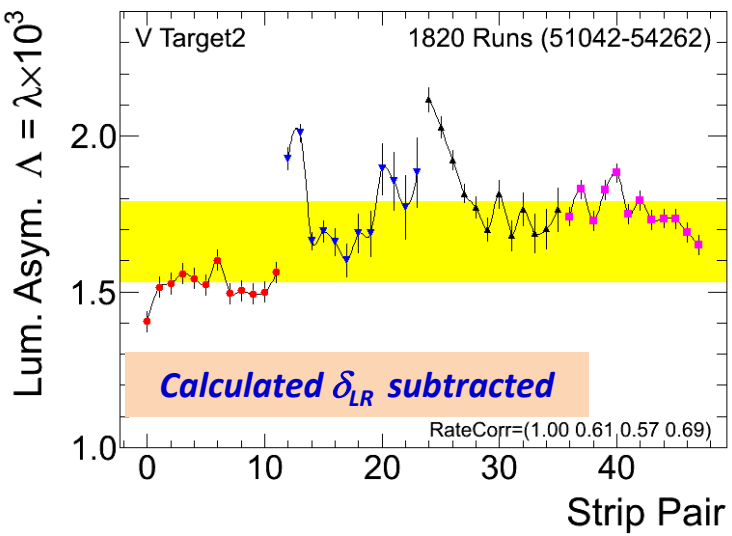
δ_{LR} – left right asymmetry of the analyzing power

Calculated δ_{LR} (from data)



Lum. asymmetry from the CBM.





- ***Variations of λ indicate uncontrolled errors in determination of the analyzing power.***
- ***There is a clear discrepancy between fast (36-47) and slow (0-11) 45 degree detectors.***
- ***CBM measurements do not allow us to resolve this discrepancy.***
- ***We observe a systematic luminosity asymmetry ~ few×10^{-3}***

Substitution of the square root formula for a general (non-symmetric) detector configuration

$$a_i = \langle A_N(t)\rangle_i \sin\theta_i$$

λ is the same for allstrips

G_i is detector acceptance

$$\begin{cases} N_i^+ = (1 + a_i P) G_i (1 + \lambda) \\ N_i^- = (1 - a_i P) G_i (1 - \lambda) \end{cases}$$

For ***n*** detectors, there are ***2n*** equations and ***n+2*** unknown values. To find ***P, λ, G_i*** we should minimize the

$$\chi^2 = \sum_{i,\pm} \frac{\left(N_i^\pm - (1 \pm a_i P) G_i (1 \pm \lambda)\right)^2}{N_i^\pm}$$

<u>Iteration method:</u>

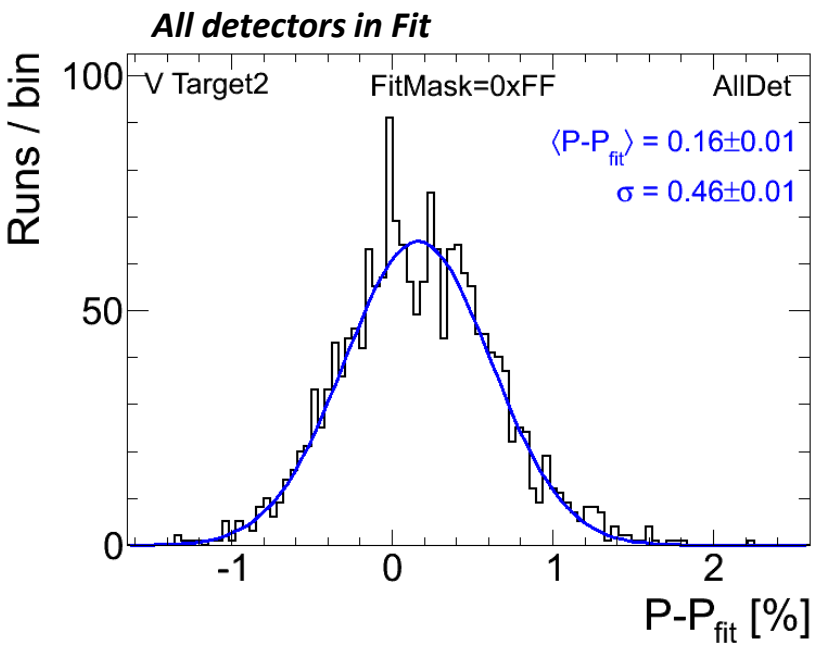
0. *P=0. λ=0*
1. $G_i = \sqrt{\frac{N_i^+ N_i^-}{(1 - a_i^2 P^2)(1 - \lambda^2)}} \approx \sqrt{N_i^+ N_i^-} \times (1 + \mathcal{O}(10^{-4}))$
2. Minimize χ^2 with fixed G_i. (Find P and λ)
3. Goto 1.

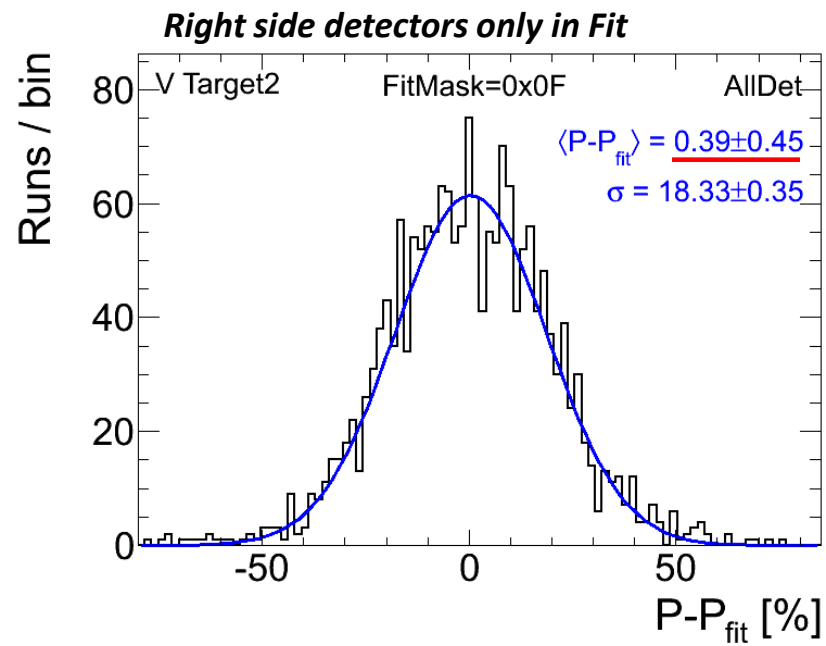
The method works well in the case of asymmetric detectors. For example, 90 degree detectors in one side and 45 degree in other side

We can determine λ using all detectors and then study every detector separately. Also, we can get λ from non-polarimeter data, e.g. from CBM.

The method might be helpful for the RHIC polarimeter !

Fit vs Square Root Formula





Statistical accuracy of the polarization measurement ≈2% (single run)

- For symmetric detectors, actual difference between the Fit and the Square Root Formula is accounting of calculated δ_{LR}, which does not affect polarization measurements.
- The Fit works well in asymmetric case.
- The Fit provide a better control on the results of measurements.
- Polarization angle φ and beam position (x,y) could be easily entered to the Fit.

An estimate of sigma for the right side only fit

$$\sigma_P/P \propto \frac{\sqrt{N}}{N(\sin\theta_R - \sin\theta_L)}$$

$$\sigma_{0x0F} \approx 2\% \frac{\sqrt{2}(\sin 90^\circ - \sin -90^\circ)}{\sin 90^\circ - \sin 45^\circ} \approx 19\%$$

Evaluation of the systematic errors in the Fit

$$\tilde{a}_i = a_i(1+\varepsilon_i)$$

$\tilde{a}, \tilde{P}, \tilde{\lambda}$ are true values of a, P, and λ

No acceptance asymmetry

$$\delta P = P_{\text{fit}} - \tilde{P} = \tilde{P}\frac{\sum_i N_i a_i^2 \varepsilon_i \sin^2\theta_i}{\sum_i N_i a_i^2 \sin^2\theta_i}$$

$$\delta\lambda = \lambda_{\text{fit}} - \tilde{\lambda} = \tilde{P}\frac{\sum_i N_i a_i \varepsilon_i \sin\theta_i}{\sum_i N_i}$$

$$\delta P_k = \tilde{P}\left(\varepsilon_k - \frac{\delta P}{\tilde{P}} - \frac{\delta\lambda}{\tilde{P} a_k \sin\theta_k}\right)$$

$$\delta a_k = -a_k\left(\varepsilon_k - \frac{\delta P}{\tilde{P}} - \frac{\delta\lambda}{\tilde{P} a_k \sin\theta_k}\right)$$

In the Fit we can evaluate values of εk up to 2 constaints:

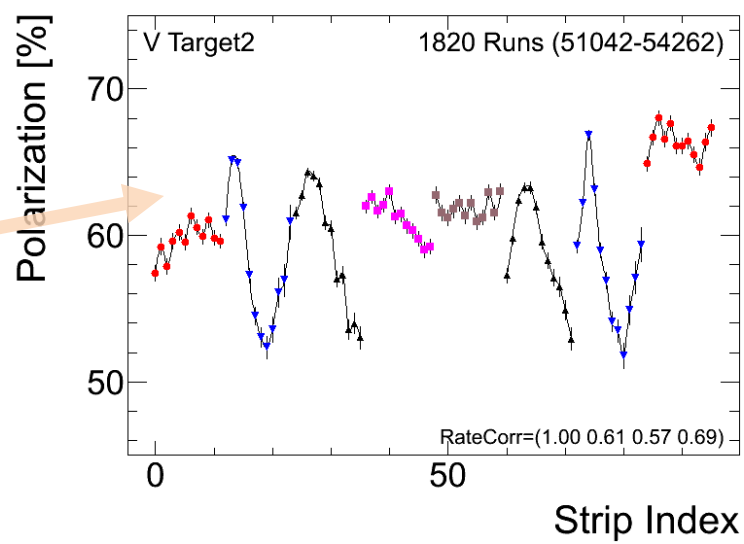
$$\varepsilon_i \to \varepsilon_i + \delta\varepsilon$$

$$\langle\varepsilon_{\text{Right Side}}\rangle = \langle\varepsilon_{\text{Left Side}}\rangle$$

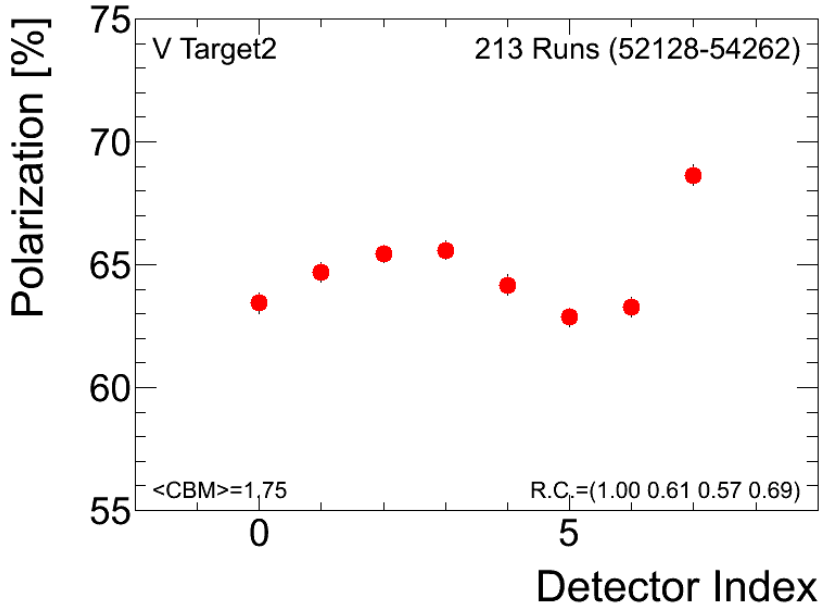
Only first constraint remains if luminosity asymmetry λ can be determined externally.

(Wall Current Monitor (CBM) measures beam intensity with accuracy about 1% (per spill) and, thus, can not serve for this purpose.

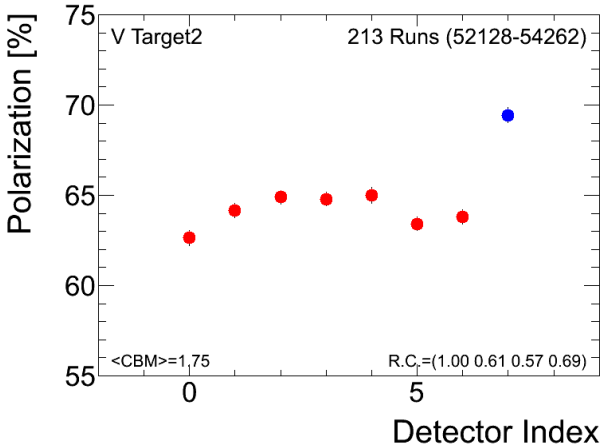
Here, luminosity asymmetry was evaluated using only BNL and fast Hamamatsu detectors. (Left/Right polarization for these detectors was equalizaed)

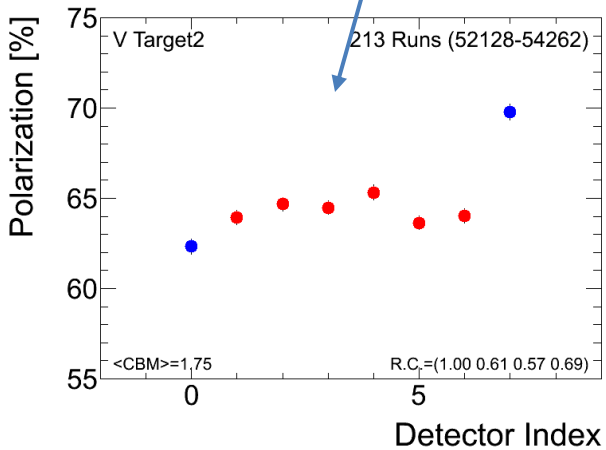


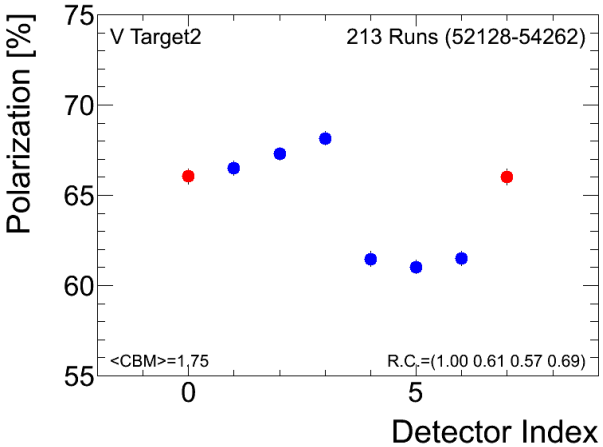
Searching for a bad detector.



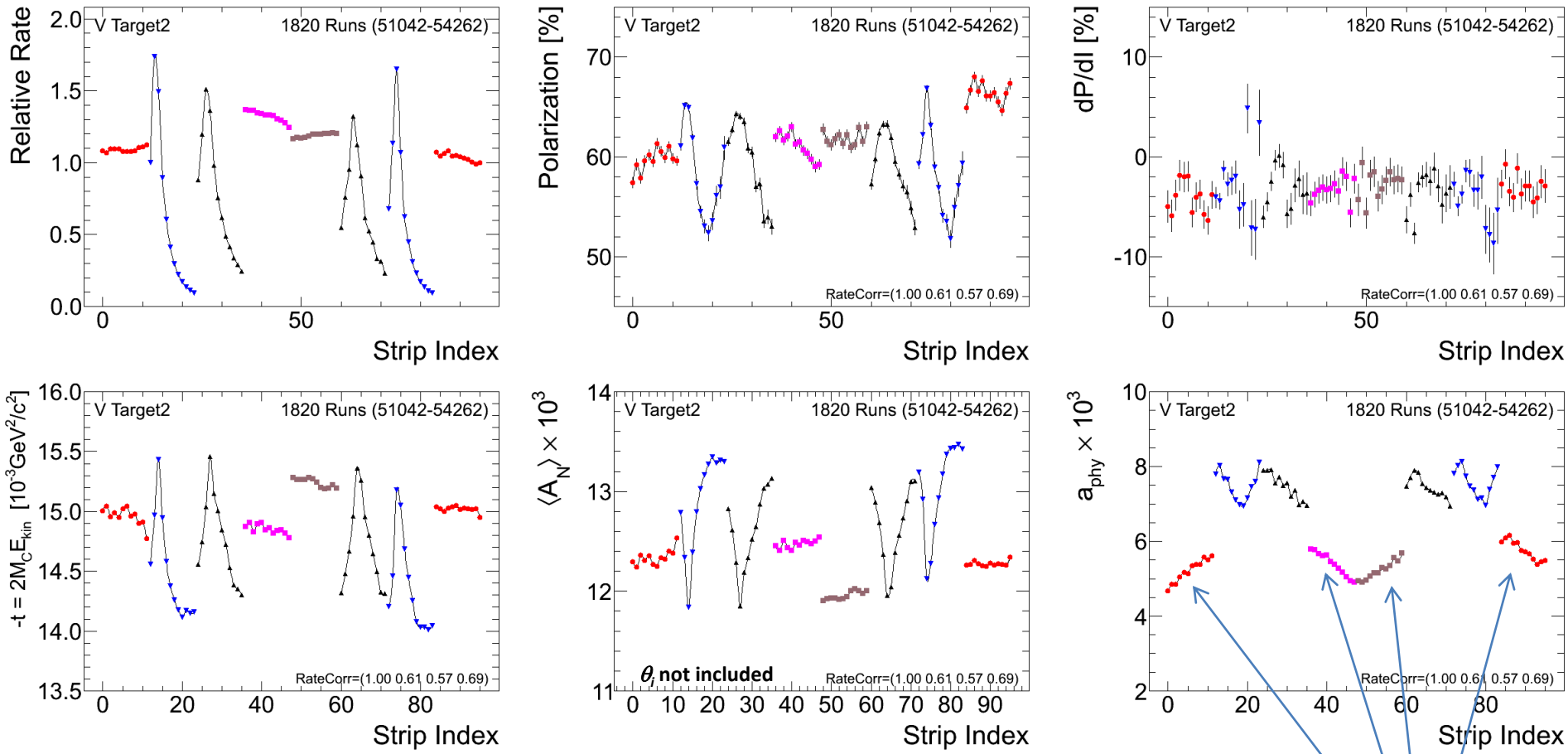
- ***Detectors used to calculate luminosity asymmetry λ are marked red.***
- ***Mathematically, neither picture has a preference.***
- ***Physically, we prefer to think that Det. 7 (and, may be, Det. 0) should be excluded.***







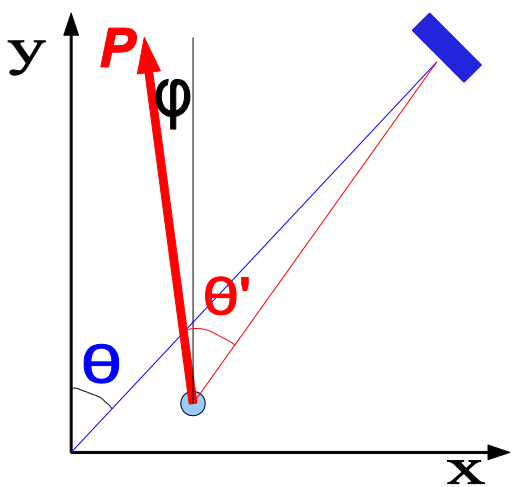
Strip Comparison. Run 2012.



- Slow Hamamatsu detectors are excluded from the determination of λ.
- No good understanding for 90 degree detectors.
  a_{phy} is not correlated with calculated A_N
- For fast Hamamatsu (45D): $A_N(L) \neq A_N(R)$ but $P(L)=P(R)$
- For slow Hamamatsu (45U): $A_N(L)=A_N(R)$ but $P(L) \neq P(R)$

Slopes due to the angle θ_i dependence on the strip number.

Beam related corrections to the analyzing power



Detector angle θ depends on beam displacement x, y and polarization angle φ, and, thus, effective analyzing power depends on x, y, and φ.

$$a_N \Rightarrow a_N^{(\mathrm{eff})} = a_N - x\frac{a_N \cos^2\theta}{L\sin\theta} + y\frac{a_N\cos\theta}{L} + \varphi\frac{a_N\cos\theta}{\sin\theta}$$

Generally, for a left/right and up/down symmetric set of 4 detectors analyzing power depends on the corrections as:

$$\begin{cases} a_{\mathrm{UR}} = a + \delta + \delta_x + \delta_y + \delta_\varphi \\ a_{\mathrm{UL}} = a + \delta - \delta_x + \delta_y - \delta_\varphi \\ a_{\mathrm{DR}} = a + \delta + \delta_x - \delta_y - \delta_\varphi \\ a_{\mathrm{DL}} = a + \delta - \delta_x - \delta_y + \delta_\varphi \end{cases} \Rightarrow \begin{aligned} \delta &= (a_{\mathrm{UR}} + a_{\mathrm{UL}} + a_{\mathrm{DR}} + a_{\mathrm{DL}})/4 - a \\ \delta_x &= (a_{\mathrm{UR}} - a_{\mathrm{UL}} + a_{\mathrm{DR}} - a_{\mathrm{DL}})/4 \\ \delta_y &= (a_{\mathrm{UR}} + a_{\mathrm{UL}} - a_{\mathrm{DR}} - a_{\mathrm{DL}})/4 \\ \delta_\varphi &= (a_{\mathrm{UR}} - a_{\mathrm{UL}} - a_{\mathrm{DR}} + a_{\mathrm{DL}})/4 \end{aligned}$$

<u>Errors in evaluation of the analyzing power may result in faked values of x, y, or φ:</u>

A "realistic" estimate (based on "Det. 7 anomaly"):

$$a_i = \begin{cases} a(1+\varepsilon), & \text{if } i \in \mathrm{UR}\ (45\ \mathrm{deg.}) \\ a, & \text{if } i \notin \mathrm{UR}\ (45\ \mathrm{deg.}) \end{cases}$$

$a = 0.013$, $\tilde{P} = 0.65$, and $\varepsilon = -0.1$

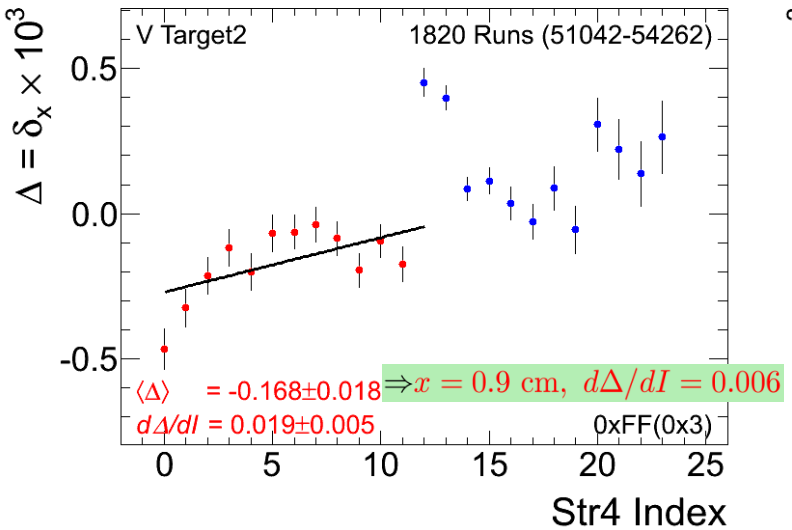
$N_{45\mathrm{deg}} \approx 2N_{90\mathrm{deg}}$

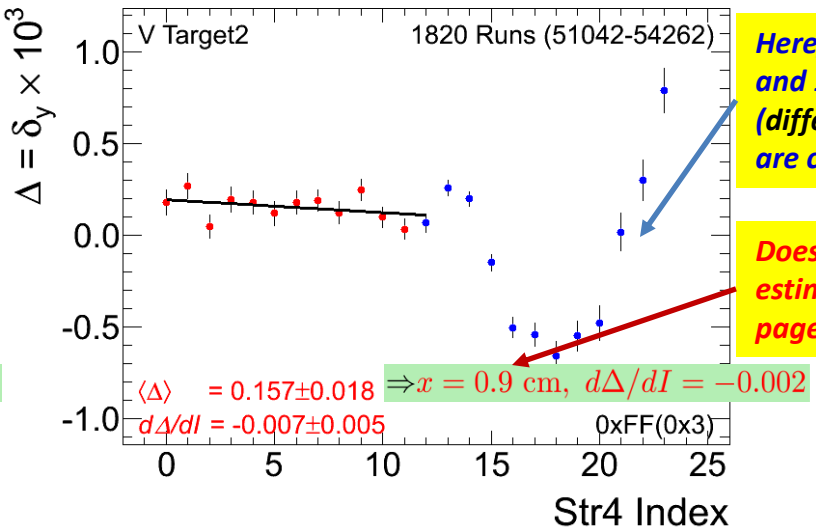
$$\begin{aligned} &\delta = \varepsilon/8, \quad \delta_x = \varepsilon/12, \quad \delta_y = \delta_\varphi = \varepsilon/4 \quad (45\ \text{degree}) \\ &\delta = -\varepsilon/8, \quad \delta_x = -\varepsilon/6, \quad \delta_y = \delta_\varphi = 0 \quad (90\ \text{degree}) \end{aligned}$$

$$\begin{aligned} \varphi &= \varepsilon/4 &&\approx -1.4^\circ \\ x &= -\frac{L\varepsilon}{6\sqrt{2}} &&\approx 0.7\ \mathrm{cm} \\ y &= \frac{L\varepsilon}{2\sqrt{2}} &&\approx -2.1\ \mathrm{cm} \end{aligned}$$

Systematic errors in the evaluation of beam position are about 1 cm and in measurement of polarization angle are about 1 deg.

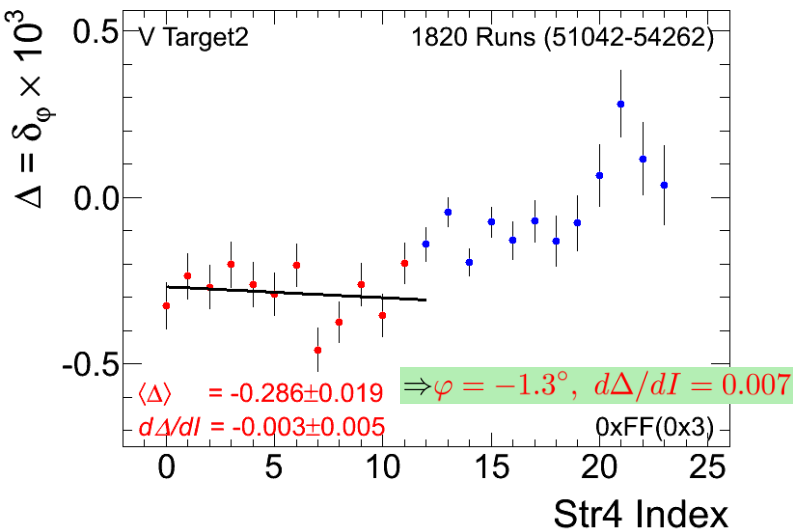
δ_x, δ_y, and δ_φ for the 4-strip sets.





Here, strips in 2 mm and 1 mm detectors (different z coordinate) are compared

Does not match estimate from previous page



- ***Red points for 45 deg. Detectors, blue for 90 deg. (naive expectation δ=0).***
- ***δ_y and δ_φ are insensitive to the δP and $\delta\lambda$.***
- ***For 90 deg., the strip angle and, thus, δ depends on the strip index (I), e.g.***

$$\delta_\varphi(I) = \tilde{\delta}_\varphi + a\varphi\left[1 - \frac{2d(I - 11/2)}{L}\right]$$

Potentially, $\tilde{\delta}_\varphi$ (systematic errors contribution) may be isolated.

δ_x and δ_φ are in a good agreement with an estimate from the "det. 7 anomaly"

Including the x, y, or φ will not improve accuracy of polarization measurements.
Inversely, an external knowledge of the x, y, φ may improve understanding of the systematic errors.

Rate corrections.

$$\begin{cases} N_R^+ = N_0(1+a)(1+\epsilon)(1+\lambda) \\ N_R^- = N_0(1-a)(1+\epsilon)(1-\lambda) \\ N_L^+ = N_0(1-a)(1-\epsilon)(1+\lambda) \\ N_L^- = N_0(1+a)(1-\epsilon)(1-\lambda) \end{cases}$$

a – polarization asymmetry
ε – acceptance asymmetry
λ – luminosity asymmetry

Rate Corrections

$$N_{\text{meas}} = N(a,\epsilon,\lambda) \times [1 - cN(a,\epsilon,\lambda)]$$

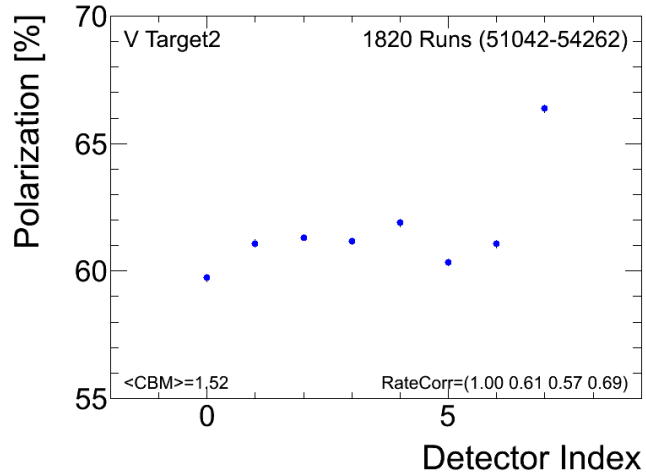
Equations are symmetri relative to the *a,ε,λ* and, thus, rate corrections are the same for the *a,ε,λ*.

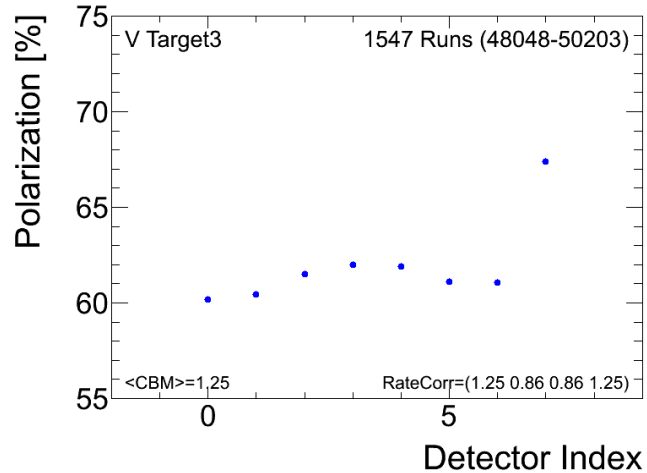
$$\begin{cases} a_{\text{meas}} = a(1-kr) \\ \epsilon_{\text{meas}} = \epsilon(1-kr) \\ \lambda_{\text{meas}} = \lambda(1-kr) \end{cases}$$

In this presentation, rate corrections are supposed to be properly applied

Single detector polarization. Run12 vs Run11.

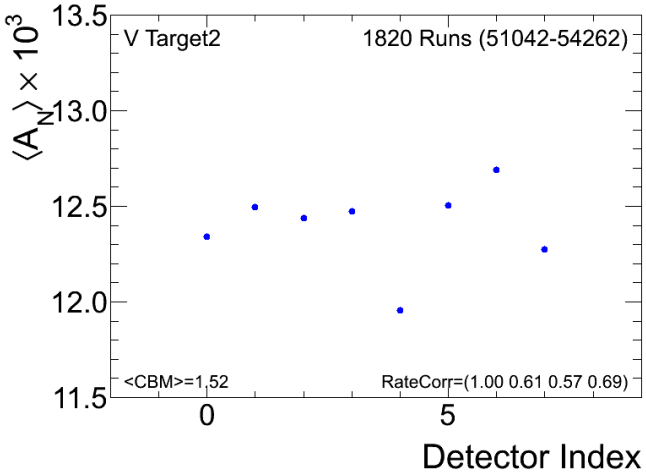
Run 2012

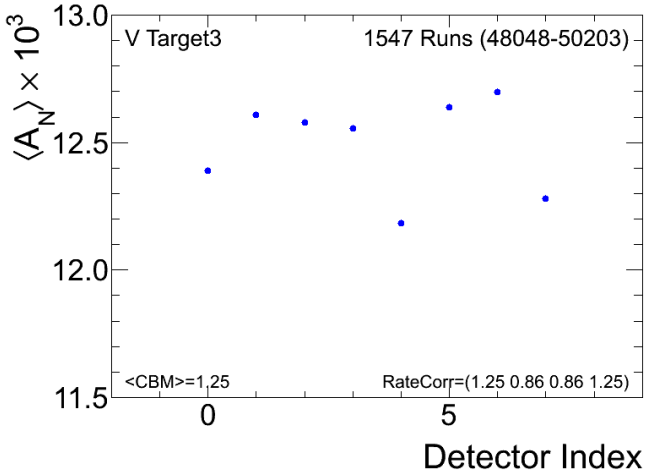




Run 2011

Run 2012



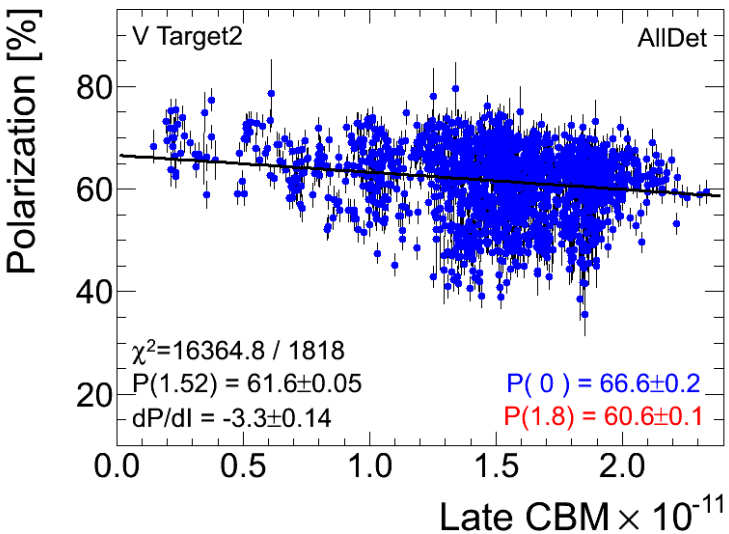


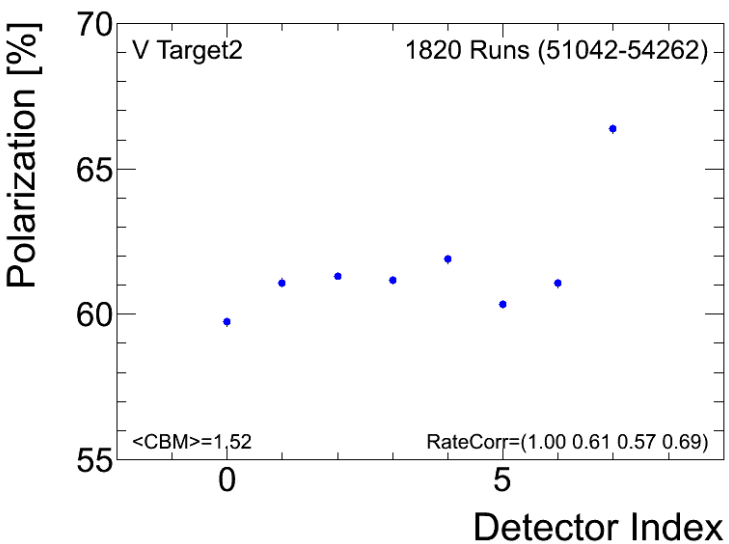
Run 2011

- ***The similar problem existed in Run 2011 (but there was no problem with dP/dI)***
- ***The effect can not be attributed to the error in measured value of analyzing power***
- ***Measured analyzing powers in the fast Hamamatsu detectors are different, but there is no difference in measured polarization.***

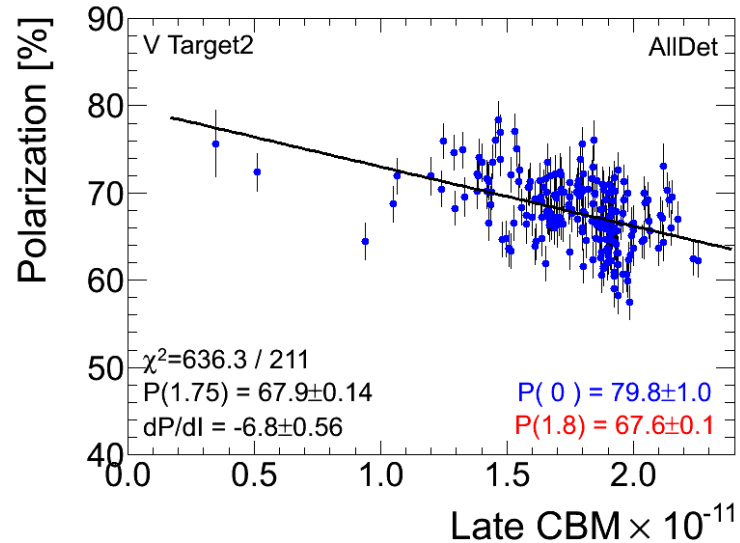
A more detailed look in Run12.

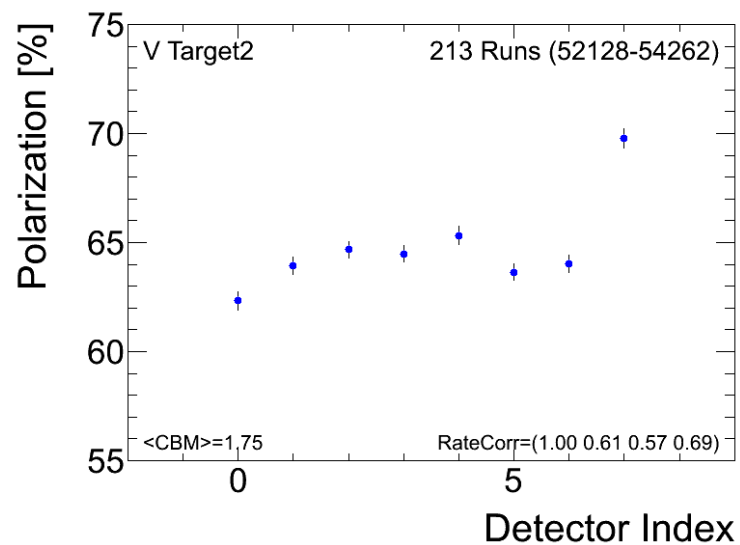
All V2 data





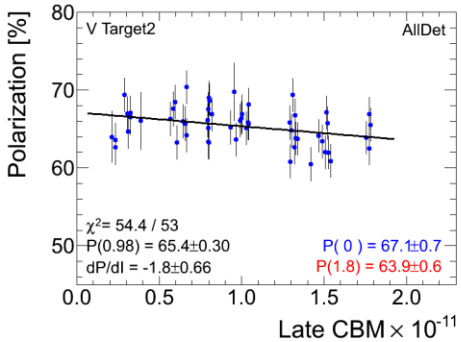
RHIC Ref. Runs



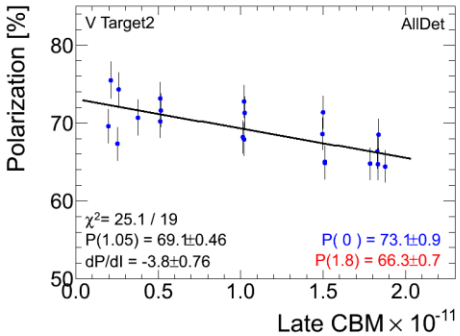


Intensity scans in Run 12

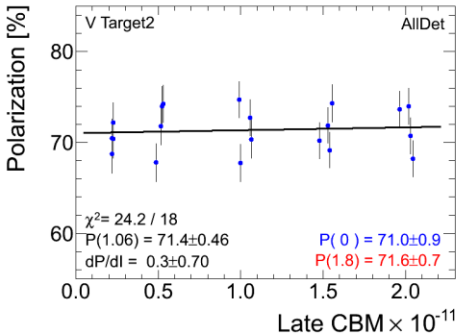
Runs 51556-51614



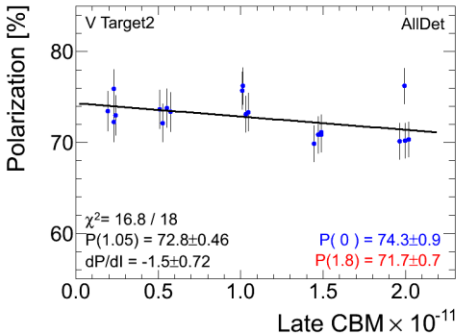
Runs 51939-51959

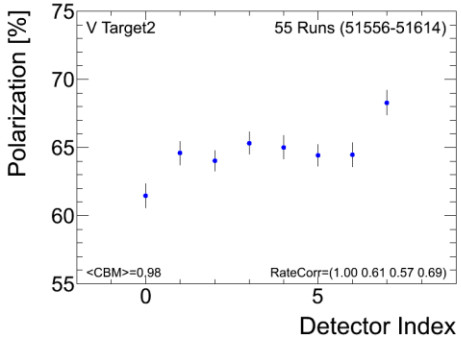


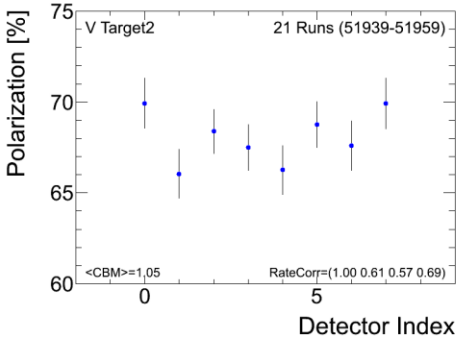
Runs 53235-53256

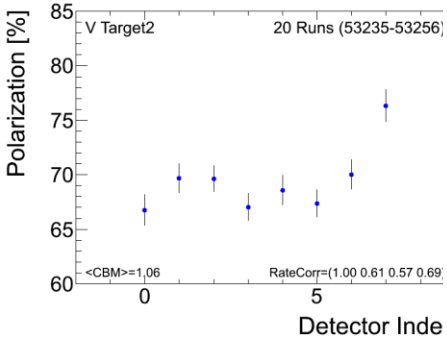


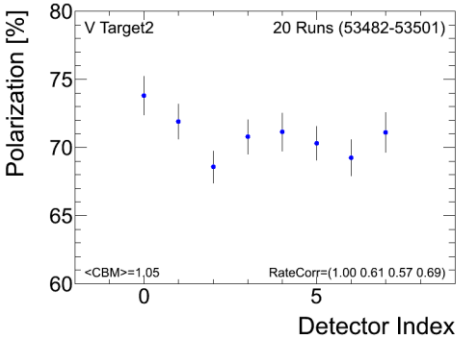
Runs 53482-53501











Sometimes detector 7 problem gone !?
Is it beam intensity dependent ?

Summary

- *Sources of the systematic errors in the "square root formula" were considered.*
- *Extension of the square root formula for the left/right asymmetric configuration of the Si detectors was discussed and tested.*
- *A systematic error of the polarization measurement is solely (?) defined by the average error ($<\delta a>$) in the calculation of the analysing power (a), e.g. $\delta P/P = -(\delta a_L+\delta a_R)/2a$.*
- *Luminosity asymmetry measurements allows us to estimate the ($\delta a_R-\delta a_L$) and, thus, get a hint about the value of $\delta P/P$.*
- *A big value of $\delta a/a \approx -10\%$ is found for the inner slow Hamamatsu detector (Det. 7).*
- *Precise knowledge of beam position and polarization angle may constraint systematic errors.*
- *There is no correlation between measured asymmetry and calculated analyzing power in 90 degree strips.*
- *There are many questions which still need to be answered.*