# Strategy for the Measurement of Vector Boson Asymmetry at RHIC

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## 1 Introduction

In this study we measure the asymmetry of the vector bosons produced in transversely polarized proton collisions. First, we focus on the $W$ boson and the lepton produced from its decay. Most of the developed formulae can be used in the measurement of $Z$ boson asymmetry, and we consider this case later. From the measured asymmetry we can also check the prediction about the sign change of the Sivers function in DY and SIDIS interactions:

$$f^{\mathrm{SIDIS}}_{q/h^{\uparrow}}(x, k_{\perp}) = -f^{\mathrm{DY}}_{q/h^{\uparrow}}(x, k_{\perp}). \tag{1}$$

The single spin asymmetry $A_N$ for the $W$ bosons and the lepton $l$ from the $W$ decay has been derived in **??**. It is parametrized based on the analyses of SIDIS data and given as a function of direction and transverse momentum. For the case of $W$ we have:

$$A^W_N = A^W_N(y_W, \phi_W, q_T) \equiv A_N(y, \phi, p_T) = A_N(\Omega, p_T), \tag{2}$$

where $\Omega = \{y, \phi\}$ is simply used as a shorthand for the direction of the particle in the lab frame. Similarly, for the lepton the expectated asymmetry depends on the direction of the lepton and its transverse momentum:

$$A^l_N = A^l_N(\eta_l, \phi_l, p_T) \equiv A_N(y, \phi, p_T) = A_N(\Omega, p_T) \tag{3}$$

## 2 Experimental Viewpoint

In the experiment we can separately measure full and differential cross sections for spin-up ($\sigma_{\uparrow}$), spin-down ($\sigma_{\downarrow}$), and unpolarized ($\sigma_0$) interactions. We are interested in the polarized

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cross sections which are given by:

$$\sigma_\uparrow = \sigma_0(1 + A_N \vec{P}_\uparrow \cdot \vec{n}), \tag{4}$$

$$\sigma_\downarrow = \sigma_0(1 - A_N \vec{P}_\downarrow \cdot \vec{n}). \tag{5}$$

*I am not sure polarization enters the expression for the cross section. It is possible that polarization appears only on the yield level...* In the following we assume that the polarization vector does not significantly deviate from the vertical direction given by the normal unit vector $\vec{n}$ along the vertical $y$ axis so, the notation is $P \equiv \vec{P} \cdot \vec{n}$. We also assume the same magnitude of the polarization vector for spin-up and spin-down bunches, *i.e.* $P = P_\uparrow = P_\downarrow$. For unpolarized cross section $\sigma_0 \equiv (\sigma_\uparrow + \sigma_\downarrow)/2$ the asymmetry $A_N$ is expressed as:

$$A_N = \frac{1}{P} \frac{\sigma_\uparrow - \sigma_\downarrow}{\sigma_\uparrow + \sigma_\downarrow} \tag{6}$$

The number of recorded events in which the particle is produced with momentum $p_T$ at angle $\Omega$ is:

$$\frac{dN_{\uparrow/\downarrow}}{d\Omega dp_T}(\Omega, p_T) = \mathcal{L}_{\uparrow/\downarrow} \frac{d\sigma_0}{d\Omega dp_T}(\Omega, p_T)\varepsilon(\Omega, p_T)\big(1 \pm A_N(\Omega, p_T)P\big), \tag{7}$$

where detection efficiency $\varepsilon$ does not depend on the spin direction of the interacting proton. In fact, every individual event can be tagged by the nominal spin of colliding protons. We thus can bin all collected data in four bins $N_{\uparrow\uparrow}$, $N_{\uparrow\downarrow}$, $N_{\downarrow\uparrow}$, and $N_{\downarrow\downarrow}$. For the single spin asymmetry the polarization of one of the beams is ignored by combining the yields with opposite spins, *e.g.*

$$N_\uparrow \equiv N_{\uparrow 0} = N_{\uparrow\uparrow} + R_{\frac{0\uparrow}{0\downarrow}} N_{\uparrow\downarrow}, \tag{8}$$

$$N_\downarrow \equiv N_{\downarrow 0} = N_{\downarrow\uparrow} + R_{\frac{0\uparrow}{0\downarrow}} N_{\downarrow\downarrow}, \tag{9}$$

where re-weighting factor $R_{\frac{0\uparrow}{0\downarrow}}$ addresses a possible relative difference in the spin-up and spin-down intensities of the other beam. Studies have shown that $R_{\frac{0\uparrow}{0\downarrow}} \approx 1$ with good precision.

We bin our data sample in three observable variables $\{y, \phi, p_T\}$ with center and width of the $i$-th bin being $\{y_i, \phi_i, p_{T,i}\}$ and $\{\Delta y_i, \Delta\phi_i, \Delta p_{T,i}\} \equiv \{\Delta\Omega_i\{y_i, \Delta\phi_i\}, \Delta p_{T,i}\} \equiv \Delta_i$ respectively. The number of events in each bin, $N_i$, is calculated by integrating both sides of (7) within the bin:

$$N_{\uparrow/\downarrow,i} = \int\limits_{\Delta_i} \frac{dN_{\uparrow/\downarrow}}{d\Omega dp_T} d\Omega dp_T. \tag{10}$$

In that bin we assume the average value:

$$A_{N,i} = \frac{1}{\Delta_i} \int\limits_{\Delta_i} A_N d\Omega dp_T, \tag{11}$$

and similarly for the cross section ($\sigma_{0,i}$) and efficiency ($\varepsilon_i$). Finally, for the yields in each bin we can write:

$$N_{\uparrow/\downarrow,i} = \mathcal{L}_{\uparrow/\downarrow}\sigma_{0,i}\varepsilon_i\Delta\Omega_i\Delta p_{T,i}\left(1 \pm A_{N,i}(\Omega, p_T)P\right) \tag{12}$$

The spacial distributions of the physical asymmetry and the cross sections are the same for the spin-up and spin-down interactions with respect to the spin direction. We can use this fact to easily get rid of the quantities of no interest in (12). This is achieved by constructing geometric means $\sqrt{N_\uparrow(\phi_i)N_\downarrow(\phi_i+\pi)}$ and $\sqrt{N_\uparrow(\phi_i+\pi)N_\downarrow(\phi_i)}$ of the yields

$$N_\uparrow(\phi_i) = \mathcal{L}_\uparrow\sigma_0(\phi_i)\varepsilon(\phi_i)\Delta\Omega_i\Delta p_T\left(1 + A_N(\phi_i)P\right) \tag{13}$$
$$N_\uparrow(\phi_i+\pi) = \mathcal{L}_\uparrow\sigma_0(\phi_i+\pi)\varepsilon(\phi_i+\pi)\Delta\Omega_i\Delta p_T\left(1 + A_N(\phi_i+\pi)P\right) \tag{14}$$

$$N_\downarrow(\phi_i+\pi) = \mathcal{L}_\downarrow\sigma_0(\phi_i+\pi)\varepsilon(\phi_i+\pi)\Delta\Omega_i\Delta p_T\left(1 - A_N(\phi_i+\pi)P\right) \tag{15}$$
$$N_\downarrow(\phi_i) = \mathcal{L}_\downarrow\sigma_0(\phi_i)\varepsilon(\phi_i)\Delta\Omega_i\Delta p_T\left(1 - A_N(\phi_i)P\right) \tag{16}$$

Using the relations for the asymmetry and cross section $A_N(\phi_i+\pi) = -A_N(\phi_i)$, $\sigma_0(\phi_i+\pi) = \sigma_0(\phi_i)$ we get for $A_N$

$$A_{N,i} = \frac{1}{P}\frac{\sqrt{N_\uparrow(\phi_i)N_\downarrow(\phi_i+\pi)} - \sqrt{N_\uparrow(\phi_i+\pi)N_\downarrow(\phi_i)}}{\sqrt{N_\uparrow(\phi_i)N_\downarrow(\phi_i+\pi)} + \sqrt{N_\uparrow(\phi_i+\pi)N_\downarrow(\phi_i)}} \tag{17}$$

# 3 Correction for Background

In this analysis an optimal set of cuts is applied to select signal enriched events without significant loss in the final statistics. The final yiels include some fraction of background events $f_B$ which affects the measured asymmetry $A_N$. In order to extract the signal asymmetry we decompose $A_N$ as following:

$$A_N = f_{\text{sig}}A_N^{\text{sig}} + f_B A_N^B, \tag{18}$$

with $f_{\text{sig}} = 1 - f_B$. The last term in (18) may include contributions from various backgrounds which will be discussed later. The background fractions and asymmetries have to be estimated in order to extract the final asymmetry of the signal:

$$A_N^{\text{sig}} = \frac{A_N + f_B A_N^B}{1 - f_B} \tag{19}$$

# 4 Sivers Sign Change Extraction

A binned likelihood method can be used to check the sensitivity of our data to the sign of the Sivers function. A direct way of doing this is to compare the measured asymmetry (17) with background corrected expectations from (18). The signal asymmetry $A_N^{\text{sig}}$ in this case directly comes from the model predictions (2) or (3). The simplest likelihood function can be constructed as a product of gaussian terms over all bins:

$$L = \prod_i G(A_{N,i}, \sigma_{A_{N,i}}; A_{N,i}^{\text{sig}}). \tag{20}$$

Alternatively, the Sivers sign can be extracted from the Poisson probabilities of measured given the expected yields.

$$L = \prod_{i,\uparrow,\downarrow} P(N_i; N_i^{\text{sig}} + B_i). \tag{21}$$

While this method is more "classic" it requires the explicit knowledge of luminosity, unpolarized cross section, and efficiencies. These values are needed to calculate the expected number of events using (12). The two methods are expected to give consistent results. However, the difference should be more perceptible through the addition of systematic effects.