

# Measurement of Vector Boson Asymmetry in Transversely Polarized $pp$ Collisions at RHIC

Salvatore Fazio\* and Dmitri Smirnov†

Physics Department, Brookhaven National Lab

August 28, 2013

---

\*fazio@bnl.gov

†d.s@plexoos.com

# Contents

1	Introduction	3
2	Experimental Viewpoint	3
3	Correction for Background	5
4	Sivers Sign Change Extraction	5
5	Reconstruction of $W$ Kinematics	5
6	Preliminary Sensitivity Studies	6

# 1 Introduction

In this study we propose to measure the asymmetry of the vector bosons produced in transversely polarized proton collisions at STAR. First, we focus on the  $W$  bosons decayed into a lepton pair ( $W^\pm \rightarrow e^\pm \nu_e$ ). However, most of the developed formulae can be used in the measurement of  $Z$  boson asymmetry, and we will consider this case later. From the measured asymmetry it is possible to verify theoretical expectations about the sign change of the Sivers function in Drell-Yan and SIDIS interactions:

$$f_{q/h^\uparrow}^{\text{SIDIS}}(x, k_\perp) = -f_{q/h^\uparrow}^{\text{DY}}(x, k_\perp). \quad (1)$$

The single spin asymmetry (SSA)  $A_N$  for the  $W$  bosons and the lepton  $l$  from the  $W$  decay has been derived in [2, ?]. It is parametrized based on the fits of SIDIS data and given as a function of direction and transverse momentum. For the case of  $W$  we have:

$$A_N^W = A_N^W(y_W, \phi_W, q_T) \equiv A_N(y, \phi, p_T) = A_N(\Omega, p_T), \quad (2)$$

where  $\Omega = \{y, \phi\}$  is simply used as a shorthand for the direction of the particle in the lab frame. Similarly, for the lepton the expected asymmetry depends on the direction of the lepton and its transverse momentum:

$$A_N^l = A_N^l(\eta_l, \phi_l, p_T) \equiv A_N(y, \phi, p_T) = A_N(\Omega, p_T) \quad (3)$$

## 2 Experimental Viewpoint

For the SSA measurements we are interested in the proton interactions  $p^{\uparrow/\downarrow} p \rightarrow W^\pm \rightarrow e^\pm \nu_e$  in which the spin direction of one of the protons is irrelevant, *i.e.* unpolarized protons. In the experiment we can separately measure full and differential cross sections for spin-up ( $\sigma_\uparrow$ ), spin-down ( $\sigma_\downarrow$ ), and unpolarized ( $\sigma_0$ ) interactions which are related as:

$$\sigma_\uparrow = \sigma_0(1 + A_N), \quad (4)$$

$$\sigma_\downarrow = \sigma_0(1 - A_N). \quad (5)$$

In the following we assume that the polarization vector does not significantly deviate from the vertical direction given by the normal unit vector  $\vec{n}$  along the vertical  $y$  axis so, the notation is  $P \equiv \vec{P} \cdot \vec{n}$ . We also assume the same magnitude of the polarization vector for spin-up and spin-down bunches, *i.e.*  $P = P_\uparrow = P_\downarrow$ . For unpolarized cross section  $\sigma_0 \equiv (\sigma_\uparrow + \sigma_\downarrow)/2$  the asymmetry  $A_N$  is expressed as:

$$A_N = \frac{\sigma_\uparrow - \sigma_\downarrow}{\sigma_\uparrow + \sigma_\downarrow}. \quad (6)$$

The number of recorded events in which the particle is produced with momentum  $p_T$  at angle  $\Omega$  is:

$$\frac{dN_{\uparrow/\downarrow}}{d\Omega dp_T}(\Omega, p_T) = \mathcal{L}_{\uparrow/\downarrow} \frac{d\sigma_0}{d\Omega dp_T}(\Omega, p_T) \varepsilon(\Omega, p_T) (1 \pm A_N(\Omega, p_T) P), \quad (7)$$

1 where detection efficiency  $\varepsilon$  does not depend on the spin direction of the interacting proton. In  
 2 fact, individual events can be tagged by the nominal spin of colliding protons. We thus can bin  
 3 all collected data in four bins  $N_{\uparrow\uparrow}$ ,  $N_{\uparrow\downarrow}$ ,  $N_{\downarrow\uparrow}$ , and  $N_{\downarrow\downarrow}$ . For the SSA measurement the polarization  
 4 of one of the beams is ignored by combining the yields with opposite spins, *e.g.*

$$N_{\uparrow} \equiv N_{\uparrow 0} = N_{\uparrow\uparrow} + R_{\frac{0\uparrow}{0\downarrow}} N_{\uparrow\downarrow}, \quad (8)$$

$$N_{\downarrow} \equiv N_{\downarrow 0} = N_{\downarrow\uparrow} + R_{\frac{0\uparrow}{0\downarrow}} N_{\downarrow\downarrow}, \quad (9)$$

5 where re-weighting factor  $R_{\frac{0\uparrow}{0\downarrow}}$  addresses a possible relative difference in the spin-up and spin-down  
 6 intensities of the other beam. Studies have shown that  $R_{\frac{0\uparrow}{0\downarrow}} \approx 1$  with good precision.

7 We bin our data sample in three observable variables  $\{y, \phi, p_T\}$  with center and width of the  
 8  $i$ -th bin being  $\{y_i, \phi_i, p_{T,i}\}$  and  $\{\Delta y_i, \Delta \phi_i, \Delta p_{T,i}\} \equiv \{\Delta \Omega_i \{y_i, \Delta \phi_i\}, \Delta p_{T,i}\} \equiv \Delta_i$  respectively. The  
 9 number of events in each bin,  $N_i$ , is calculated by integrating both sides of (7) within the bin:

$$N_{\uparrow/\downarrow,i} = \int_{\Delta_i} \frac{dN_{\uparrow/\downarrow}}{d\Omega dp_T} d\Omega dp_T. \quad (10)$$

10 In that bin we assume the average value:

$$A_{N,i} = \frac{1}{\Delta_i} \int_{\Delta_i} A_N d\Omega dp_T, \quad (11)$$

11 and similarly for the cross section ( $\sigma_{0,i}$ ) and efficiency ( $\varepsilon_i$ ). Finally, for the yields in each bin we  
 12 can write:

$$N_{\uparrow/\downarrow,i} = \mathcal{L}_{\uparrow/\downarrow} \sigma_{0,i} \varepsilon_i \Delta \Omega_i \Delta p_{T,i} (1 \pm A_{N,i}(\Omega, p_T) P) \quad (12)$$

13 The spacial distributions of the physical asymmetry and the cross sections are the same for  
 14 the spin-up and spin-down interactions with respect to the spin direction. We can use this fact to  
 15 easily get rid of the quantities of no interest in (12). This is achieved by constructing geometric  
 16 means  $\sqrt{N_{\uparrow}(\phi_i) N_{\downarrow}(\phi_i + \pi)}$  and  $\sqrt{N_{\uparrow}(\phi_i + \pi) N_{\downarrow}(\phi_i)}$  of the yields

$$N_{\uparrow}(\phi_i) = \mathcal{L}_{\uparrow} \sigma_0(\phi_i) \varepsilon(\phi_i) \Delta \Omega_i \Delta p_T (1 + A_N(\phi_i) P) \quad (13)$$

$$N_{\uparrow}(\phi_i + \pi) = \mathcal{L}_{\uparrow} \sigma_0(\phi_i + \pi) \varepsilon(\phi_i + \pi) \Delta \Omega_i \Delta p_T (1 + A_N(\phi_i + \pi) P) \quad (14)$$

$$N_{\downarrow}(\phi_i + \pi) = \mathcal{L}_{\downarrow} \sigma_0(\phi_i + \pi) \varepsilon(\phi_i + \pi) \Delta \Omega_i \Delta p_T (1 - A_N(\phi_i + \pi) P) \quad (15)$$

$$N_{\downarrow}(\phi_i) = \mathcal{L}_{\downarrow} \sigma_0(\phi_i) \varepsilon(\phi_i) \Delta \Omega_i \Delta p_T (1 - A_N(\phi_i) P) \quad (16)$$

17 Using the relations for the asymmetry and cross section  $A_N(\phi_i + \pi) = -A_N(\phi_i)$ ,  $\sigma_0(\phi_i + \pi) =$   
 18  $\sigma_0(\phi_i)$  we get for  $A_N$

$$A_{N,i} = \frac{1}{P} \frac{\sqrt{N_{\uparrow}(\phi_i) N_{\downarrow}(\phi_i + \pi)} - \sqrt{N_{\uparrow}(\phi_i + \pi) N_{\downarrow}(\phi_i)}}{\sqrt{N_{\uparrow}(\phi_i) N_{\downarrow}(\phi_i + \pi)} + \sqrt{N_{\uparrow}(\phi_i + \pi) N_{\downarrow}(\phi_i)}} \quad (17)$$

### 3 Correction for Background

In this analysis an optimal set of cuts is applied to select signal enriched events without significant loss in the final statistics. The final yields include some fraction of background events  $f_B$  which along with the signal asymmetry contribute to the measured asymmetry  $A_N$ . In order to extract the signal asymmetry we decompose  $A_N$  as following:

$$A_N = f_{\text{sig}} A_N^{\text{sig}} + f_B A_N^B, \quad (18)$$

with  $f_{\text{sig}} = 1 - f_B$ . The last term in (18) may include contributions from various backgrounds which will be discussed later. The background fractions and asymmetries have to be estimated in order to extract the final asymmetry of the signal:

$$A_N^{\text{sig}} = \frac{A_N + f_B A_N^B}{1 - f_B} \quad (19)$$

### 4 Sivers Sign Change Extraction

A binned likelihood method can be used to check the sensitivity of our data to the sign of the Sivers function. A direct way of doing this is to compare the measured asymmetry (17) with background corrected expectations from (18). The signal asymmetry  $A_N^{\text{sig}}$  in this case directly comes from the model predictions (2) or (3). The simplest likelihood function can be constructed as a product of gaussian terms over all bins:

$$L = \prod_i G(A_{N,i}, \sigma_{A_{N,i}}; A_{N,i}^{\text{sig}}). \quad (20)$$

Alternatively, the Sivers sign can be extracted from the Poisson probabilities of measured given the expected yields.

$$L = \prod_{i,\uparrow,\downarrow} P(N_i; N_i^{\text{sig}} + B_i). \quad (21)$$

While this method is more “classic” it requires the explicit knowledge of luminosity, unpolarized cross section, and efficiencies. These values are needed to calculate the expected number of events using (12). The two methods are expected to give consistent results. However, the difference can be more perceptible when systematic effects are taken into account.

### 5 Reconstruction of $W$ Kinematics

In order to fully reconstruct the  $W$  kinematics the momenta of all  $W$  decay products must be measured. The neutrino produced in the leptonically decayed  $W$  cannot be measured in our detector. The momentum of the neutrino can only be indirectly deduced from conservation of the total momentum. In the transverse plane the initial momentum of the system of interacting partons is negligible and so must be the vector sum of all final particles momenta. We define the missing transverse energy as a vector restoring the balance in the event:

$$\vec{\cancel{E}}_T = - \sum_{\substack{i \in \text{jets,} \\ \text{tracks,} \\ \text{clusters}}} \vec{P}_{i,T}. \quad (22)$$

1 In a typical collider detector like STAR the problem with measuring the longitudinal component  
 2 of the missing energy is that the total particle momentum along the beam direction is not measured  
 3 due to particles with very high rapidities escape the detector. However, in the  $W$  events produced  
 4 at 250 GeV at STAR we can assume that most of the  $\vec{\cancel{E}}_T$  is due to the neutrino from the  $W$   
 5 decay. The assumption  $\vec{p}_{\nu,T} \approx \vec{\cancel{E}}_T$  is based on the fact that only very little energy is left for  
 6 anything other than  $W$  production from the primary hard scattering. At the same time, the beam  
 7 remnants with high longitudinal momentum carry away only a little portion of the total transverse  
 8 momentum. With an assumption that the boson is produced with mass  $M_W$  we can reconstruct  
 9 the  $z$  component of the neutrino's momentum. For the invariant mass we have:

$$M_W^2 = (E_l + E_\nu)^2 - (\vec{p}_l + \vec{p}_\nu)^2, \quad (23)$$

$$M_W^2/2 = |\vec{p}_l| |\vec{p}_\nu| - \vec{p}_{l,T} \cdot \vec{p}_{\nu,T} - \vec{p}_{l,z} \cdot \vec{p}_{\nu,z}, \quad (24)$$

10 where we neglected the masses of the both neutrino and lepton. Introducing a shorthand expression  
 11 for  $A = M_W^2/2 + \vec{p}_{l,T} \cdot \vec{p}_{\nu,T}$ , after trivial arithmetics we arrive to a quadratic equation for  $p_{\nu,z}$ :

$$|\vec{p}_{l,T}|^2 p_{\nu,z}^2 - 2A p_{l,z} p_{\nu,z} + |\vec{p}_{\nu,T}|^2 |\vec{p}_l|^2 - A^2 = 0. \quad (25)$$

12 This equation has two solutions:

$$p_{\nu,z} = \frac{A p_{l,z} \pm \sqrt{A^2 p_{l,z}^2 - |\vec{p}_{l,T}|^2 (|\vec{p}_{\nu,T}|^2 |\vec{p}_l|^2 - A^2)}}{|\vec{p}_{l,T}|^2} \quad (26)$$

## 13 6 Preliminary Sensitivity Studies

14 In 2011 transversely polarized proton-proton beams were brought into collisions at STAR with a  
 15 center of mass energy of 500 GeV. In this regime the  $W$  is expected to have a relatively small  
 16  $P_T \sim 2$  GeV as confirmed by a Monte-Carlo simulation in Figure 1. We use PYTHIA 6.8 to  
 17 simulate  $W^\pm \rightarrow e^\pm \nu_e$  to the LO with unpolarized beams. Expected kinematic distributions of the  
 18 lepton coming from the  $W$  decay is shown in Figure 2.

19 Most of the recoil tracks in the BARREL region are expected to carry a very small fraction of  
 20 the energy as shown in fig. 3.

21 We can use MC to correct for the missing  $P_T$  in the recoil tracks due to the limited acceptance  
 22 of the STAR detector. *Such a procedure will introduce a model-dependent systematic which will*  
 23 *grow with the value of the correction.*

24 We estimate the statistical power of the  $A_N$  measurement for an integrated luminosity of  
 25  $300 \text{ pb}^{-1}$ . As a basis we use the total  $W^\pm$  and  $Z^0$  yields observed at STAR in Run 9. The  $W$   
 26 and  $Z$  candidate events,  $N_{\text{obs}}$ , along with the background numbers,  $N_{\text{bkg}}$ , are borrowed from the  
 27 earlier STAR analysis [1] that reported the production cross section using  $\approx 13 \text{ pb}^{-1}$  of integrated  
 28 luminosity:

$$N_{W^+} = 496 - 37 = 459,$$

$$N_{W^-} = 148 - 26 = 125,$$

$$N_Z = 13 - 0 = 13.$$

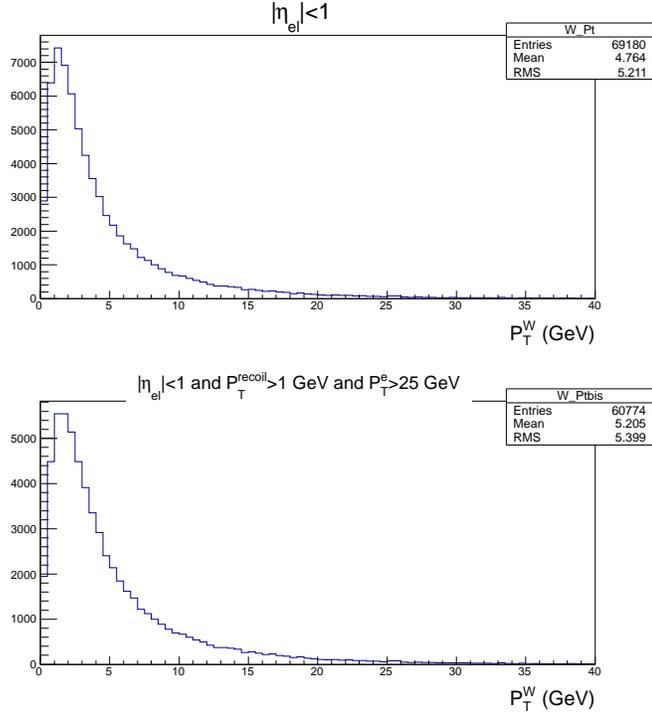


Figure 1: Expected distribution of the transverse momentum of the produced W boson,  $P_T^W$ .

- 1 To reflect the expected increase in the integrated luminosity we scale the above numbers a
- 2 factor  $\approx 23$ . In order to illustrate the sensitivity of the future measurement to the non-vanishing
- 3  $W$  and  $Z$   $A_N$  we calculate the relative yields in bins of the boson rapidity from the MC sample.
- 4 The expected statistical power of  $A_N$  in bins of  $W$  rapidity is shown in Figure 4 for  $W^+$  and  $W^-$
- 5 respectively compared with theoretical prediction from [2].

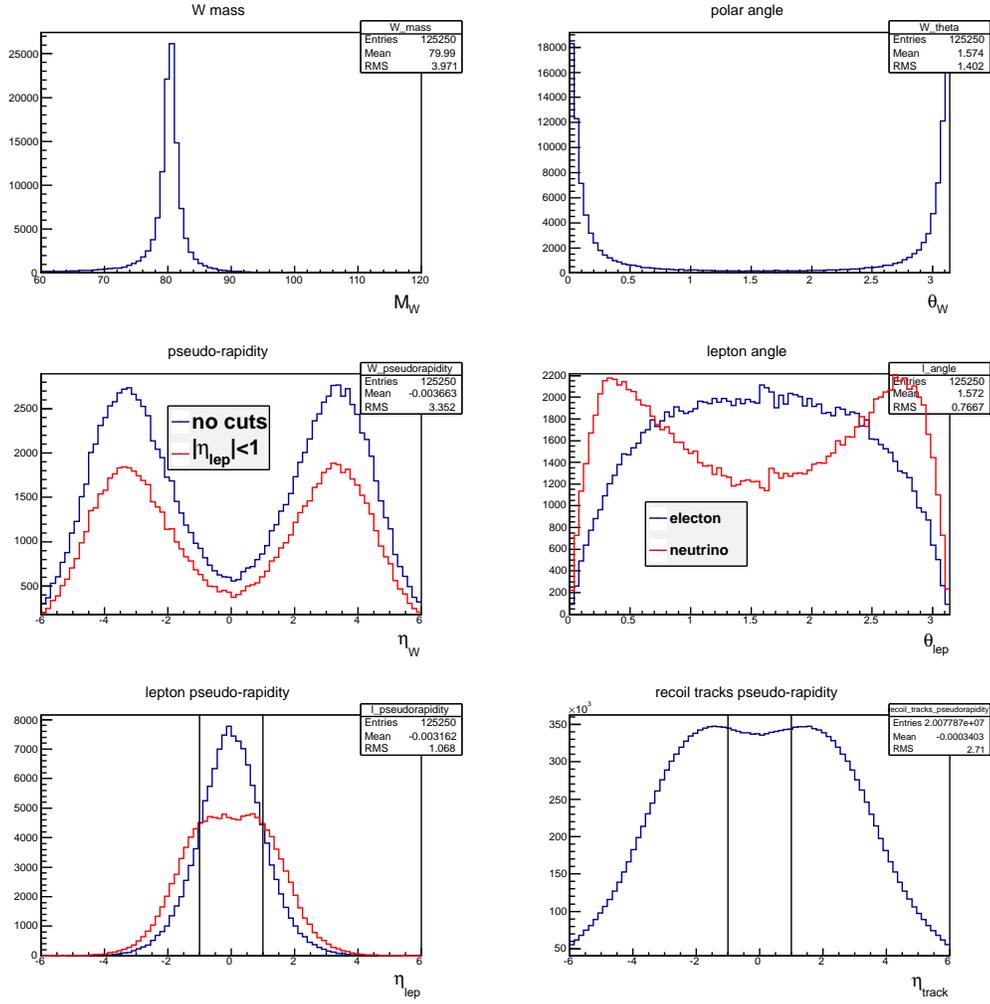


Figure 2: W-mass; polar angles and pseudo-rapidity distributions of the produced W, the decay leptons and the recoil tracks.

## 1 References

- 2 [1] “Measurement of the W and Z Production Cross Sections at Mid-rapidity in Proton-Proton
- 3 Collisions at  $\sqrt{s} = 500$  GeV in Run 9,” STAR Note 0546.
- 4 [2] Z. -B. Kang and J. -W. Qiu, “Testing the Time-Reversal Modified Universality of the Siverson
- 5 Function,” Phys. Rev. Lett. **103**, 172001 (2009) [arXiv:0903.3629 [hep-ph]].

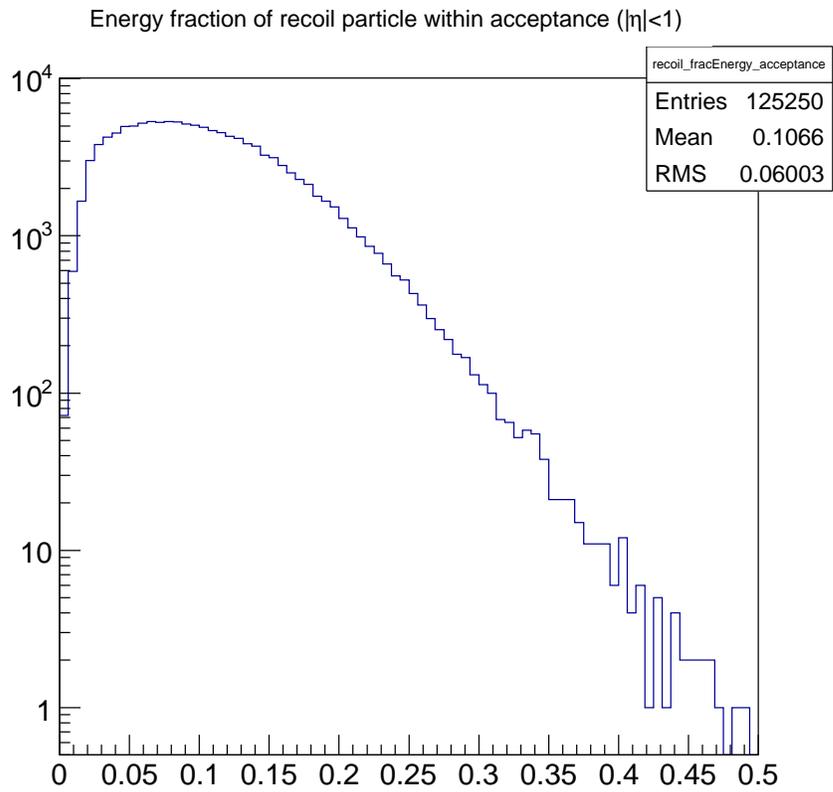


Figure 3: Expected fraction of energy carried by recoil particles within the BARREL region ( $|\eta| < 1$ ).

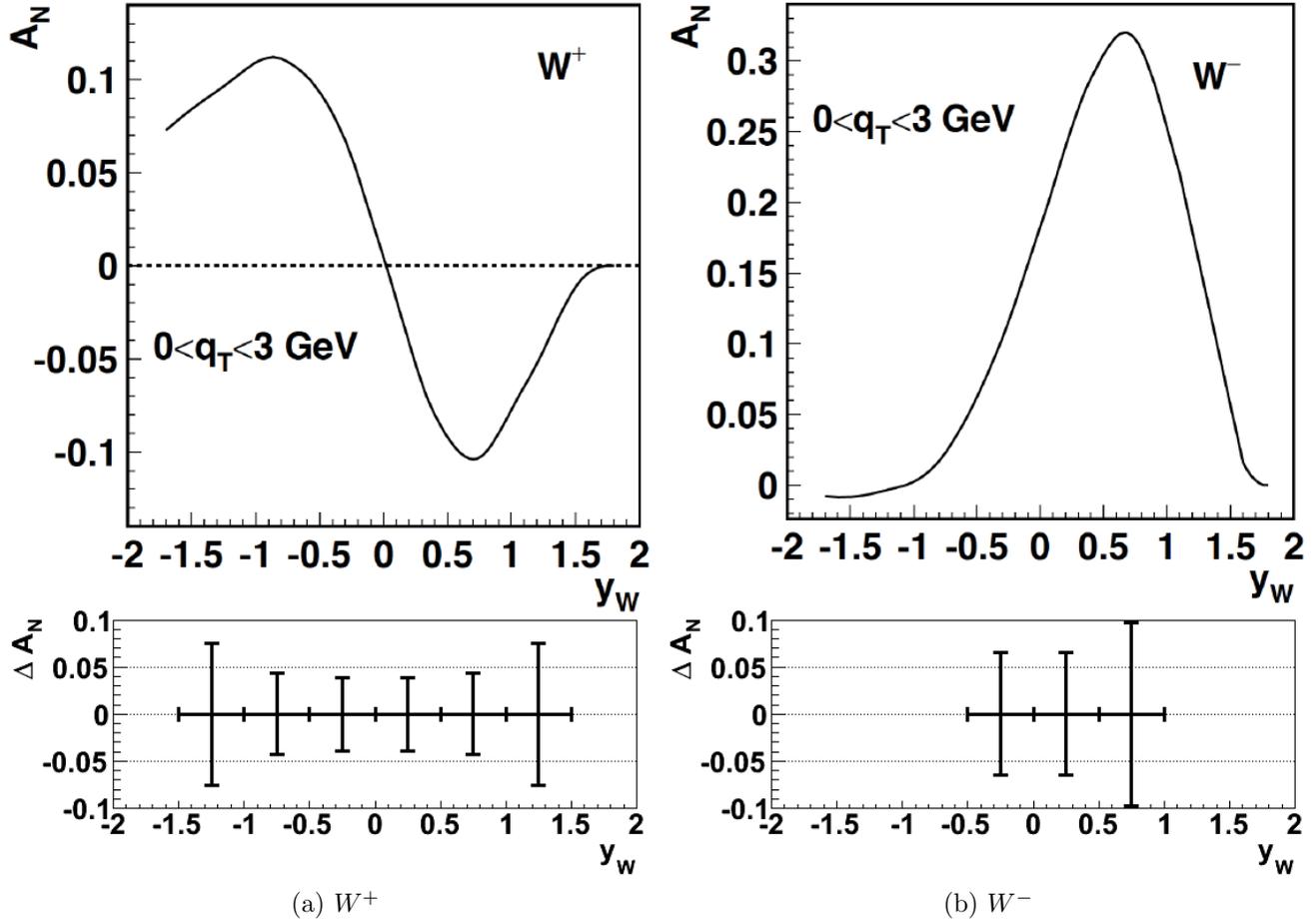


Figure 4: Expected statistical uncertainties for measured asymmetry  $A_N$  of  $W^+$  (a) and  $W^-$  (b) decaying leptonically at STAR as a function of the boson's rapidity.